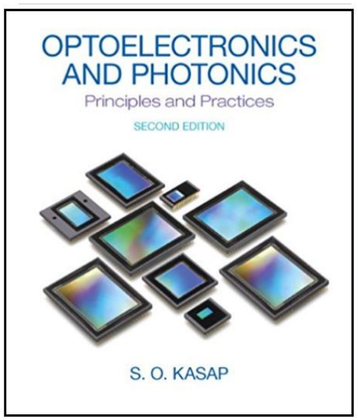
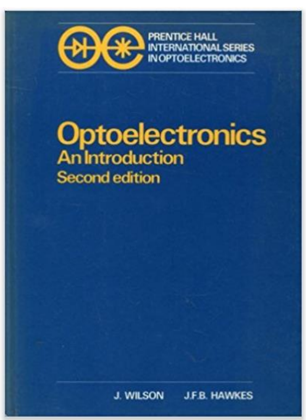
Optoelectronics-I

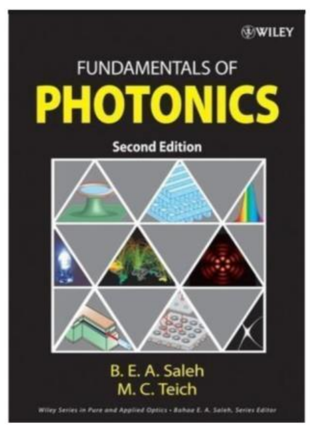
Assoc. Prof. Dr. Isa NAVRUZ

Lecture Notes - 2018

Recommended books







Department of Electrical and Electronics Enginnering, Ankara University
Golbasi, ANKARA

Tutorial-1

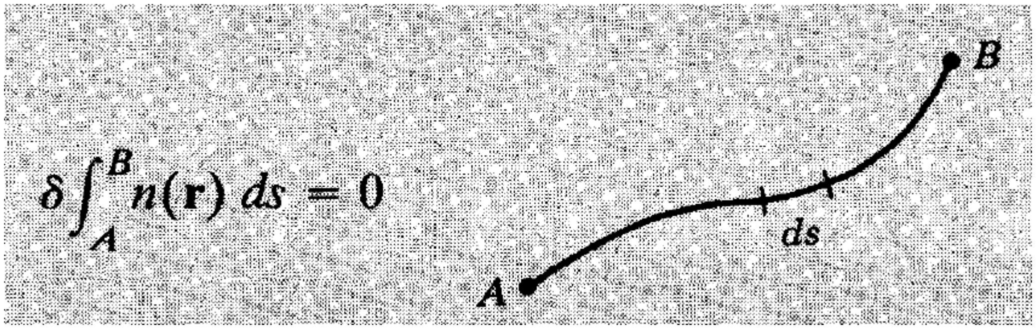
Objectives

When you finish this lesson you will be able to:

- ✓ Fermat’s Principle and Reflection
- ✓ Reflection and refraction in multi layer
- ✓ Wavelength, frequency and speed
- ✓ Wave optics
- ✓ Standing wave
- ✓ Electric and Magnetic Fields

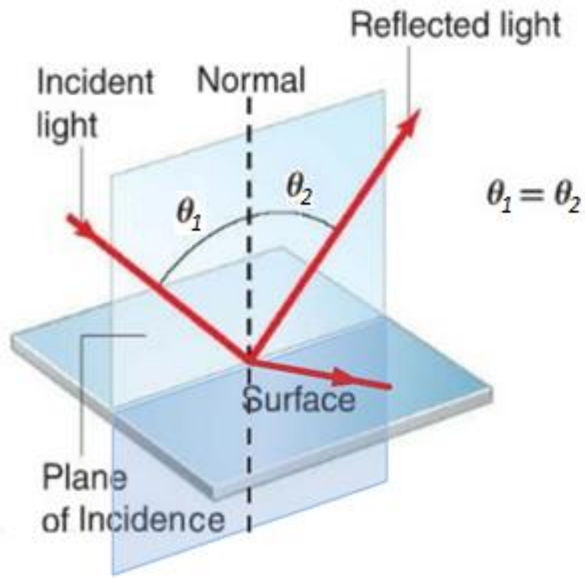
Fermat's principles:

$$\delta \int_A^B n(\mathbf{r})\, ds = 0$$



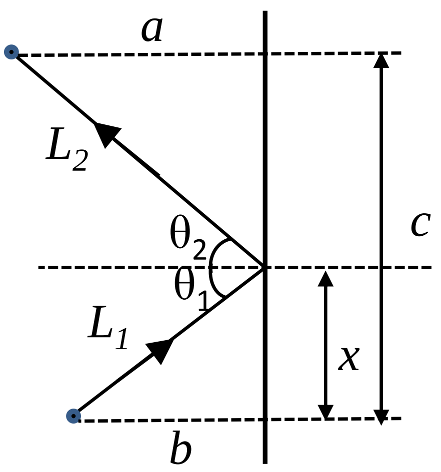
light rays travel along the path of least time

