Physics 122: Electricity & Magnetism – Lecture 13
DC Circuits

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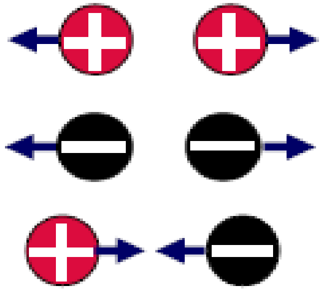
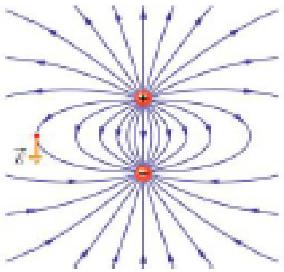
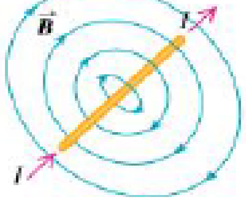
Electric Field & Magnetic Field

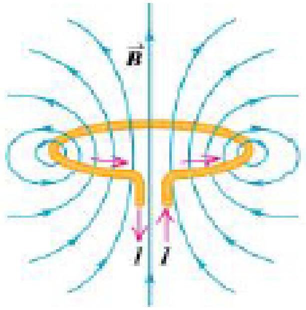
- ❑ Electric forces acting at a distance through electric field.
- ❑ Vector field, **E**.
- ❑ Source: electric charge.
- ❑ Positive charge (+) and negative charge (-).
- ❑ Opposite charges attract, like charges repel.
- ❑ Electric field lines visualizing the direction and magnitude of **E**.

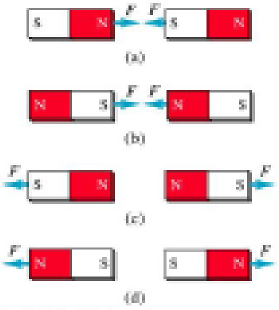
- ❑ Magnetic forces acting at a distance through Magnetic field.
- ❑ Vector field, **B**
- ❑ Source: **moving** electric charge (current or magnetic substance, such as permanent magnet).
- ❑ North pole (N) and south pole (S)
- ❑ Opposite poles attract, like poles repel.
- ❑ Magnetic field lines visualizing the direction and magnitude of **B**.











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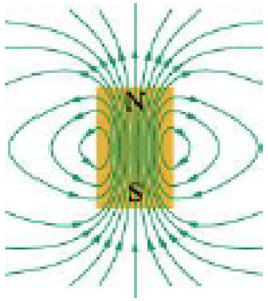
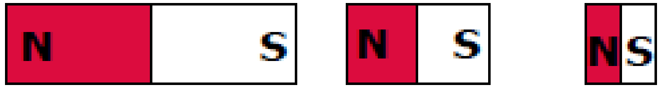


Definition of $\vec{B}$

- Test charge and electric field

$$\vec{E} = \frac{\vec{F}_E}{q}$$

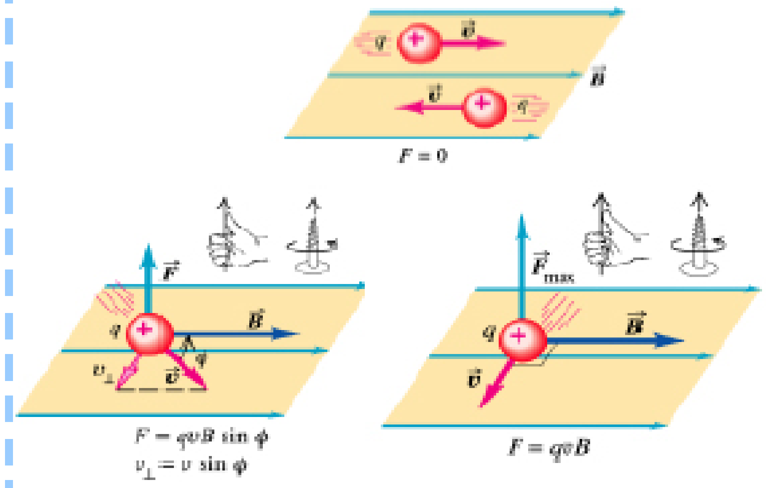


- Test monopole and magnetic field ?

$$\cancel{\vec{B} = \frac{\vec{F}_B}{p}}$$

- Magnetic poles are always found in pairs. A single magnetic pole has never been isolated.

