Optoelectronics-I

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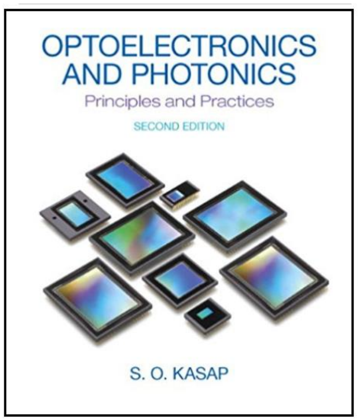
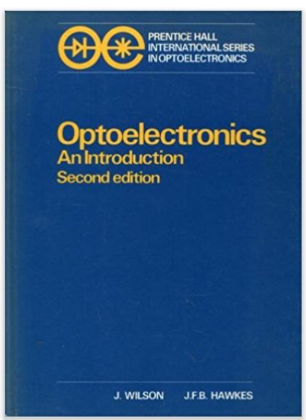
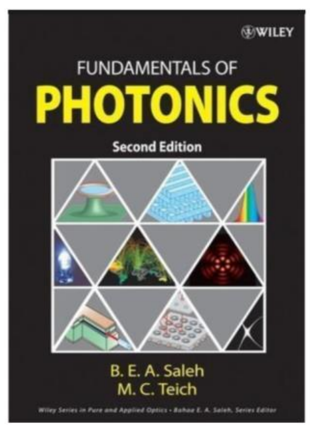
Lecture Notes - 2018

Recommended books







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Tutorial-2

Objectives

When you finish this lesson you will be able to:

- ✓ Refractive index of conductive materials
- ✓ Intensity of Light
- ✓ Power of Light
- ✓ Reflection and Refraction
- ✓ Brewster angle,
- ✓ Reflectance (R) and Transmittance (T)
- ✓ Circular and Elliptical polarization

Refractive index of conductive materials:

Q1. The complex refractive index of germanium at 400 nm is given by $\hat{n}$ =4.141 + i 2.215. Calculate for germanium at 400 nm:

(a) the phase velocity of light,
(b) the absorption coefficient
(c) the penetration depth at which light intensity falls to I_0 / e^2.

Refractive index of conductive materials:

A1.

$$v = \frac{c}{n} = \frac{2.998 \times 10^8}{4.141}\ \mathrm{m\,s^{-1}} = 7.24 \times 10^7\ \mathrm{m\,s^{-1}}$$

$\hat{n} = 4.141 + i\ 2.215 \longrightarrow n = 4.141 \quad \kappa = 2.215$

$\lambda = 400\ \mathrm{nm}$

$$\vec{E}(z,t) = \vec{E}_o e^{-\frac{\omega}{c}Kz} e^{i\left[\frac{\omega}{c}nz - \omega t + \phi\right]}$$

$$I(z) = I_o e^{-\alpha z}$$

Intensity is proportional to the square of the electric field

$$\alpha = \frac{2\kappa\omega}{c} = \frac{4\pi\kappa}{\lambda}$$

$$\alpha = \frac{4\pi \times 2.215}{400 \times 10^{-9}}\ \mathrm{m^{-1}} = 6.96 \times 10^7\ \mathrm{m^{-1}}$$

$$\frac{I(z)}{I_o} = e^{-\alpha z} = \frac{1}{e^2} \Longrightarrow \alpha z = 2 \Longrightarrow z = 28.74\mathrm{nm}$$

intensity

Q2. The intensity (irradiance) of the red laser beam from a He-Ne laser in air has been measured to be about 1 mW cm^{-2}.

What are the magnitudes of the electric and magnetic fields?

