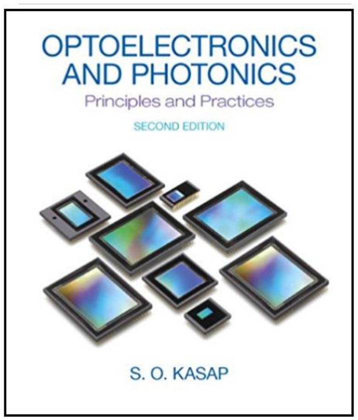
Optoelectronics-I

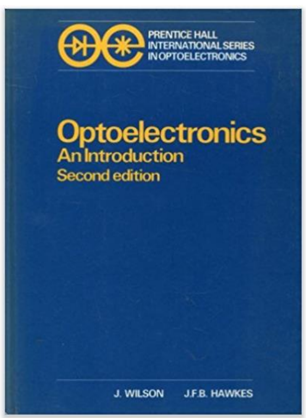
Chapter-6

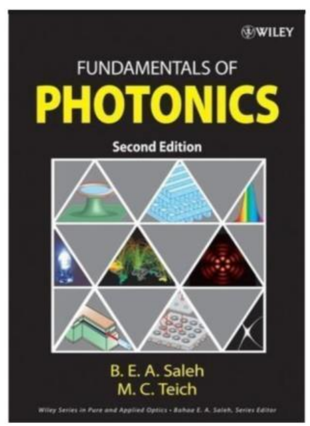
Assoc. Prof. Dr. Isa NAVRUZ

Lecture Notes - 2018

Recommended books







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Polarization Optics

Objectives

When you finish this lesson you will be able to:

- ✓ Describe the Polarization
- ✓ Define polarization types
- ✓ Explain the linear, circular and elliptical polarizations
- ✓ Define the mathematical description of polarizations
- ✓ Explain the matrix representation with the Jones Vector
- ✓ Define the orthogonal polarizations

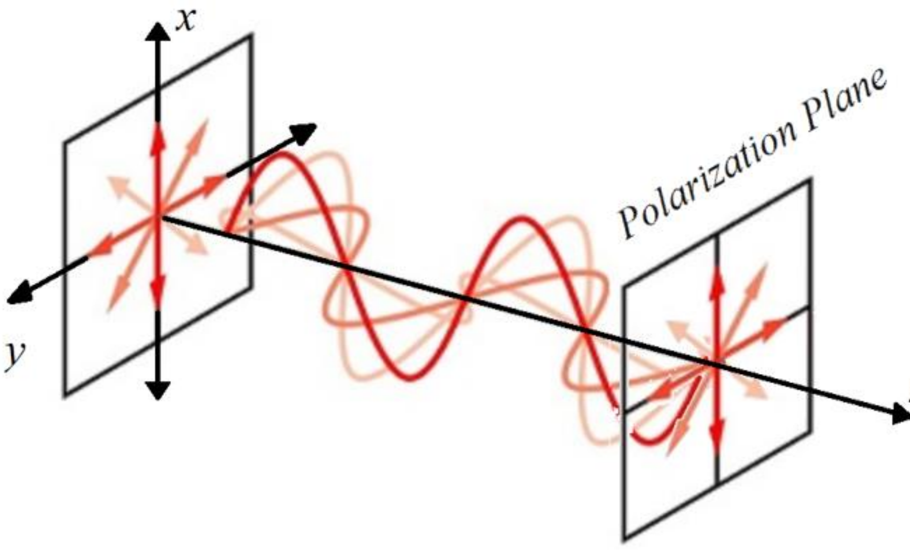
Polarization

The meaning of the Polarization

Polarization is a fundamental property of light . The polarization of the wave is the description of the behaviour of the vector E in the plane x,y, perpendicular to the direction of propagation z.

The plane of polarization is defined as the plane containing the propagation vector, i.e. the z axis, and the electric field vector.

Consider a plane wave traveling z direction. The electric field lies in the x-y plane. If the direction of E changes randomly with time, the wave is said to be randomly polarized, or unpolarized.