- Define ***B*** at some point in space in terms of the magnetic force ***F_B*** that the field exerts on a ***charged*** particle ***moving*** with a velocity ***v*** :
- The magnitude F_B is proportional to the charge q and to the speed v of the particle.
- $F_B = 0$ when the charged particle moves parallel to the magnetic field vector.
- When velocity vector makes any angle $\theta \neq 0$ with the magnetic field, ***F_B*** is perpendicular to both ***B*** and ***v***.
- F_B on a positive charge is opposite on a negative charge.
- The magnitude F_B is proportional to $\sin\theta$.



Magnetic Fields

- Magnetic force $\boxed{\vec{F}_B = q\,\vec{v}\times\vec{B}}$
- Right-hand rule determine the direction of magnetic force. So the magnetic force is always perpendicular to **v** and **B**.
- The magnitude of the magnetic force is

$$F_B = |q| vB\sin\theta$$

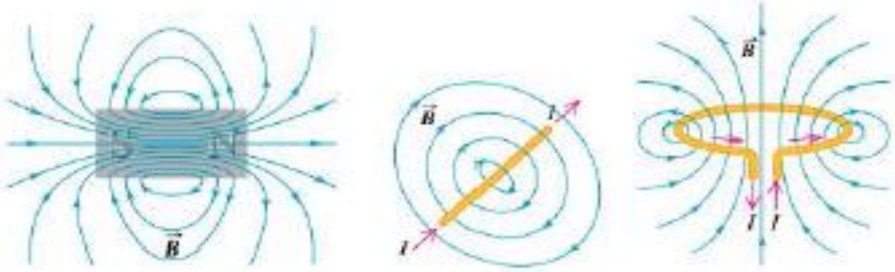
$$\boxed{\vec{F}_E = q\,\vec{E}} \quad \Longleftrightarrow \quad \boxed{\vec{F}_B = q\,\vec{v}\times\vec{B}}$$

- The electric force is along the direction of the electric field, the magnetic force is perpendicular to the magnetic field.
- The electric force acts on a charged particle regardless of whether the particle is moving, the magnetic force acts on a charged particle only when the particle is in motion.
- The electric force does work in displacing a charged particle, the magnetic force does no work when a particle is displaced.

Magnetic Fields

- Magnetic field: $$B = \frac{F_B}{|q|v}$$
- SI unit of magnetic field: tesla (T)
  - 1T = 1 N/[Cm/s] = 1 N/[Am] = 10^4 gauss
- Magnetic field lines with similar rules:
  - The direction of the tangent to a magnetic field line at any point gives the direction of **B** at that point;
  - The spacing of the lines represents the magnitude of **B** – the magnetic field is stronger where the lines are closer together, and conversely.

At surface of neutron star	10^8 T
Near big electromagnet	1.5 T
Inside sunspot	10^{-1} T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T



CONVENTION
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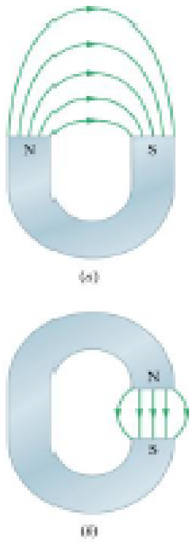
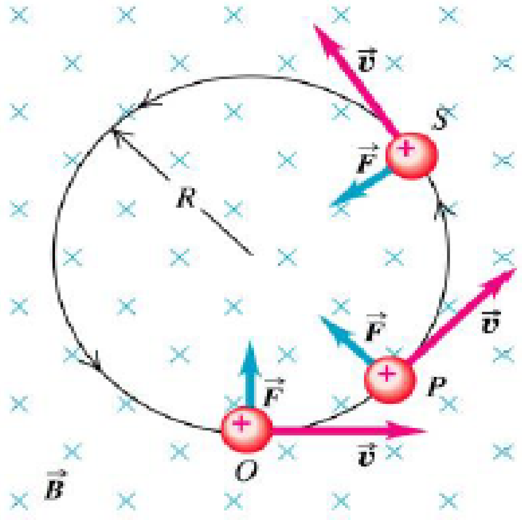