What are the magnitudes if this 1 mW cm^{-2} beam were in a glass medium with a refractive index n = 1.45 and still had the same intensity?

intensity

A2.

$$I = \frac{1}{2} c\,\varepsilon_o n E_o^2$$

$$E_o = \sqrt{\frac{2I}{c\varepsilon_o n}} = \sqrt{\frac{2(10\ \mathrm{W\,m^{-2}})}{(3\times10^8\ \mathrm{m\,s^{-1}})(8.85\times10^{-12}\ \mathrm{F\,m^{-1}})(1)}} = \mathbf{86.772\ V\,m^{-1}}$$

$$B_o = \frac{nE_o}{c} = \frac{(1)(86.772\ \mathrm{V\,m^{-1}})}{(3\times10^8\ \mathrm{m\,s^{-1}})} = \mathbf{2.892\ \ 10^{-7}\ \ V\,m^{-2}s}$$

$$B_0 = \mu_0 H_0 \qquad H_0 = \frac{B_0}{\mu_0} = \frac{2.892\ 10^{-7}}{4\pi 10^{-7}} = \mathbf{0.230\ A/m}$$

in a glass medium of $n = 1.45$

$$E_o = \sqrt{\frac{2I}{c\varepsilon_o n}} = \sqrt{\frac{2(10\ \mathrm{W\,m^{-2}})}{(3\times10^8\ \mathrm{m\,s^{-1}})(8.85\times10^{-12}\ \mathrm{F\,m^{-1}})(1.45)}} = \mathbf{72.06\ V\,m^{-1}}$$

$$B_o = \frac{n E_o}{c} = \frac{(1.45)(\ 72.06\ \mathrm{V\,m^{-1}})}{(3\times10^8\ \mathrm{m\,s^{-1}})} = \mathbf{3.483\ 10^{-7}\ V\,m^{-2}s}$$

Intensity and Power

Q3.

The power of a laser beam of light is P = 2 mW. The distribution of the light intensity over a certain cross section of the beam is given by the Gaussian function,

$$I(x, y) = I_o e^{-\frac{x^2+y^2}{w^2}} \quad \text{W/m}^2$$

Determine the intensity I_0 and the amplitude E_0 of the electric field strength in the beam center if the beam radius w = 1 mm and the refractive index of the medium n = 1.33.

Intensity and Power

A3.

$$I = \varepsilon_o c \left\langle \left|\vec{E}\right|^2 \right\rangle = \frac{1}{2}\varepsilon_o c \left|\vec{E}\right|^2$$ in free space

$$I(x,y) = I_o e^{-\frac{x^2+y^2}{w^2}}$$

w : beam radius (spot size)

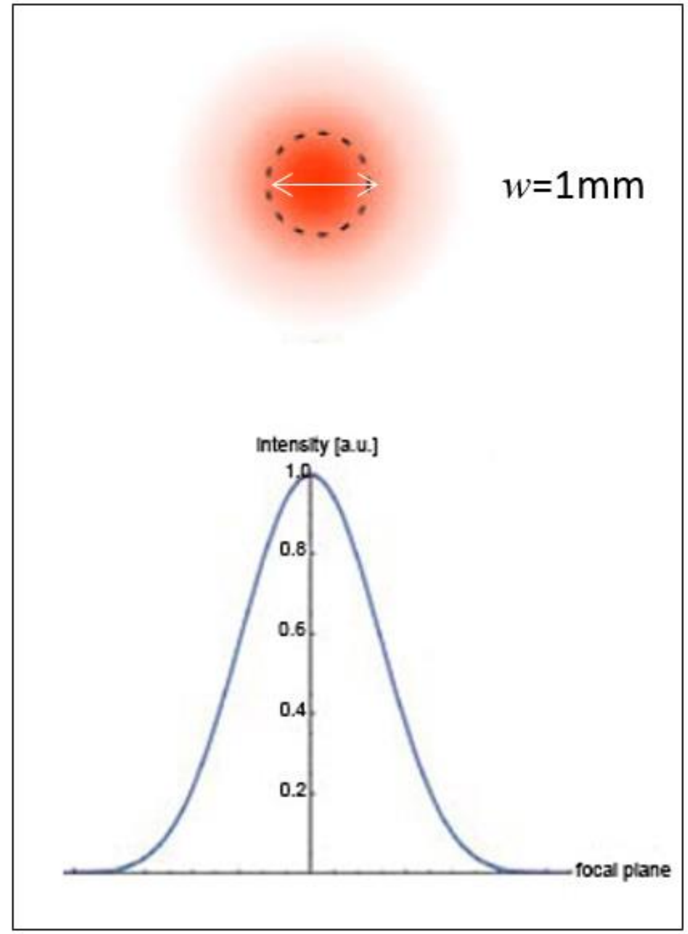
$$x^2+y^2=r^2$$

$$P = \int_A I(r)\cdot dA$$

$$P = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_o e^{-\frac{x^2+y^2}{w^2}} dx\,dy$$

transfering to polar coordinates $\Rightarrow$

$$P = \int_0^{2\pi}\int_0^{\infty} I_o e^{-\frac{r^2}{w^2}} \cdot r \cdot dr \cdot d\theta$$