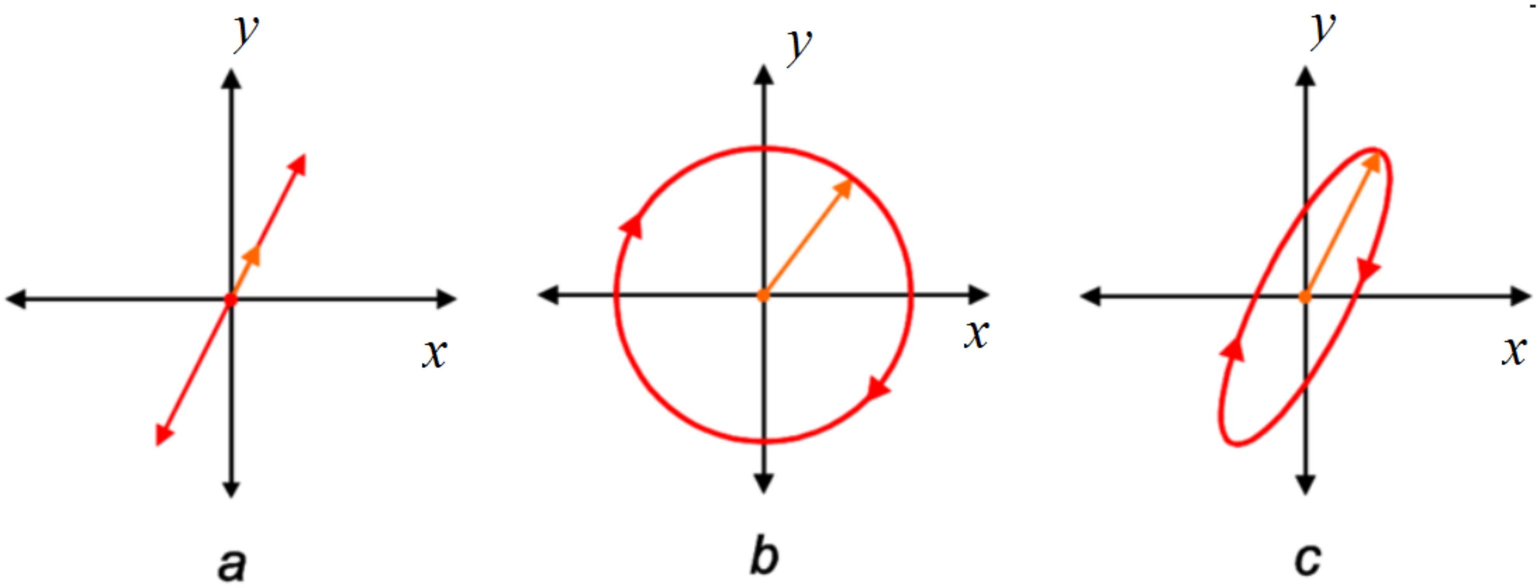
Polarization

The meaning of the Polarization

Polarization is controlled by the electric field direction in the x-y plane of the light traveling in z direction.

There are three different types of polarization.

1-Linear Polarization ***2-Circular Polarization*** ***3-Elliptical Polarization***



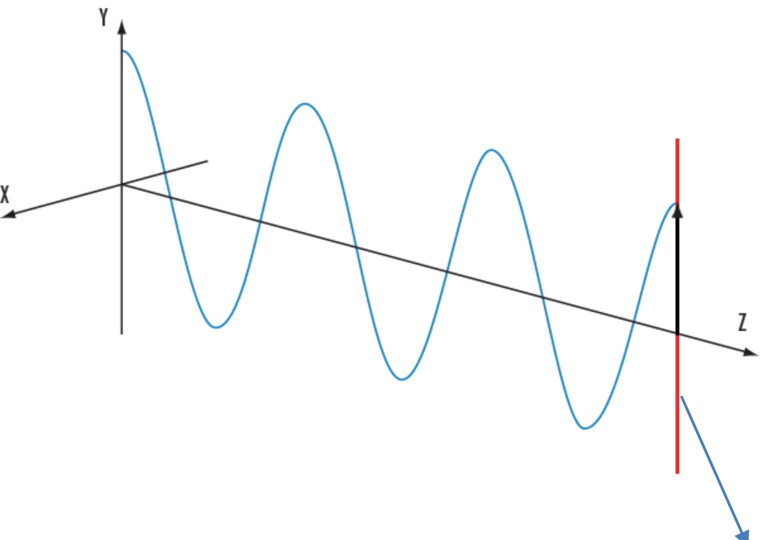
Note that it is not mandatory for the wave to travel only in z direction. The wave (Electric field) can propagate in any ***r*** direction. However, we chose the light traveling in +z direction for to be easily understood.