Motion of a Charged Particle in a Uniform Magnetic Field

- F_B never has a component parallel to **v** and can't change the particle's kinetic energy. The force can change only the direction of **v**.
- Charged particle moves in a circle in a plane perpendicular to the magnetic field.
- Start with $\sum F = F_B = ma$
- Then, we have $$F_B = qvB = \frac{mv^2}{r}$$
- The radius of the circular path: $r = \frac{mv}{qB}$
- The angular speed: $$\omega = \frac{v}{r} = \frac{qB}{m}$$
- The period of the motion: $$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$



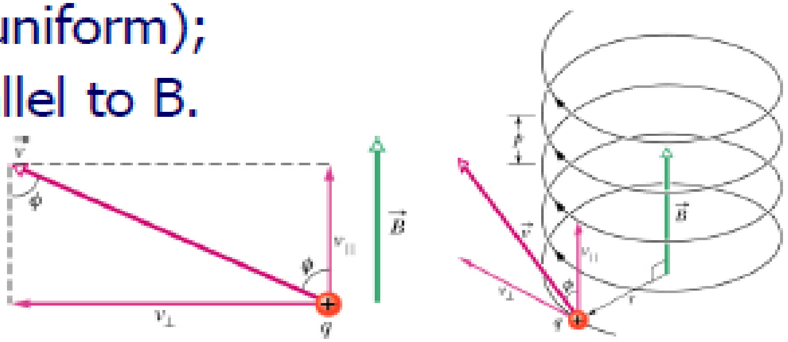
- T and ω do not depend on v of the particle. Fast particles move in large circles and slow ones in small circles, but all particles with the same charge-to-mass ratio take the same time T to complete one round trip.
- The direction of rotation for a positive particle is always counterclockwise, and the direction for a negative particle is always clockwise.

Motion of a Charged Particle in Magnetic Field

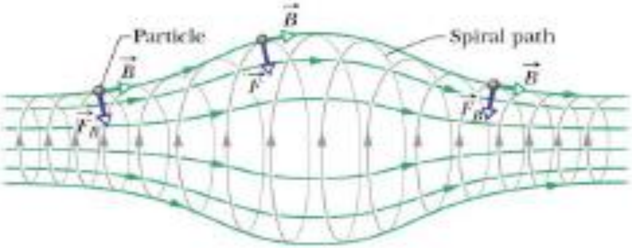
- ☐ Circle Paths: **v** is perpendicular to **B** (uniform);
- ☐ Helical Paths: v has a component parallel to B.

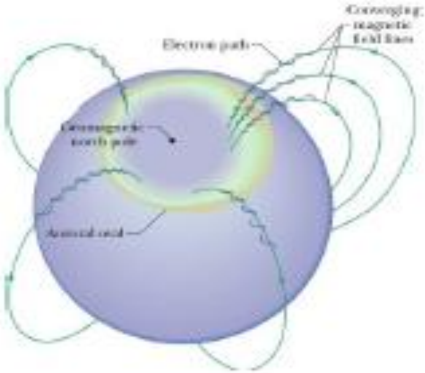
$$v_{\parallel} = v \cos \phi$$

$$v_{\perp} = v \sin \phi$$



- ☐ Motion in a nonuniform magnetic field: strong at the ends and weak in the middle;
  - Magnetic bottle
  - Aurora





Motion of a Charged Particle in a Uniform Electric Field and Magnetic Field

- ☐ Charged particle in both electric field and magnetic field

$$\boxed{\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}}$$

- ☐ Velocity Selector:

$$qE = qvB \qquad v = \frac{E}{B}$$

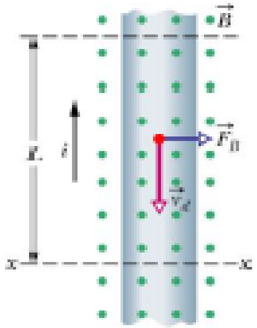
- ☐ The Mass Spectrometer:

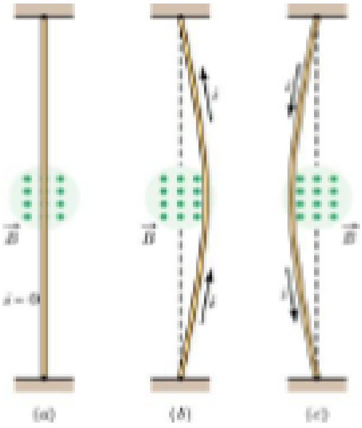
$$r = \frac{mv}{qB} \qquad \frac{m}{q} = \frac{rB_0}{v} \qquad \frac{m}{q} = \frac{rB_0B}{E}$$

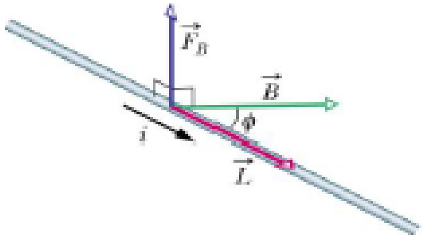
- ☐ The Cyclotron:

$$T = \frac{2\pi m}{|q|B} \qquad f = f_{osc} = \frac{1}{T} \qquad |q|B = 2\pi m f_{osc}$$

Magnetic Force on a Current-Carrying Wire







- Free electrons (negative charges) move with drift velocity v_d opposite to the current.
- Electrons in this section feel Lorentz force:

$$\vec{F}_B = (q\vec{v}_d \times \vec{B})nAL$$

- We have $i = nqv_d A$
- So,

$$\boxed{\vec{F}_B = i\vec{L} \times \vec{B}}$$

- Wire is pushed/pulled by the charges. **L** is a length vector that points in the direction of *i* and has a magnitude equal to the length.

- Arbitrarily shaped wire segment of uniform cross section in a magnetic field.

$$d\vec{F}_B = I\,d\vec{s} \times \vec{B}$$

$$\boxed{\vec{F}_B = I\int_a^b d\vec{s} \times \vec{B}}$$

Torque on a Current Loop

- Loop rotates. Calculate force for each side of the loop:

$$F_1 = F_3 = ibB\sin(90^\circ - \theta) = ibB\sin\theta$$

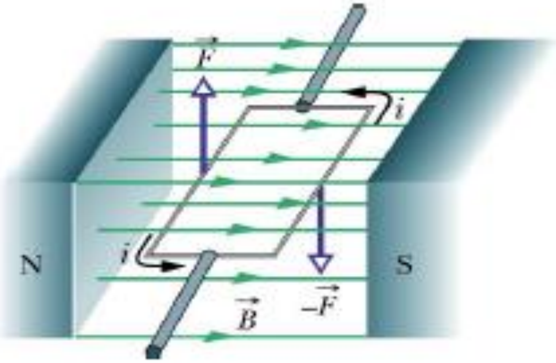
$$F_2 = F_4 = iaB$$

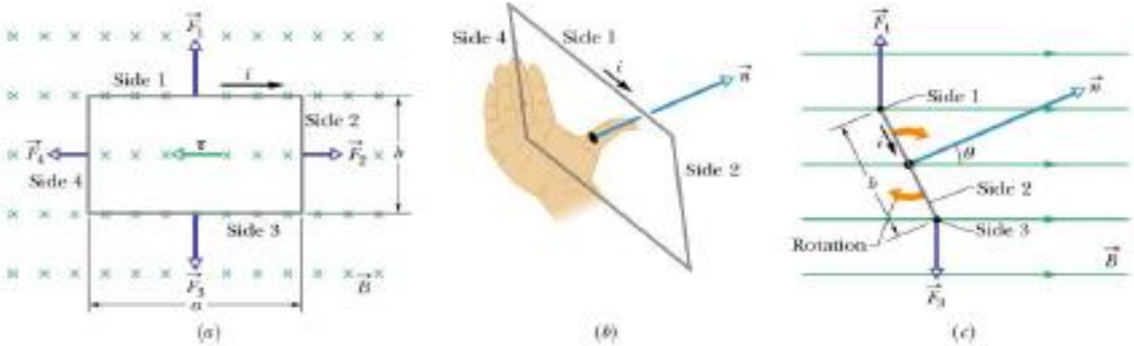
- Torque: $\tau = F_2 \frac{b}{2}\sin\theta + F_4 \frac{b}{2}\sin\theta$