Intensity and Power

A3.

$$P = -\frac{1}{2} I_o e^{-r^2}\Big|_0^{\infty} \cdot \theta \Big|_0^{2\pi}$$

$$P = -\frac{1}{2} I_o (0-1)\cdot(2\pi - 0) \Rightarrow P = \pi \cdot I_o$$

$$P = 2\,mW \Rightarrow 2 = \pi \cdot I_o \Rightarrow$$

$$I_o = 0{,}637\, mW/mm^2 = 637\, W/m^2$$

$$I = \varepsilon_o c \left\langle \left|\vec{E}\right|^2 \right\rangle = \frac{1}{2}\varepsilon_o c \left|\vec{E}\right|^2$$ in free space

$$I = \frac{1}{2}\varepsilon \cdot v \left|\vec{E}\right|^2$$

$$I = \frac{1}{2} n^2 \varepsilon_o \cdot \frac{c}{n} \left|\vec{E}\right|^2$$

$$n = \left(\frac{\varepsilon}{\varepsilon_o}\right)^{1/2}$$

$$\varepsilon = n^2 \cdot \varepsilon_o$$

$$v = \frac{c}{n}$$

$$I = \frac{1}{2} n \varepsilon_o \cdot c \left|\vec{E}\right|^2$$

$$637 = \frac{1}{2} \cdot 1{,}33 \cdot 8{,}854 \cdot 10^{-12} \cdot 3 \cdot 10^{8} \left|E_o\right|^2$$

$$E_o = 600\, V/m$$

Reflection and Refraction

Q4.

A ray of light which is traveling in a glass medium of refractive index $n1 = 1.450$ becomes incident on a less dense glass medium of refractive index $n2 = 1.430$. Suppose that the free space wavelength (λ) of the light ray is 1 μm.

a) What should be the minimum incidence angle for TIR?

b) What is the phase change in the reflected wave when $\theta i = 85°$ and when $\theta i = 90°$?

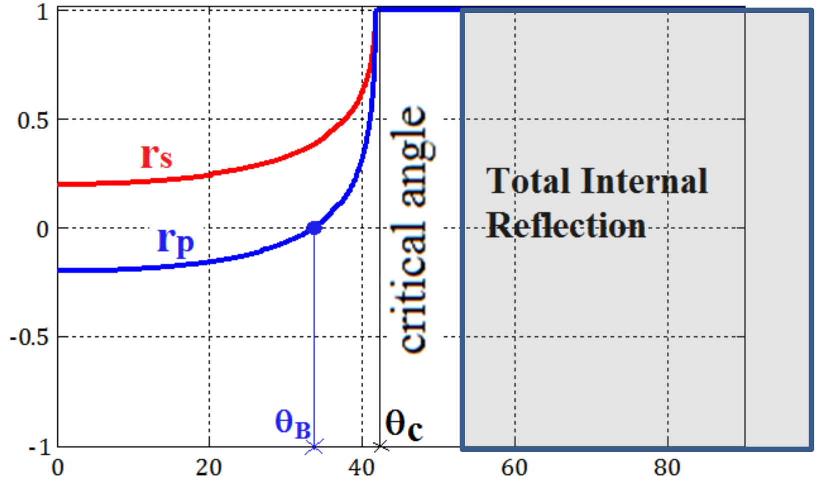
Reflection and Refraction

A4.