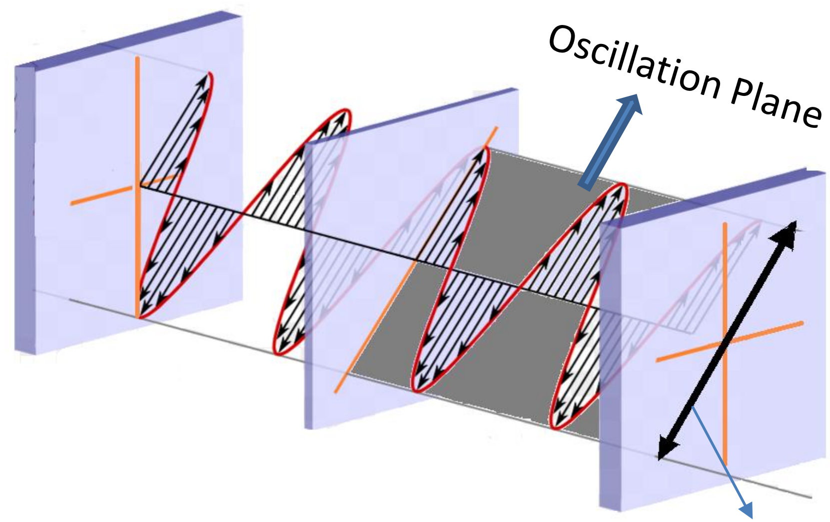
Polarization

Linear Polarization

The electric field of light is confined to a single plane along the direction of propagation. All the electric field vectors oscillate in the same plane. They parallel to a fixed direction referred to as the polarization direction.

Direction of the electric field vector=direction of polarization

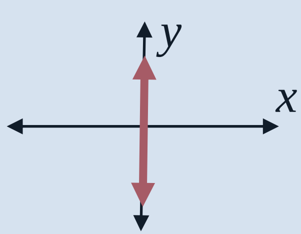




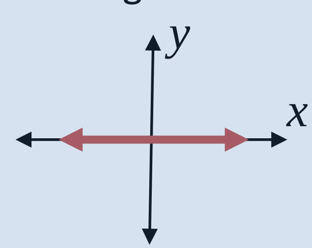
polarization direction

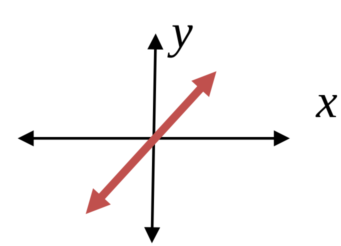
polarization direction

Linear polarized light in *y* direction



Linear polarized light in *x* direction





Polarization

Linear Polarization

Exercise:

Suppose that an electric field propagating in the positive z direction and linearly polarized in the x direction. Calculate the magnetic filed vector $\vec{H}$ and pointing vector $\vec{S}$.

$$\vec{E} = E_0 \sin(kz - \omega t)\hat{x} \quad \text{or} \quad \vec{E} = E_0 \sin(kz - \omega t)\hat{i}$$

$$\vec{\nabla} \times \vec{\boldsymbol{E}} = \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \hat{\boldsymbol{i}}(0-0) - \hat{\boldsymbol{j}}(0 - \frac{\partial}{\partial z}E_x) + \hat{\boldsymbol{k}}(0 - \frac{\partial}{\partial y}E_x)$$

$$\vec{\nabla} \times \vec{\boldsymbol{E}} = \hat{\boldsymbol{j}}(\frac{\partial E_x}{\partial z}) = \hat{\boldsymbol{j}}\left[kE_{ox}\cos(kz - \omega t + \phi)\right]$$

$$\vec{\boldsymbol{H}} = -\frac{1}{\mu_o}\int\left[kE_o\cos(kz - \omega t + \phi)\,\hat{j}\right]dt = \frac{k}{\omega}(\frac{E_o}{\mu_o})\sin(kz - \omega t + \phi)\,\hat{j}$$

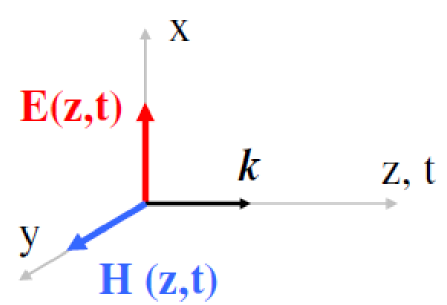
Polarization

Linear Polarization

Remember that $c = \frac{\omega}{k} = \frac{1}{(\varepsilon_o \mu_o)^{1/2}}$

$$\vec{H} = \left(\frac{\varepsilon_o}{\mu_o}\right)^{1/2} E_o \sin(kz - \omega t + \phi)\,\hat{j} = H_o \sin(kz - \omega t + \phi)\,\hat{j}$$

$$H_o \equiv (\varepsilon_o / \mu_o)^{1/2} E_o$$



$$\frac{|\vec{E}_o|}{|\vec{H}_o|} = \left(\frac{\mu_o}{\varepsilon_o}\right)^{1/2} \equiv \eta_o$$

