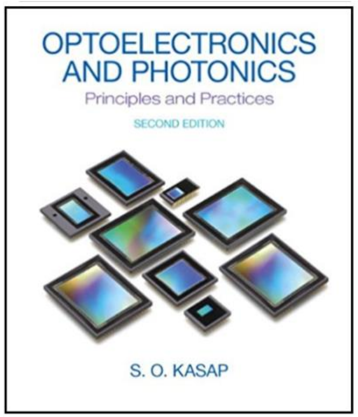
Optoelectronics-I

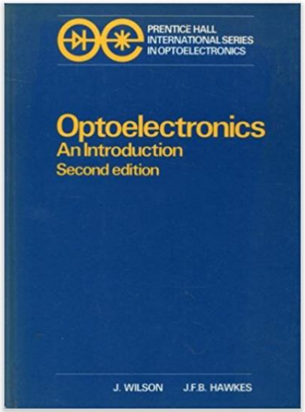
Chapter-8

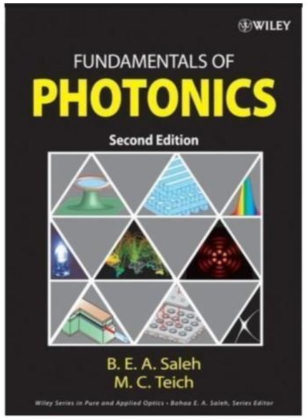
Assoc. Prof. Dr. Isa NAVRUZ

Lecture Notes - 2018

Recommended books







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Refraction & Reflection-2

Objectives

When you finish this lesson you will be able to:

- ✓ Describe the Fresnel Equations
- ✓ Define Total Internal Reflection
- ✓ Explain the Fresnel coefficients in the Internal Reflection
- ✓ Explain the Fresnel coefficients in the External Reflection
- ✓ Describe the Brewster Angle

Fresnel Equations

Reflection coefficient for s polarization

$$r_s = \frac{E_r}{E_i} = \frac{n_i cos\theta_i - n_t cos\theta_t}{n_i cos\theta_i + n_t cos\theta_t}$$

Transmission coefficient for s polarization

$$t_s = \frac{E_t}{E_i} = \frac{2 n_i cos\theta_i}{n_i cos\theta_i + n_t cos\theta_t}$$

For the parallel polarization case, using similar methods, the result are

Reflection coefficient for p polarization

$$r_p = \frac{E_r}{E_i} = \frac{n_t cos\theta_i - n_i cos\theta_t}{n_t cos\theta_i + n_i cos\theta_t}$$

Transmission coefficient for p polarization

$$t_p = \frac{E_t}{E_i} = \frac{2 n_i cos\theta_i}{n_i cos\theta_t + n_t cos\theta_i}$$

Fresnel Equations

Using Snell's law, we can re-write:

$$r_s = r_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$r_p = r_\parallel = +\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$t_s = t_\perp = +\frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}$$

$$t_p = t_\parallel = +\frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

Fresnel Equations

Reflectance (R) and Transmittance (T)