The critical angle θ*c* for TIR,

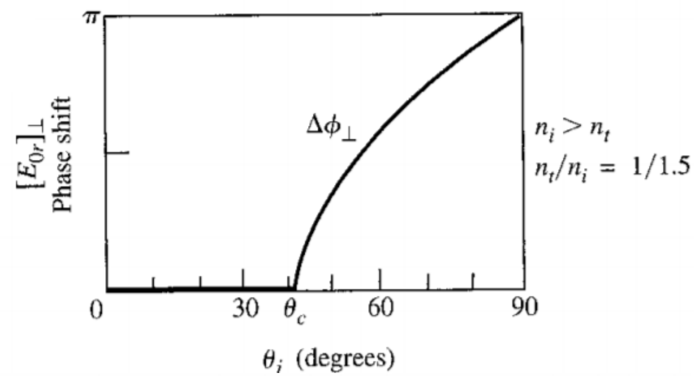
sinθ*c* = *n*2/*n*1 =1.430/1.450

θ*c* = **80.47°**



Remember that since the incidence angle θ*i* > θ*c*, the magnitudes of $\mathbf{r_s}$ and $\mathbf{r_p}$ equal to 1 but there is a phase shift in the reflected wave.

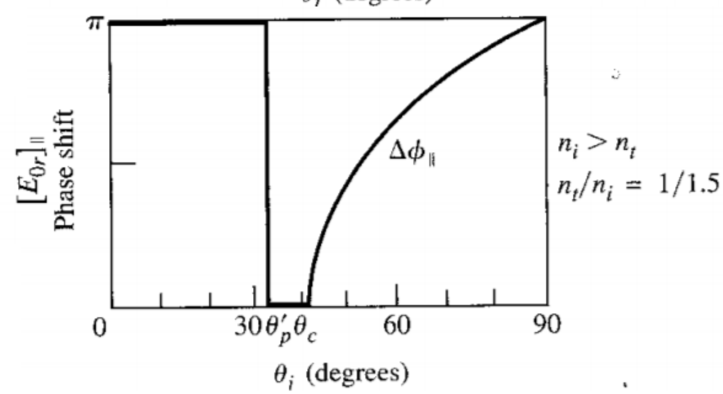
$$r_s = r_\perp = \frac{E_r}{E_i} = \frac{n_i cos\theta_i - n_t cos\theta_t}{n_i cos\theta_i + n_t cos\theta_t}$$



$$r_\perp = \frac{cos\theta_i - \frac{n_t}{n_i} cos\theta_t}{cos\theta_i + \frac{n_t}{n_i} cos\theta_t}$$

$$n_i sin\theta_i = n_t sin\theta_t$$

$$cos\theta_t = \sqrt{1 - sin^2\theta_t}$$



$$r_\perp = \frac{\cos\theta_i - (n^2 - \sin^2\theta_i)^{1/2}}{\cos\theta_i + (n^2 - \sin^2\theta_i)^{1/2}}$$

$$n = \frac{n_t}{n_i}$$

Reflection and Refraction

A4.

when θ*i* = 85°, *n*1 = 1.45 and *n*2 = 1.43

$$r_{\perp} = \frac{cos(85) - \left(\left(\frac{1.43}{1.45}\right)^2 - sin^2(85)\right)^{1/2}}{cos(85) + \left(\left(\frac{1.43}{1.45}\right)^2 - sin^2(85)\right)^{1/2}} = \frac{0.08716 - j0.140712}{0.08716 + j0.140712}$$

$$\phi_{\perp} = \frac{-58.225°}{58.225°}$$

$$\Delta\phi_{\perp} = 116.45°$$

We can repeat the calculation for θ*i* = 90° to find the phase change,

$$\Delta\phi_{\perp} = 180°$$

The phase change for θ*i* = 90°

Reflection and Refraction

Q5.

Consider the reflection of light at ***normal incidence*** on a boundary between a GaAs crystal medium of refractive index 3.6 and air of refractive index 1.

a) If light is traveling from air to GaAs, what is the reflection coefficient and the intensity of the reflected light in terms of the incident light?

b) If light is traveling from GaAs to air, what is the reflection coefficient and the intensity of the reflected light in terms of the incident light?

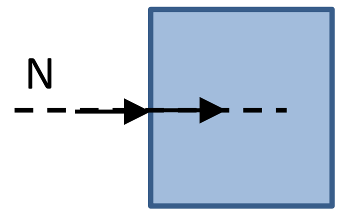
Reflection and Refraction

A5.