Free space impedance → $\eta_o = \left(\frac{\mu_o}{\varepsilon_o}\right)^{1/2} \cong 120\pi \cong 377\Omega$

$$\vec{S} = \vec{E} \times \vec{H} \Rightarrow \vec{S} = \left|\vec{E}_o \times \vec{H}_o\right| \hat{k} \qquad \left|\vec{S}\right| = \frac{\left|\vec{E}\right|^2}{\eta_o} = \sqrt{\frac{\varepsilon_o}{\mu_o}} \left|\vec{E}\right|^2 = c\varepsilon_o \left|\vec{E}\right|^2$$

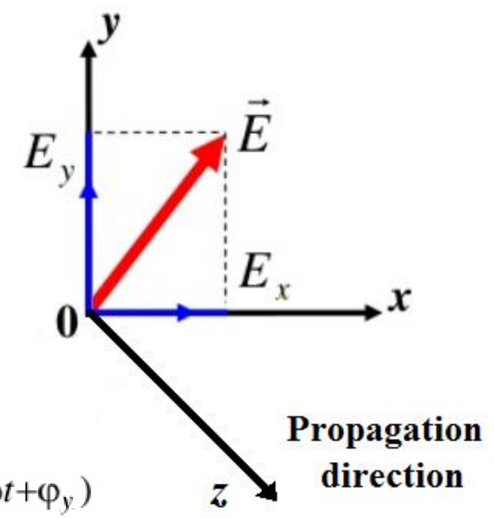
→ $$\vec{S} = \varepsilon_0 c\, E_0^2 \sin^2(kz - \omega t)\,\hat{z}$$

Polarization

The Mathematical description of Polarization

Assume that an EM wave propagating along z direction shown in the figure.
The electric field of this wave at the origin of the axis system can be defined as,

$$\vec{E} = E_x \hat{x} + E_y \hat{y}$$



The complex amplitude vectors,

$$E_x = E_{0x} e^{i(kz-\omega t+\varphi_x)} \quad \text{and} \quad E_y = E_{0y} e^{i(kz-\omega t+\varphi_y)}$$

The real amplitude vector of electric field

$$\vec{E} = E_{ox} \cos(kz - \omega t + \varphi_x)\, \hat{x} + E_{oy} \cos(kz - \omega t + \varphi_y)\, \hat{y}$$

$$\vec{E} = E_{ox} \cos(kz - \omega t) \hat{x} + E_{oy} \cos(kz - \omega t \pm \delta) \hat{y}$$

$$\delta \equiv \varphi_y - \varphi_x$$

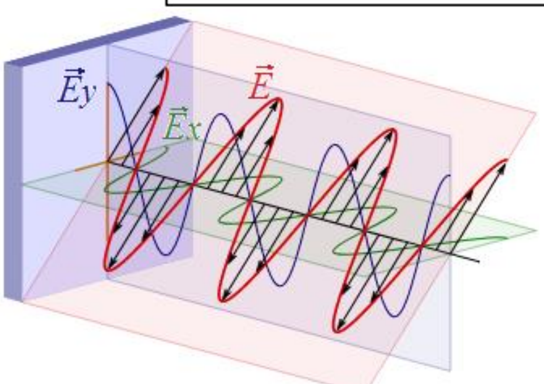
Polarization

The Mathematical description of Polarization

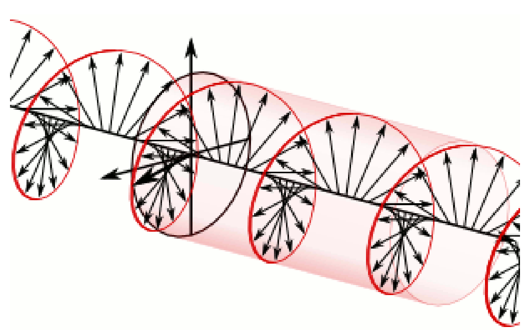
$$\vec{E} = E_{ox}\cos(kz-\omega t)\hat{x} + E_{oy}\cos(kz-\omega t \pm \delta)\hat{y} \qquad \delta \equiv \varphi_y - \varphi_x$$

When amplitudes and phase differences are compared,

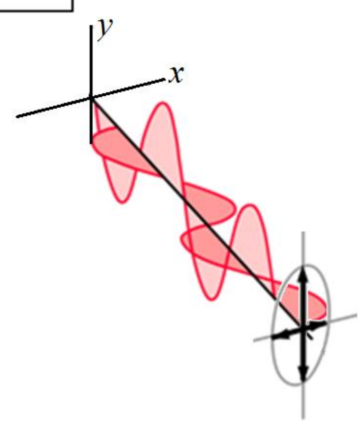
Amplitude	Phase Difference	
doesn't matter	$\delta = 0$ *or* π	***Linear Polarization***
$E_{ox} = E_{oy} = E_o$	and $\delta = \mp\pi/2$	***Circular Polarization***
$E_{ox} \neq E_{oy}$	$\delta = \mp\pi/2$	***Elliptical Polarization***