Q1. Show that the light travels along the path of least time in the case of $\theta_1 = \theta_1$ at reflection medium.



Fermat's principles:

A1.



$$\theta_1 = \theta_2$$

$$\frac{c-x}{a} = \frac{x}{b}$$

$$L = L_1 + L_2 = \sqrt{(c-x)^2 + a^2} + \sqrt{x^2 + b^2}$$

$$L = \sqrt{c^2 - 2cx + x^2 + a^2} + \sqrt{x^2 + b^2}$$

$$L' = \frac{\frac{1}{2}(-2c + 2x)}{\sqrt{c^2 - 2cx + x^2 + a^2}} + \frac{\frac{1}{2}(2x)}{\sqrt{x^2 + b^2}} = 0$$

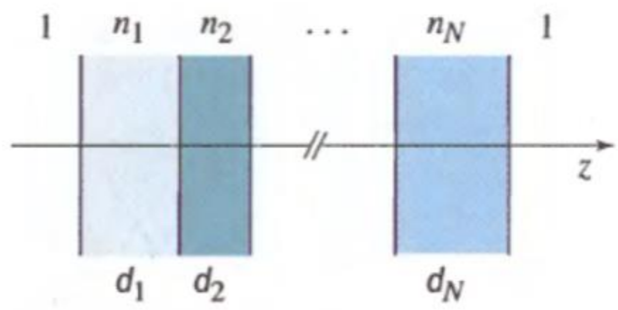
$$\frac{(c^2 - 2cx + x^2)}{c^2 - 2cx + x^2 + a^2} = \frac{x^2}{x^2 + b^2}$$

$$\boxed{x = \frac{bc}{a+b}} \Leftarrow \boxed{\frac{c-x}{a} = \frac{x}{b}}$$

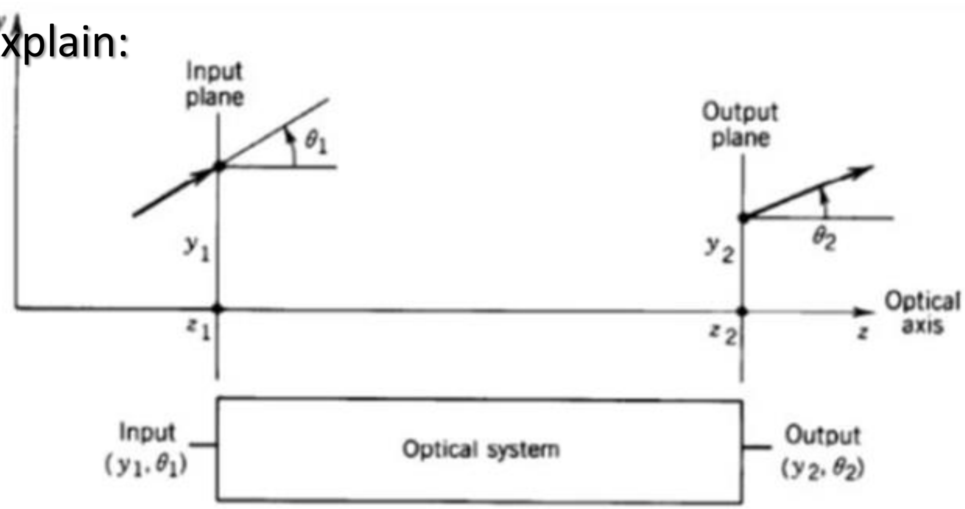
Reflection and refraction in multi layer

Q2. Consider a set of N parallel planar transparent plates of refractive indexes n_1, n_2,.., n_N and thicknesses d_1, d_2,. . . d_N , placed in air (n = 1) normal to the z axis. Using induction, show that the ray-transfer matrix is

$$\mathbf{M} = \begin{bmatrix} 1 & \sum_{i=1}^{N} \frac{d_i}{n_i} \\ 0 & 1 \end{bmatrix}$$