The light travels in air and becomes partially reflected at the surface of the GaAs crystal which corresponds to external reflection. Thus *n*1 = 1 and *n*2 = 3.6. Reflection coefficient is given by,

$$r_s = \frac{E_r}{E_i} = \frac{n_i cos\theta_i - n_t cos\theta_t}{n_i cos\theta_i + n_t cos\theta_t}$$

$$\theta_i = \theta_t = 0°$$



$$r_s = \frac{n_i - n_t}{n_i + n_t} = \frac{n_1 - n_2}{n_1 + n_2}$$

We can repeat the calculation for r_p with light at ***normal incidence***

$$r_p = \frac{E_r}{E_i} = \frac{n_t cos\theta_i - n_i cos\theta_t}{n_t cos\theta_i + n_i cos\theta_t}$$

Same

$$r_p = \frac{n_i - n_t}{n_i + n_t} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$r_{//} = r_{\perp} = \frac{1-3.6}{1+3.6} = -\mathbf{0.565}$$

This is negative which means that there is a 180° phase shift.

$$R = (r_{\perp})^2 = (-0.565)^2 = \mathbf{0.319 = 31.9\,\%}$$

Reflection and Refraction

A5.

If light is traveling from GaAs to air, thus $n1 = 3.6$ and $n2 = 1$.

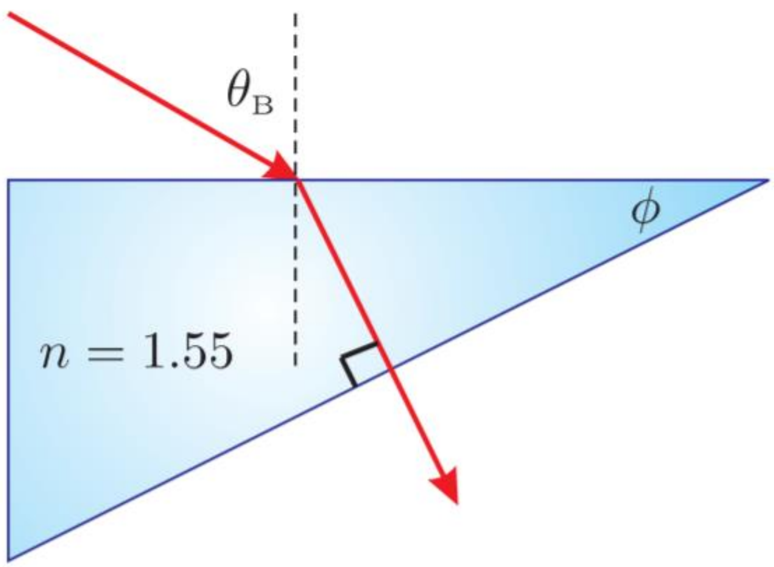
$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{3.6 - 1}{3.6 + 1} = \mathbf{0.565}$$

There is no phase shift. The reflectance is again **0.319** or **31.9%**. In both cases, **a** and **b**, the amount of reflected light is the same.

Brewster angle, Reflectance (R) and Transmittance (T)

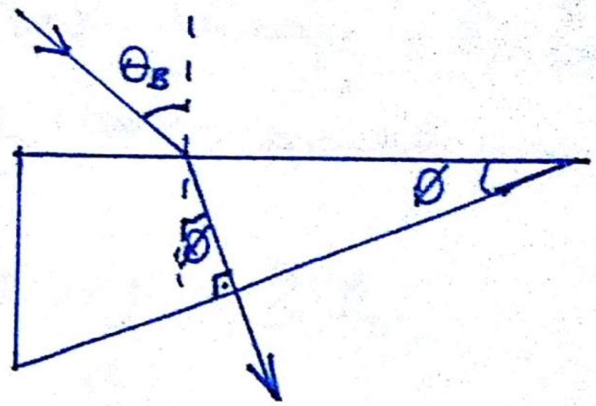
Q6. Light goes through a glass prism with optical index n = 1.55. The light enters at Brewster's angle and exits at normal incidence.

a) Calculate Brewster's angle θ_B
b) Calculate ϕ
c) What percent of the light (power) goes all the way through the prism if it is p-polarized?
d) What percent for s-polarized light?