Linear Polarization



Circular Polarization



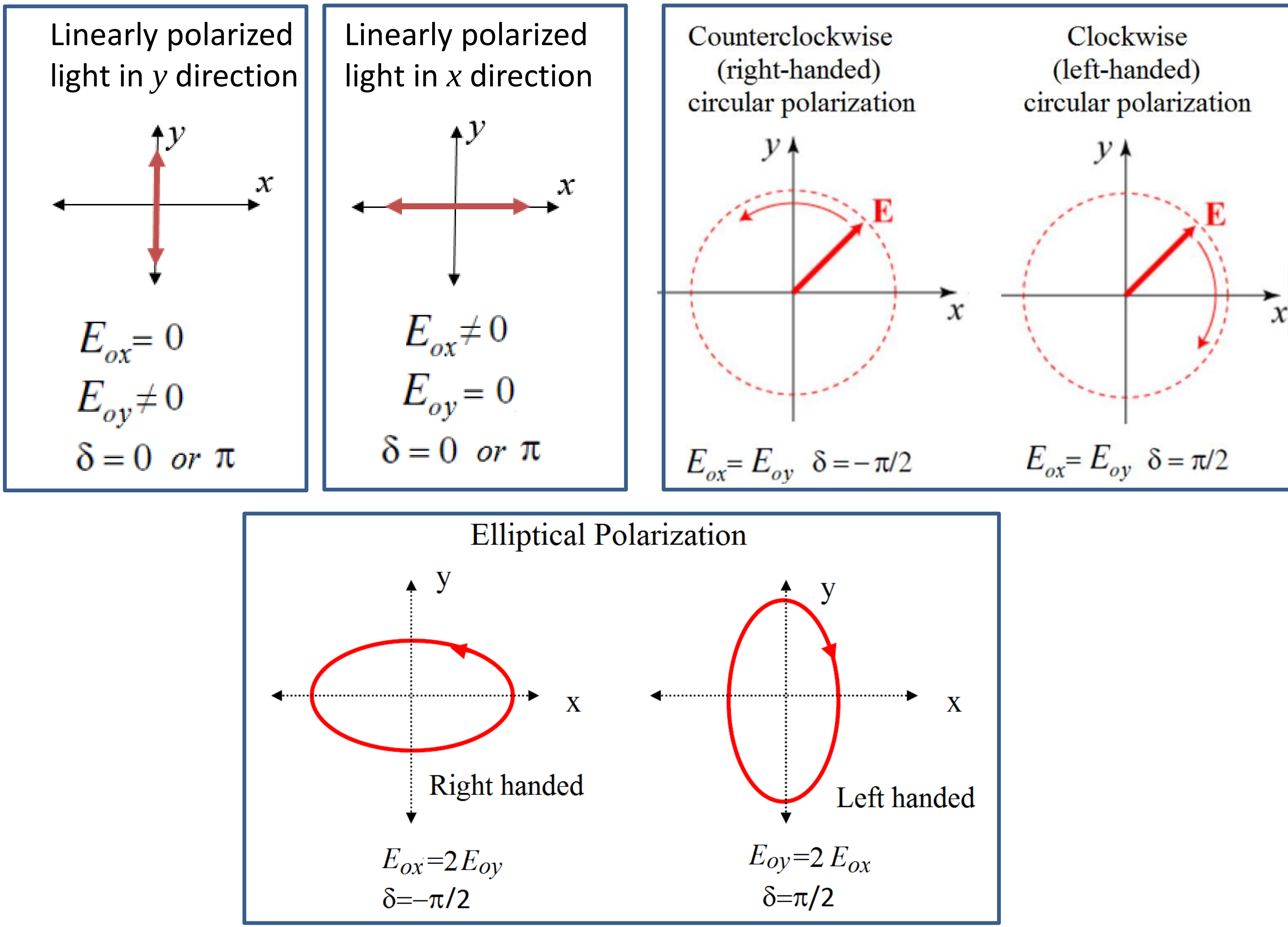
Elliptical Polarization

Polarization

Exercise:

The frequency of a plane electromagnetic wave is 600x10^{12} Hz. It is propagating along positive x direction in vacuum and has an electric field amplitude of 43.42 V/m The wave is linearly polarized such that the oscillation plane is at 45° to the x-z plane. Obtain the vector $\vec{E}$ and $\vec{H}$.