Explain:



In the paraxial approximation,

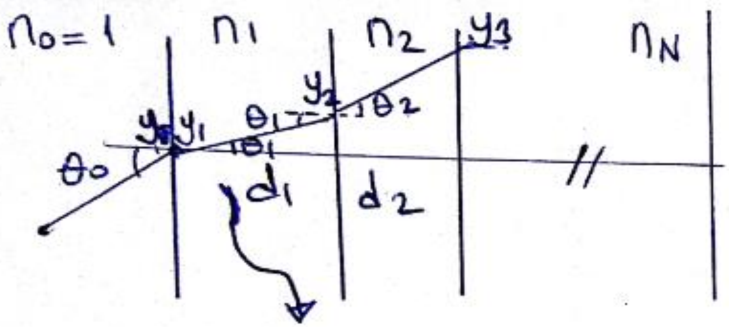
$$y_2 = Ay_1 + B\theta_1$$

$$\theta_2 = Cy_1 + D\theta_1$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

Reflection and refraction in multi layer

A2.



$$n_0 \cdot \sin\theta_0 = n_1 \cdot \sin\theta_1$$

$$1 \cdot \sin\theta_0 = n_1 \cdot \sin\theta_1$$

Using paraxial approximatio

$$1 \cdot \theta_0 = n_1 \cdot \theta_1$$

$$\boxed{\theta_1 = \frac{1}{n_1}\theta_0}$$

$$y_1 = y_0$$

$$\begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1/n_1 \end{bmatrix} \begin{bmatrix} y_0 \\ \theta_0 \end{bmatrix}$$

$$n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2$$

$$n_1 \cdot \theta_1 = n_2 \cdot \theta_2$$

$$\boxed{\theta_2 = \frac{n_1}{n_2}\theta_1}$$

$$y_2 = y_1 + d_1 \cdot \tan\theta_1$$

$$\boxed{y_2 = y_1 + d_1 \cdot \theta_1}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & d_1 \\ 0 & n_1/n_2 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

Reflection and refraction in multi layer

A2.

$$\theta_N = \frac{n_{N-1}}{n_N}\theta_{N-1} \quad \text{and} \quad y_N = y_{N-1} + d_{N-1}\,\theta_{N-1}$$

For last layer, $\nearrow$ air

$$n_N \cdot \sin\theta_N = n_{N+1}\cdot \sin\theta_{N+1}$$

$$n_N \cdot \theta_N = \theta_{N+1}$$

$$\boxed{\theta_{N+1} = n_N \cdot \theta_N}$$

$$\theta_{N+1} = \underbrace{n_N}_{\text{last layer}} \cdot \underbrace{\left[\frac{n_{N-1}}{n_N} \cdots \frac{n_1}{n_2}\right]}_{\text{other layers}} \cdot \underbrace{\frac{1}{n_1}}_{\text{first layer}}\theta_0$$

$$\boxed{\theta_{N+1} = \theta_0}$$

$$\begin{bmatrix} y_{N+1} \\ \theta_{N+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \sum_{i=1}^{N} \frac{d_i}{n_i} \\ 0 & 1 \end{bmatrix}}_{M} \begin{bmatrix} y_0 \\ \theta_0 \end{bmatrix}$$

$$\boxed{y_{N+1} = y_N + d_N \cdot \theta_N}$$

$y_1 = y_0$ first layer

$$y_2 = y_1 + d_1\theta_1 = y_0 + d_1 \cdot \frac{1}{n_1}\theta_0$$

$$y_3 = y_2 + d_2\theta_2 = y_0 + \frac{d_1}{n_1}\theta_0 + \frac{d_2}{n_2}\cdot\frac{n_1\cdot\theta_0}{n_1}$$

$$y_3 = y_0 + \left(\frac{d_1}{n_1} + \frac{d_2}{n_2}\right)\theta_0$$

$\vdots$

$$y_{N+1} = y_0 + \left(\frac{d_1}{n_1} + \frac{d_2}{n_2} + \cdots\right)\theta_0$$

Wavelength, frequency and speed

Q3. Monochromatic light of wavelength 589nm is incident from air on a water surface. What are the wavelength, frequency and speed of
(a) Reflected light (b) refracted light?