$$R = |r|^2$$

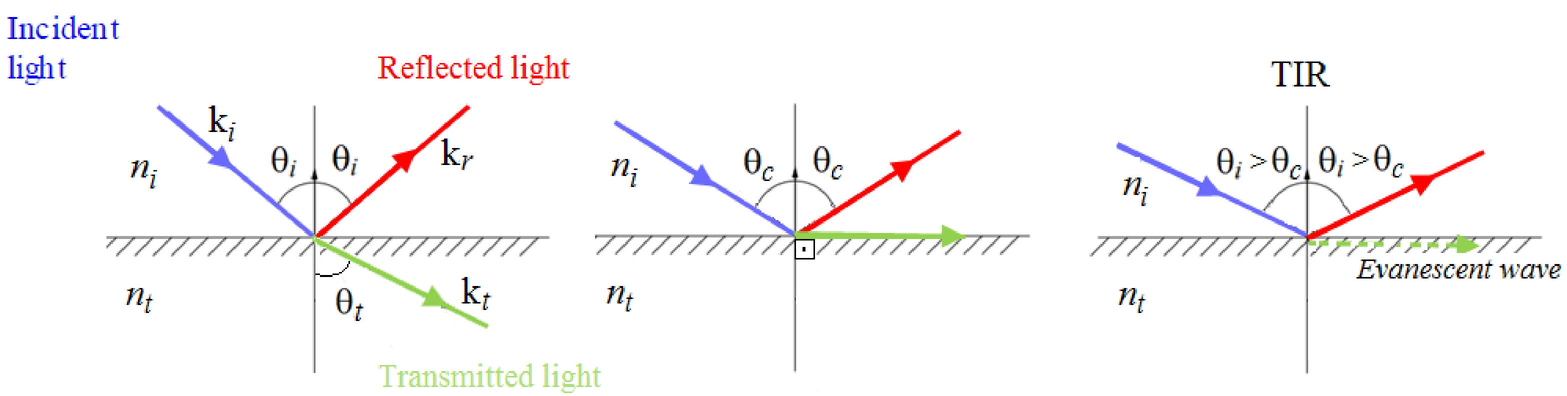
$$T = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right)|t|^2$$

Total Internal Reflection

When $n_i > n_t$, the transmitted angle is bigger than incidence angle. In this case, if $\theta t = 90°$, then the incidence angle is called the critical angle

$$\sin(\theta_c) = \frac{n_t}{n_i}$$

If $\theta i > \theta c$, Total Internal Reflection (TIR) occurs and an evanescent wave propagates along the boundary (i.e. high loss electric field propagating along the surface).



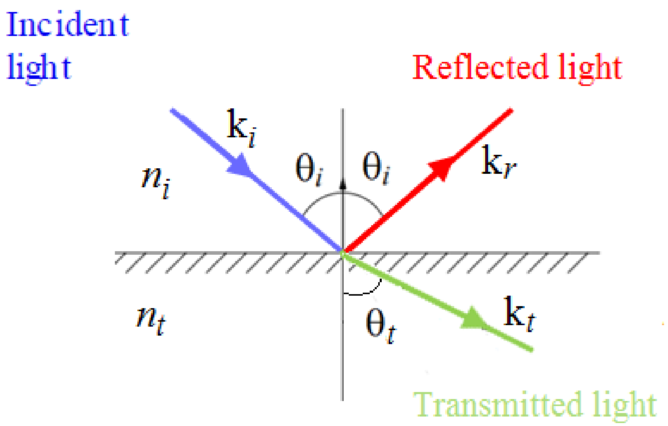
(a) (b) (c)

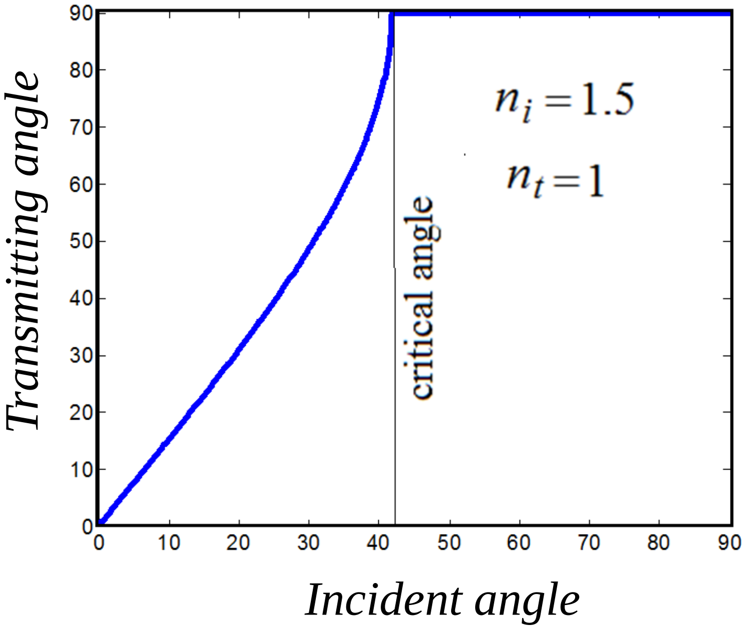
Fresnel Equations

Internal Reflection (n1>n2)