Polarization

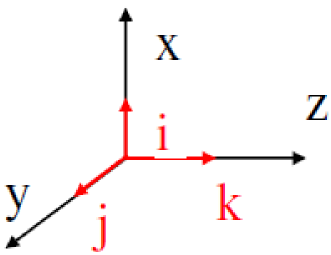


Polarization

Matrix Representation with the Jones Vector

A monochromatic plane wave traveling in the z direction is completely characterized by the complex envelopes given below,

$$\vec{E} = \hat{i}E_{ox} + \hat{j}E_{oy}$$



$$E_{ox} = |E_{ox}|e^{i\varphi_x} \qquad E_{oy} = |E_{oy}|e^{i\varphi_y}$$

This monochromatic plane wave can be expressed in matrix form.

$$\vec{E} = \begin{bmatrix} E_{ox} \\ E_{oy} \end{bmatrix} = \begin{bmatrix} |E_{ox}|e^{i\varphi_x} \\ |E_{oy}|e^{i\varphi_y} \end{bmatrix}$$

The form of column matrix known as Jones vector. we can determine the total intensity of the wave,

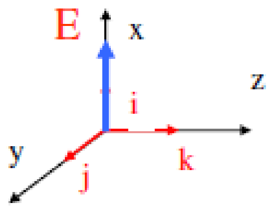
$$I = \left(|E_{ox}|^2 + |E_{oy}|^2\right)/2\eta$$

Where, η is the impedance of the medium.

Polarization

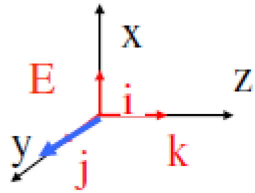
Matrix Representation with the Jones Vector

Linearly polarized light in x direction



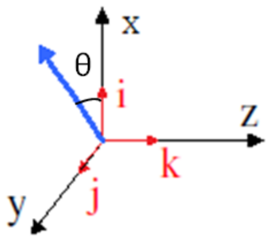
$$\vec{E} = \begin{bmatrix} E_o \\ 0 \end{bmatrix} = E_o \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Linearly polarized light in y direction



$$\vec{E} = \begin{bmatrix} 0 \\ E_o \end{bmatrix} = E_o \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Linearly polarized wave, plane of polarization making angle θ with x axis



$$\vec{E} = E_o \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

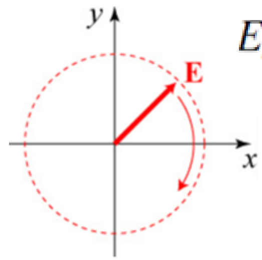
For example, the Jones vector for linearly polarized wave with θ=45°

$$\vec{E} = \begin{bmatrix} E_o \\ E_o \end{bmatrix} = \frac{E_o}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Polarization

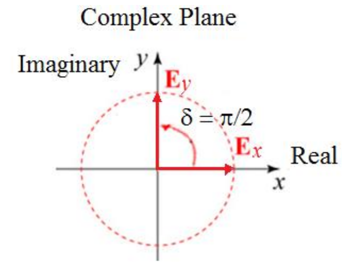
Matrix Representation with the Jones Vector

Clockwise (left-handed) circular polarization

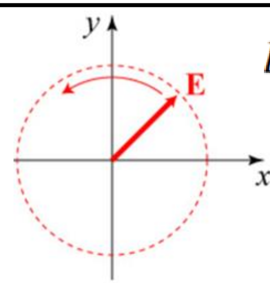


$E_{ox} = E_{oy}$ $\delta = \pi/2$

$$\vec{E} = \frac{E_o}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$$



Counterclockwise (right-handed) circular polarization

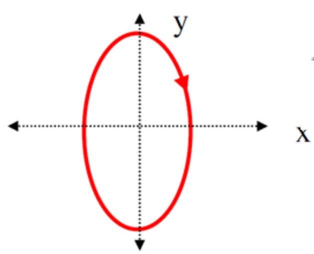


$E_{ox} = E_{oy}$ $\delta = -\pi/2$

$$\vec{E} = \frac{E_o}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

Elliptical Polarization

Left handed



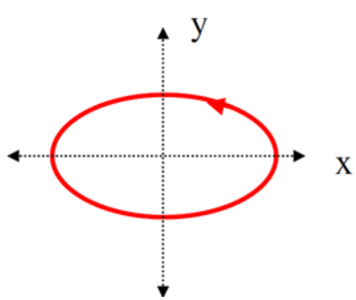
$E_{oy} = 2\,E_{ox} = 2\,E_o$

$\delta = \pi/2$

$$\vec{E} = \frac{E_o}{\sqrt{5}} \begin{bmatrix} 1 \\ 2i \end{bmatrix}$$

Elliptical Polarization

Right handed



$E_{ox} = 2E_{oy} = 2E_o$

$\delta = -\pi/2$

$$\vec{E} = \frac{E_o}{\sqrt{5}} \begin{bmatrix} 2 \\ -i \end{bmatrix}$$

Polarization

Orthogonal Polarizations

Suppose that two polarization states represented by the Jones vectors J_1 and J_2.
If the inner product between J_1 and J_2 is zero, this state is called as *orthogonal*.