A3. Wavelength of incident monochromatic light,
$\lambda = 589 \text{ nm} = 589 \times 10^{-9}$ m
Speed of light in air, $c = 3 \times 108$ m/s and Refractive index of water, $n = 1.33$

The ray will reflect back in the same medium as that of incident ray. Hence, the wavelength, speed, and frequency of the reflected ray will be the same as that of the incident ray.

$$\begin{aligned} \nu &= c / \lambda \\ &= 3 \times 10^{8} / 589 \times 10^{-9} \\ &= 5.09 \times 10^{14} \text{ Hz} \end{aligned}$$

Wavelength, frequency and speed

A3.

Frequency of light does not depend on the property of the medium in which it is travelling. Hence, the frequency of the refracted ray in water will be equal to the frequency of the incident or reflected light in air.

So Refracted frequency, $\nu = 5.09 \times 10^{14}$ Hz

Speed of light in water is related to the refractive index of water as:

$v = c / n$

$v = 3 \times 10^{8} / 1.33 = 2.26 \times 10^{8}$ m/s

Wavelength of light in water is given by the relation,

$\lambda = v / \nu$

$\lambda = 2.26 \times 10^{8}$ **/** $5.09 \times 10^{14} = 444.007 \times 10^{-9}$ m

$= 444.01$ nm

Hence, the speed, frequency, and wavelength of refracted light are 2.26×10^{8} m/s, 444.01nm, and 5.09×10^{14} Hz respectively.

Wave optics

Q4. Find, using algebraic addition, the amplitude and phase resulting from the addition of the two superposed waves $\psi_1 = E_1 \sin(kx - \omega t + \varepsilon_1)$ and $\psi_2 = E_2 \sin(kx - \omega t + \varepsilon_2)$, where ε_1=0, ε_2=π/2, E_1=8, E_2=6, and x=0.

A4.

$$\alpha_1 = kx + \varepsilon_1 = 0 \qquad \alpha_2 = kx + \varepsilon_2 = \pi/2 \qquad E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos(\alpha_2 - \alpha_1)} = 10$$

$$\alpha = \arctan\frac{E_1 \sin\alpha_1 + E_2 \sin\alpha_2}{E_1 \cos\alpha_1 + E_2 \cos\alpha_2} = \arctan 0.75 = 36.87^\circ$$

$$\psi = 10\sin(kx - \omega t + 0.6435)$$

Q5. Two waves $\psi_1 = E_1 \sin(kx - \omega t)$ and $\psi_2 = E_2 \sin(kx - \omega t + \pi)$ are coplanar and overlap. Calculate the resultant's amplitude if E_1=3 and E_2=2.

A5.

$$E^2 = E_1^2 + E_2^2 + 2E_1E_2\cos(\alpha_2 - \alpha_1)$$

$$= 3^2 + 2^2 + 2\times 3\times 2\times \cos\pi = 1$$

$$E = 1$$

Standing wave

Q6.

Two plane waves with electric fields E1=A sin(ωt−kx) and E2=A sin(ωt+kx) have the same angular frequencies $\omega = 4.00 \times 10^{15}\ s^{-1}$ propagate in opposite directions in an isotropic medium with index of refraction $n = 1.50$. The waves interfere with each other and form a standing wave. Find the magnitude of the net electric field vector at the instant $t_0 = (\pi/16) \times 10^{-15}$ s and the amplitude of the standing wave's electric field at all points between nodes separated by a distance $l = \lambda/8$ from each other.

A6. A standing wave is formed

$$E(t,x)=E1(t,x)+E2(t,x)=-2A \sin kx \cos \omega t.$$

From this expression for standing wave, we can see that at each point, the electric field oscillates with angular frequency ω. The oscillation amplitude at point x is

$$A(x)=2A|\sin kx|.$$

Therefore, at points where $\sin kx = 0$, the oscillation amplitude vanishes. These points are the electric field nodes of standing wave.

$k = 2\pi/\lambda$ first node $kx_1 = 0$ and the second node at $kx_2 = \pi$.

$\lambda=v\times T=2\pi c \omega n$ and $\Delta x=\pi c \omega n$.

$$A_m=2A|\sin km\lambda 8|=2A|\sin(m\pi 4)|=2A\sin(m\pi 4),$$

$$A0=0,\ A1=2A,\ A2=2A,\ A3=2A,\ A4=0.$$

$$E(t0,xm)=2A\sin(m\pi 4)\cos(\omega t0)=2A\sin(m\pi 4).$$

Electric and Magnetic Fields

Q7.