This is case of traveling the light from a more dense medium into a less dense one

$$n_i > n_t \Rightarrow \theta_t > \theta_i$$



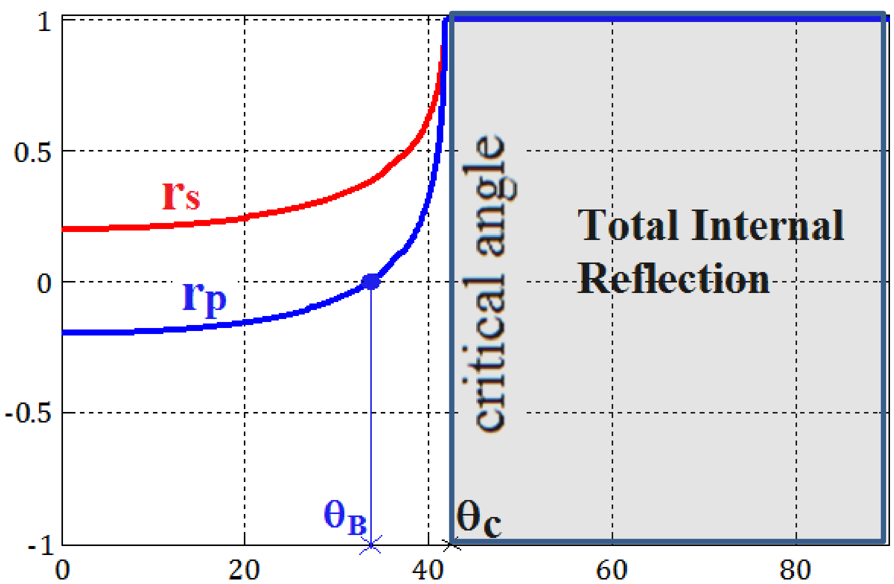


How much of the light is reflected ?
How much of the light is transmitted?
How about the phase of reflected and transmitted light?

Now let's calculate the reflection and transmission coefficients in both the s and p polarizations.

Fresnel Equations

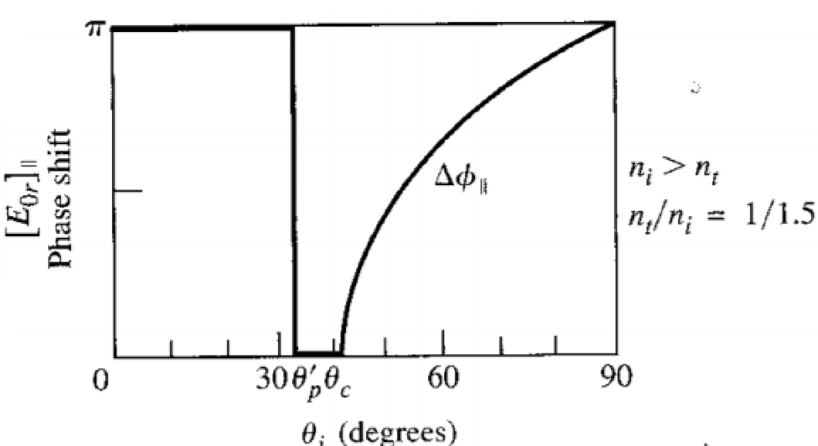
Internal Reflection (n1>n2)