Brewster angle, Reflectance (R) and Transmittance (T)

A6.



a) $\tan\theta_B = \frac{n_t}{n_i} = 1{,}55$

$\theta_B = 57.17°$

b) $\emptyset + \theta_B = 90°$

$\emptyset = 90 - 57.17° = 32.83°$

c) if the light is P-polarized, in the first interface $\Rightarrow$ $t_p = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$

$$t_p = \frac{2\cos(57.17)}{\cos(32.83) + 1.55\cos(57.17)} = 0{,}645$$

$$(\text{Transmittance})\ T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}|t|^2 = \frac{1.55 \cdot \cos(32.83)}{1 \cdot \cos(57.17)} \cdot 0.645^2$$

$$T = 0{,}999 \approx 1$$

Brewster angle, Reflectance (R) and Transmittance (T)

A6.

(Transmittance) $T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} |t|^2 = \frac{1.55 \cdot \cos(32.83)}{1 \cdot \cos(57.17)} \cdot 0.645^2$

$T = 0,999 \approx 1$

When the incident light is P-Polarized, if $\theta_i = \theta_B$, there is no reflected light. This event only occurs in P-Polarized case.

The Second interface: $\theta_i = \theta_t = 0°$ (the light is normal to the surface)

$t = \frac{2 \cdot 1.55}{(1.55 + 1)} = 1.2156$ $T = 0.9534$ $T_{total} = T_1 \cdot T_2 = 0,9525$

d) When the light is S-Polarized

$t_1 = 0,5932$ $T_1 = 0,8398$
$t_2 = 0,2156$ $T_2 = 0,9533$
$\Rightarrow T = T_1 \cdot T_2 \simeq 0,8$

Circular and Elliptical polarization

Q7. Two electric fields, which are linearly polarized in the x and y axes respectively, have π/2 phase difference.

$$E_x(z,t) = E_{ox}\cos(kz-\omega t) \qquad E_y(z,t) = E_{oy}\cos(kz-\omega t+\frac{\pi}{2})$$

a) If these two electric fields have the different amplitude, show that the superposition of two fields provides elliptical polarization given as below.

$$\left(\frac{E_x}{E_{ox}}\right)^2 + \left(\frac{E_y}{E_{oy}}\right)^2 = 1$$

b) If the phase difference differ from π/2, show that the superposition of two fields provides general ellipse equation given as below.

$$\left(\frac{E_x}{E_{ox}}\right)^2 + \left(\frac{E_y}{E_{oy}}\right)^2 - 2\left(\frac{E_x}{E_{ox}}\right)\left(\frac{E_y}{E_{oy}}\right)\cos(\delta) = \sin^2(\delta)$$

Circular and Elliptical polarization

A.7

a)
$$E_x(z,t) = E_{ox}\cos(kz-\omega t) \qquad E_y(z,t) = E_{oy}\cos(kz-\omega t+\frac{\pi}{2})$$

$$\frac{E_x}{E_{ox}} = \cos(kz-\omega t)$$

$$\frac{E_y}{E_{oy}} = \sin(kz-\omega t)$$

$$\left(\frac{E_x}{E_{ox}}\right)^2 + \left(\frac{E_y}{E_{oy}}\right)^2 = 1$$

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b)
$$\frac{E_x}{E_{ox}} = \cos(kz-\omega t) \qquad \frac{E_y}{E_{oy}} = \cos(kz-\omega t)\cos\delta - \sin(kz-\omega t)\sin\delta$$

$$\frac{E_y}{E_{oy}} - \frac{E_x}{E_{ox}}\cos\delta = -\sin(kz-\omega t)\sin\delta$$

$$\sin(kz-\omega t) = \left[1-\left(\frac{E_x}{E_{ox}}\right)^2\right]^{1/2} \Rightarrow \left(\frac{E_y}{E_{oy}} - \frac{E_x}{E_{ox}}\cos\delta\right)^2 = \left(1-\left(\frac{E_y}{E_{oy}}\right)^2\right)\sin^2\delta$$

$$\Rightarrow \left(\frac{E_x}{E_{ox}}\right)^2 + \left(\frac{E_y}{E_{oy}}\right)^2 - 2\left(\frac{E_x}{E_{ox}}\right)\left(\frac{E_y}{E_{oy}}\right)\cos(\delta) = \sin^2(\delta)$$