The electric field of an electromagnetic plane wave is given (in S.I. units) by:

$$\vec{E}(x,y,z,t) = \left(5\hat{e}_x + 7.5\hat{e}_y\right)\cos\left[6\times10^8\,t - \left(3x - 2y + \sqrt{3}z\right)\right]$$

a) Find the Cartesian components of the $\vec{k}$-vector.

b) Prove that the $\vec{k}$-vector and the electric field $\vec{E}$ are perpendicular to each other.

c) Calculate the magnitude of the $\vec{k}$- vector.

d) Determine the angular frequency ω.

e) Calculate the phase velocity for this electromagnetic wave.

f) Calculate the refractive-index of the medium in which this electromagnetic wave is propagating (remember: speed of light in vacuum is $c = 3\times10^8$ in S.I. units).

g) Find the Cartesian components of the magnetic field associated with this electromagnetic wave.

Electric and Magnetic Fields

A7.

a) $\vec{E}(x,y,z,t) = \left(5\hat{e}_x + 7.5\hat{e}_y\right)\cos\left[6\times10^8\,t - k\,r\right]$ $\quad r^2 = x^2+y^2+z^2$

$\vec{k} = \left(3, -2, \sqrt{3}\right)$ 1/m $\Longrightarrow$ $\vec{k} = \left(3\hat{e}_x, -2\hat{e}_y + \sqrt{3}\hat{e}_z\right)$ 1/m

b) $\vec{k}\cdot\vec{E} = \left(3, -2, \sqrt{3}\right)\cdot(5, 7.5, 0) = (15 - 15) = 0$

c) $\left|\vec{k}\right| = \left(3^2, (-2)^2, \sqrt{3}^2\right)^{1/2} = 4$ 1/m

d) $\omega = 6\text{x}10^8$ rad/s

e) $v = \dfrac{\omega}{k} = \dfrac{6.10^8}{4} = 1.5\ 10^8$ m/s

f) $n = \dfrac{c}{v} = \dfrac{3.10^8}{1.5\ 10^8} = 2$

Electric and Magnetic Fields

A7.

g) $$\vec{\nabla}\times\vec{E} = -\mu_o \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \hat{e}_x\left(0-\frac{\partial}{\partial z}E_y\right) + \hat{e}_y\left(\frac{\partial}{\partial z}E_x - 0\right) + \hat{e}_z\left(\frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x\right)$$

$$\begin{aligned} &-7.5\sqrt{3}\sin[6.10^8 t - (3x - 2y + \sqrt{3}z)]\hat{e}_x \\ = \quad &+5\sqrt{3}\sin[6.10^8 t - (3x - 2y + \sqrt{3}z)]\hat{e}_y \\ &+32.5\sin[6.10^8 t - (3x - 2y + \sqrt{3}z)]\hat{e}_z \end{aligned}$$

$$\boxed{H = -\frac{1}{\mu_o}\int\left[\quad\right]dt = \frac{1}{\mu_0\, 6.10^8}(7.5\sqrt{3}\hat{e}_x + 5\sqrt{3}\hat{e}_y + 32.5\hat{e}_z)\cos\left[6.10^8 t - (3x - 2y + \sqrt{3}z)\right]}$$

Electric and Magnetic Fields

Homework

A plane wave is described by $\psi(x, y, z, t) = A\,e^{i(2x-3y+5z-7t)}$ in S.I. units.

a) What are the planes of constant phase?

b) In which direction the planes of constant phase propagate?

c) What is the wavelength λ (make sure to provide proper unit)?

d) What is the speed v that a plane with constant phase propagates (make sure to provide proper unit)?

e) What is the frequency ν (make sure to provide proper unit)?

Electric and Magnetic Fields

Homework

(a) A plane, harmonic linearly polarised light light wave has an electric field

$$E_z = E_0 \cos[\pi 10^{15}(t - \frac{x}{0.65c})]$$

Write down the frequency and wavelength of the light wave and obtain the index of refraction of the glass.

(b) Consider electromagnetic wave of wavelength $\lambda = 30cm$ in air. What is the frequency of such waves? If such waves waves pass from air into the block of quartz, for which the dielectric constant $K = 4.3$, what is their new speed, frequency, and wavelength?