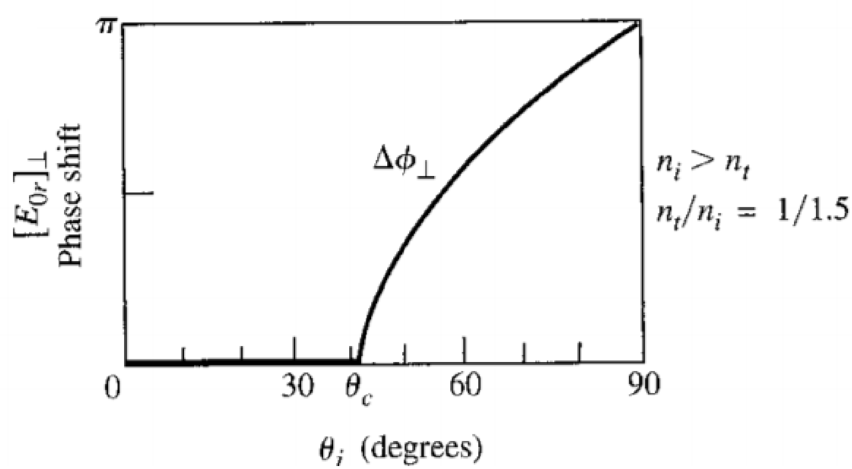


$n_i = 1.5 \quad n_t = 1$ (air)

Matlab Code

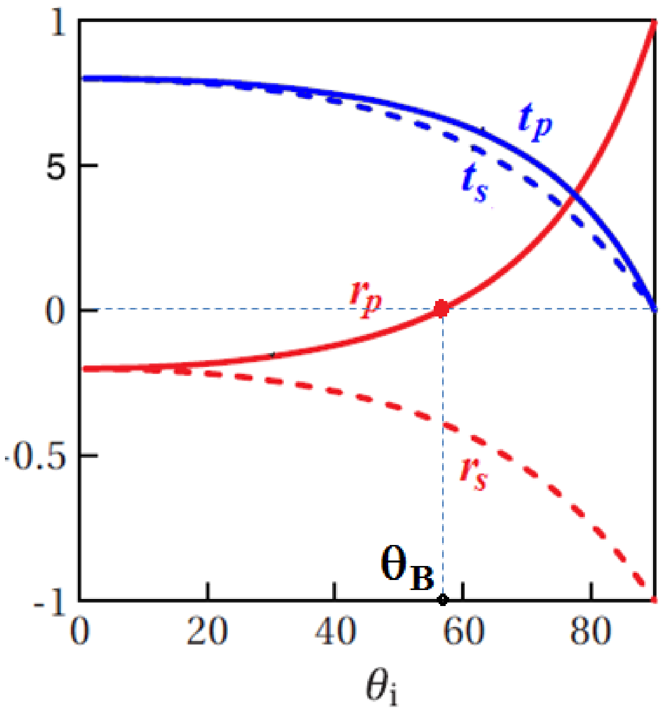
```
clear;
clc;
tetai=0:0.01:89.99;
n1=1.5;
n2=1;
tetac=asind(n2/n1);
tetat=asind(n1*sind(0:0.01:tetac)/n2);
plot(0:0.01:tetac,tetat);
L=length(tetat);
L2=length(tetai);
tetat(L+1:1:L2)=90;
rs=(n1*cosd(tetai)-n2*cosd(tetat))./(n1*cosd(tetai)+n2*cosd(tetat));
figure;
plot(tetai,rs);
hold on
rp=(n2*cosd(tetai)-n1*cosd(tetat))./(n1*cosd(tetat)+n2*cosd(tetai));
plot(tetai,rp);
ylim([-1-1]);
```





Fresnel Equations

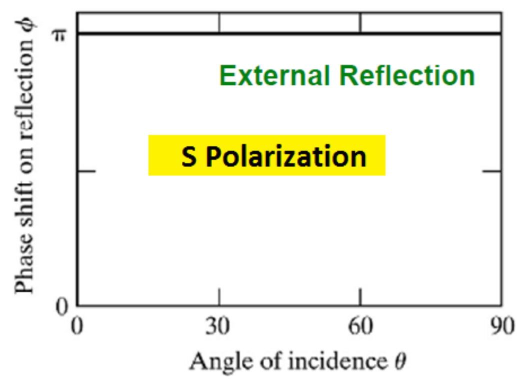
External Reflection (n1<n2)