The inner product is defined as,

$$(\mathbf{J}_1, \mathbf{J}_2) = A_{1x}A_{2x}^* + A_{1y}A_{2y}^*$$

$$\mathbf{J} = \begin{bmatrix} A_x \\ A_y \end{bmatrix}$$

Here, the symbol (*) represents the complex conjugate.

Polarization

Exercise:

Show that the linearly polarized wave with plane of polarization making an angle θ with the *x* axis is equivalent to a superposition of right and left circularly polarized waves with weights $(1/\sqrt{2})e^{-j\theta}$ and $(1/\sqrt{2})e^{j\theta}$, respectively.

Solution:

Right circularly polarized waves have a Jones vector given by $J_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$

And left circularly polarized waves have $J_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$

The weight α_1 attached to J_1 $\alpha_1 = \frac{1}{\sqrt{2}}\left(e^{-i\theta}\right)$

The weight α_2 attached to J_2 $\alpha_2 = \frac{1}{\sqrt{2}}\left(e^{i\theta}\right)$

$\Rightarrow$ $J = \alpha_1 J_1 + \alpha_2 J_2$

$$J = \frac{1}{2}\begin{pmatrix} e^{-i\theta} \\ ie^{ie^{-i\theta}} \end{pmatrix} + \frac{1}{2}\begin{pmatrix} e^{i\theta} \\ -ie^{i\theta} \end{pmatrix} \Rightarrow J = \frac{1}{2}\begin{pmatrix} e^{-i\theta} + e^{i\theta} \\ ie^{-i\theta} - ie^{i\theta} \end{pmatrix}$$

$$J = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

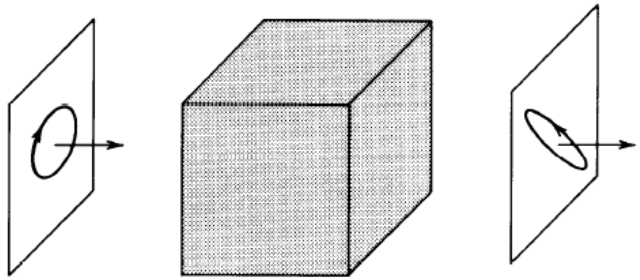
Remember that

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos\theta = \frac{e^{-i\theta} + e^{i\theta}}{2}$$

Polarization

The Jones Matrix Representation of Polarization Devices



Optical system

Consider the transmission of a plane wave of arbitrary polarization through an optical system. The optical system can alter the polarization of a plane wave as shown in the figure.

The relationship between the polarization states at the input and output of the system can be defined by a matrix as follows.

$$\begin{bmatrix} A_{2x} \\ A_{2y} \end{bmatrix} = \underbrace{\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}}_{\boldsymbol{T}} \begin{bmatrix} A_{1x} \\ A_{1y} \end{bmatrix}$$

T is the Jones Matrix of the optical system

If the input and output waves are described by the Jones vectors J_1 and J_2, we can write the equation as follows

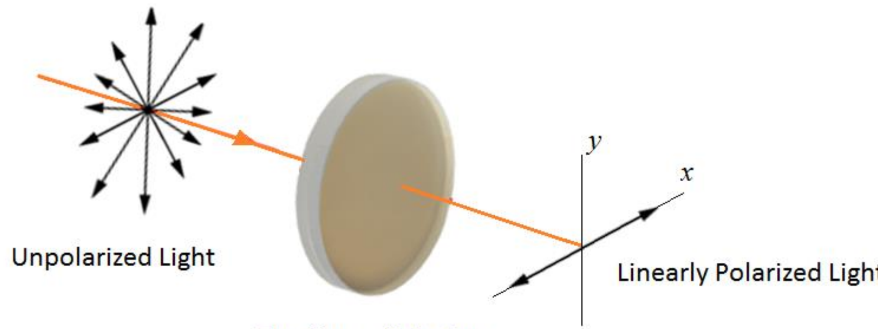
$$\boldsymbol{J_2 = T\, J_1}$$

Polarization

The Jones Matrix Representation of Polarization Devices

Linear Polarizer along x Direction

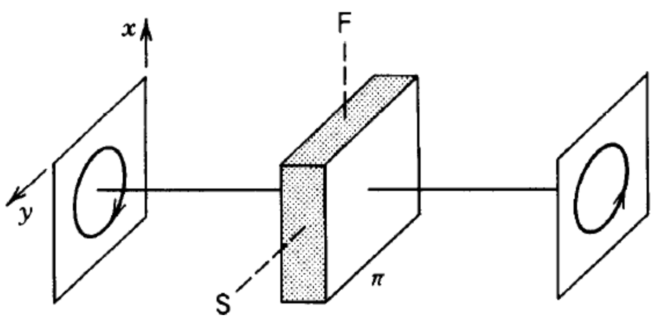
$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$