$$= iaB(\frac{b}{2}\sin\theta) + iaB(\frac{b}{2}\sin\theta)$$

$$= iabB\sin\theta = iAB\sin\theta$$

$$\boxed{\vec{\tau} = i\,\vec{A} \times \vec{B}}$$





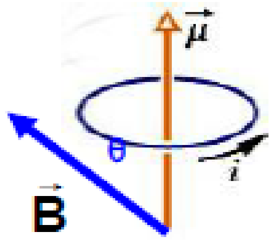
- Maximum torque $\tau_{\max} = iAB$
- Sinusoidal variation $\tau(\theta) = \tau_{\max}\sin\theta$
- Stable when n parallels B.
- Restoring torque: oscillations.

The Magnetic Dipole Moment

- Magnetic dipole moment $\boxed{\vec{\mu} = i\vec{A}}$
- SI unit: Am², Nm/T = J/T
- A coil of wire has N loops of the same area:

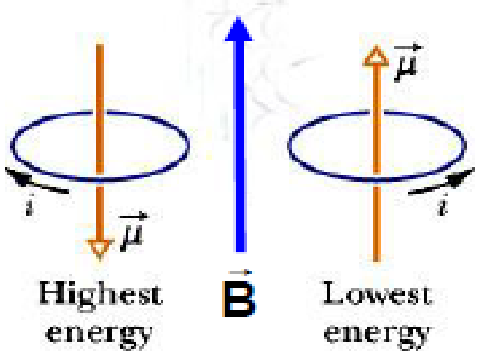
$$\vec{\mu}_{coil} = Ni\vec{A}$$

- Torque $\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$
- Magnetic potential $\boxed{U = -\vec{\mu} \cdot \vec{B}}$
- Electric dipole and magnetic dipole



Small bar magnet	5 J/T
Earth	8.0×10^{22} J/T
Proton	1.4×10^{-26} J/T
Electron	9.3×10^{-24} J/T

	Electric Dipole	Magnetic Dipole
Moment	$p = qd$	$\mu = NiA$
Torque	$\vec{\tau} = \vec{p} \times \vec{E}$	$\vec{\tau} = \vec{\mu} \times \vec{B}$
Potential Energy	$U = -\vec{p} \cdot \vec{E}$	$U = -\vec{\mu} \cdot \vec{B}$



Summary

- A magnetic field **B** is defined in terms of the force **F$_B$** acting on a test particle with charge q moving through the field with velocity **v**,

$$\boxed{\vec{F}_B = q\vec{v}\times\vec{B}}$$

- The SI unit for **B** is the tesla (T): 1T = 1 N/(Am).
- A charged particle with mass m and charge magnitude q moving with velocity v perpendicular to a uniform magnetic field B will travel in a circle. Applying Newton's second law to the circular motion yields
- From which we find the radius r of the circle to be

$$F_B = qvB = \frac{mv^2}{r} \qquad r = \frac{mv}{qB}$$

- The frequency of revolution f, the angular frequency, and the period of the motion T are given by

$$\omega = \frac{v}{r} = \frac{qB}{m} \qquad T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

- A straight wire carrying a current I in a uniform magnetic field experiences a sideways force

$$\boxed{\vec{F}_B = i\vec{L}\times\vec{B}}$$

- The force acting on a current element i**dL** in a magnetic field is
- The direction of the length vector L or dL is that of the current i.

$$d\vec{F}_B = I\,d\vec{s}\times\vec{B}$$

- A coil in a uniform magnetic field **B** will experience a torque given by $\vec{\tau} = i\vec{A}\times\vec{B}$
- Here is the magnetic dipole moment of the coil, with magnitude $\mu = NiA$ and direction given by the right-hand rule.
- The magnetic potential energy of a magnetic dipole in a magnetic field is $U = -\vec{\mu}\cdot\vec{B}$