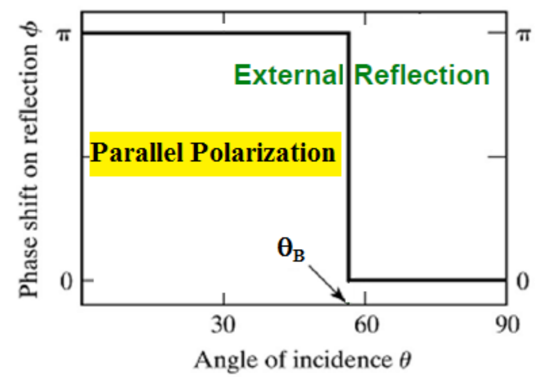


n_i = 1 (air) n_t = 1.5

If r is real and r >0 then there are no phase changes after reflection.

If r < 0 then there are π (180°) phase changes after reflection.





For s polarization, π phase shift for all incident angles

For "p" case, ***π phase shift for $\theta < \theta_B$***
No phase shift for $\theta > \theta_B$

Fresnel Equations

Brewster's Angle (θ_B) :

Note that r_p is zero at a certain angle. This angle only occurs when the p-polarized (TM mode) light is reflected in both the internal and external reflection cases.

Brewster's angle equals to 56.3° for $n_i = 1$ and $n_t = 1.5$ values.

$$r_p = r_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} = 0$$

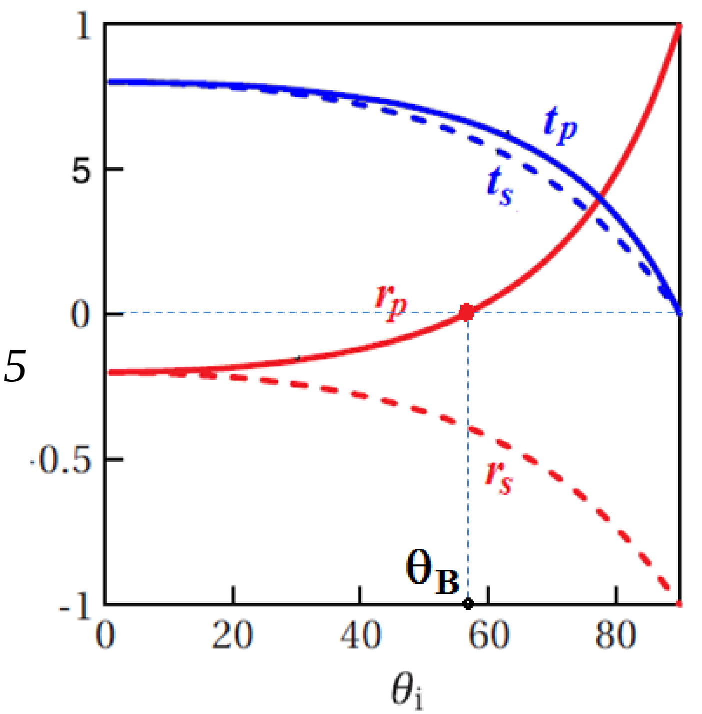
$$\theta_i + \theta_t = \frac{\pi}{2}$$

$$n_i \sin\theta_i = n_t \sin\left(\frac{\pi}{2} - \theta_i\right) = n_t \cos\theta_i$$

Brewter's angle is the angle that satisfies this equation,

$$\theta_B = \tan^{-1}\frac{n_t}{n_i}$$

Brewster's angle (or, polarizing angle)
(No reflection of TM mode)