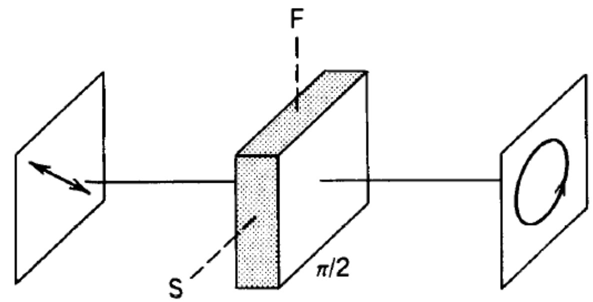
The Linear Polarizer

Wave-Retarder (Fast Axis along x Direction)

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & \exp(-j\Gamma) \end{bmatrix}$$



π wave retarder (half wave)

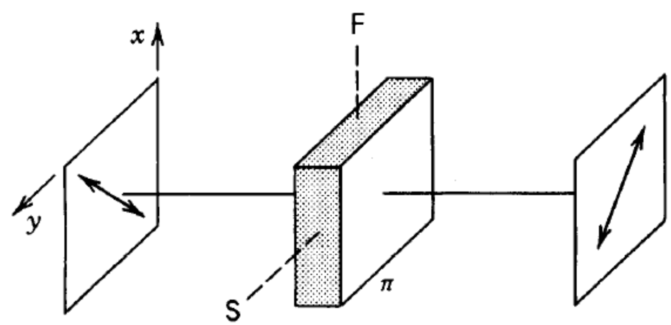


π/2 wave retarder (quarter wave)

Rotation the plane of polarization
of a linearly polarized wave by an angle θ

$$\mathbf{T} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

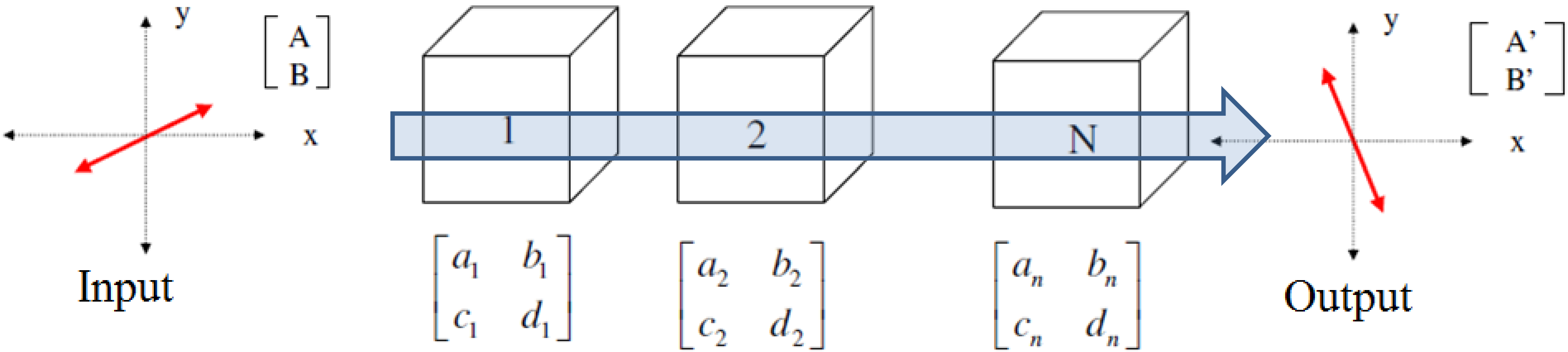
Polarization Rotator



π wave retarder (half wave)

Polarization

The Jones Matrix Representation of Cascaded Systems



$$J_{output} \equiv \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} = \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix} \cdots\cdots \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

Polarization

The Jones Matrix Representation of Polarization Devices

Linear Polarizer along x Direction $\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	Linear Polarizer along y Direction $\mathbf{T} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
Quarter-wave retarder Axis x is the fast axis of the retarder. $\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	Quarter-wave retarder Axis y is the slow axis of the retarder. $\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$
Circular Polarizer Left handed $\mathbf{T} = \frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$	Circular Polarizer Right handed $\mathbf{T} = \frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$

Exercise:

Show that the Jones matrix of a polarizer making an angle θ=45º with the x axis is equivalent to

$$\mathbf{T} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$