$n_i = 1$ (air) $n_t = 1.5$

Fresnel Equations

Brewster's Angle (θ_B) :

Example:

Show that polarizing (Brewter's) angles for internal and external reflection between the same two media must be complementary of π/2

For external reflection ⟹ θ_B

For internal reflection ⟹ θ'_B

$$\theta_B + \theta'_B = \pi/2$$

Fresnel Equations

Example:

For an air-glass interface (n_i = 1 and n_t = 1.5), suppose that the incident light is perpendicularly (S) polarized Light. When θ_i = 0,

a) Calculate the reflection and transmission coefficients (r, t)

b) Calculate the reflectance (R) and Transmittance (T)

c) Explain the phase change of reflected light and transmitted light.

Fresnel Equations

Example:

Show analytically that $R_p + T_p = 1$, where R_p and T_p is given by

$$r_p = \frac{E_r}{E_i} = \frac{n_t cos\theta_i - n_i cos\theta_t}{n_t cos\theta_i + n_i cos\theta_t}$$

$$t_p = \frac{E_t}{E_i} = \frac{2n_i cos\theta_i}{n_i cos\theta_t + n_t cos\theta_i}$$

Fresnel Equations

Example:
a)Consider three dielectric media with flat and parallel boundaries with refractive indices n1, n2, and n3. Show that for normal incidence the reflection coefficient between layers 1 and 2 is the same as that between layers 2 and 3 if n2 = n1 n3 . What is the significance of this?

b)Consider a Si photodiode that is designed for operation at 900 nm. Given a choice of two possible antireflection coatings, SiO2 with a refractive index of 1.5 and TiO2 with a refractive index of 2.3 which would you use and what would be the thickness of the antireflection coating you chose? The refractive index of Si is 3.5.

Fresnel Equations

Example:
Consider that light propagates at normal incidence from air, n_1 = 1, to semiconductor likes a photocell with a refractive index of n_3=3.5 as given in Fig-a

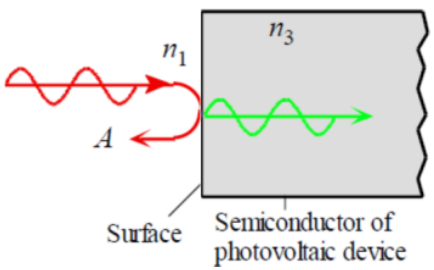


Fig-(a)

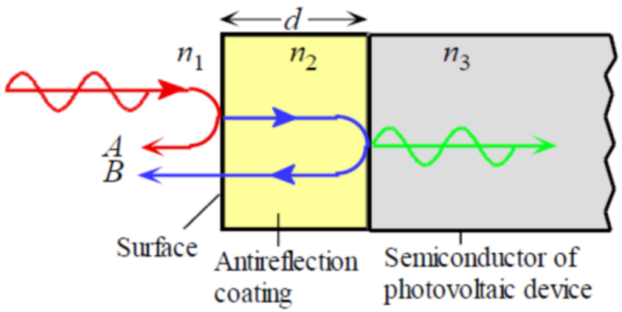


Fig-(b)

a) What is the reflection coefficient (r) and the reflectance (R) with respect to the incident beam?
b) When the semiconductor material is coated with thin layer of electric material such as Si_3N_4 (silicon nitride) that has an intermediate refractive index of n_2=1.9 as given in Fig-(b), the loss can be reduced. In this case, calculate the reflection coefficient (r) and the reflectance (R) and discuss the loss.
c) In this system, how can you explain the phase matching relation to thickness of antireflective layer?

Fresnel Equations

Example:

A ray of light which is traveling in a glass medium of refractive index n1 = 1.450 becomes incident on a less dense glass medium of refractive index n2 = 1.430. Suppose that the free space wavelength (λ) of the light ray is 1 µm. a.

a)What should be the minimum incidence angle for TIR?

b) What is the phase change in the reflected wave when θi = 85° and when θi = 90°? c.