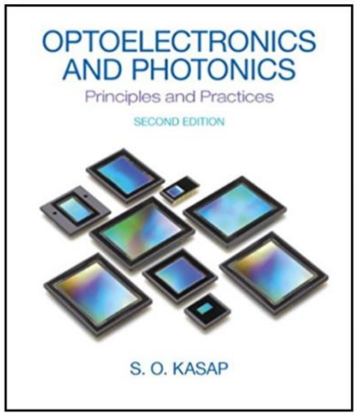
Optoelectronics-I

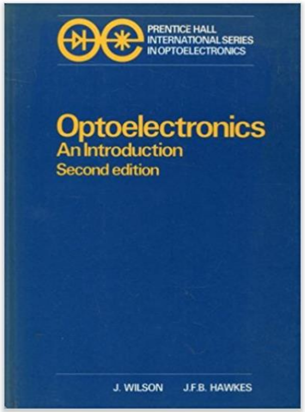
Chapter-11

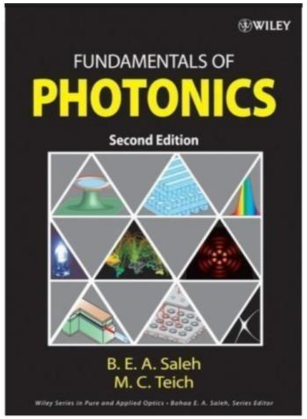
Assoc. Prof. Dr. Isa NAVRUZ

Lecture Notes - 2018

Recommended books







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Interference & Diffraction

Objectives

When you finish this lesson you will be able to:

- ✓ Describe the Interference fringe
- ✓ Define constructive interference and Destructive Interference
- ✓ Explain the Conditions for Interference
- ✓ Explain the interference on Thin Film
- ✓ Describe the Diffraction
- ✓ Explain the Fresnel and Fraunhofer diffractions

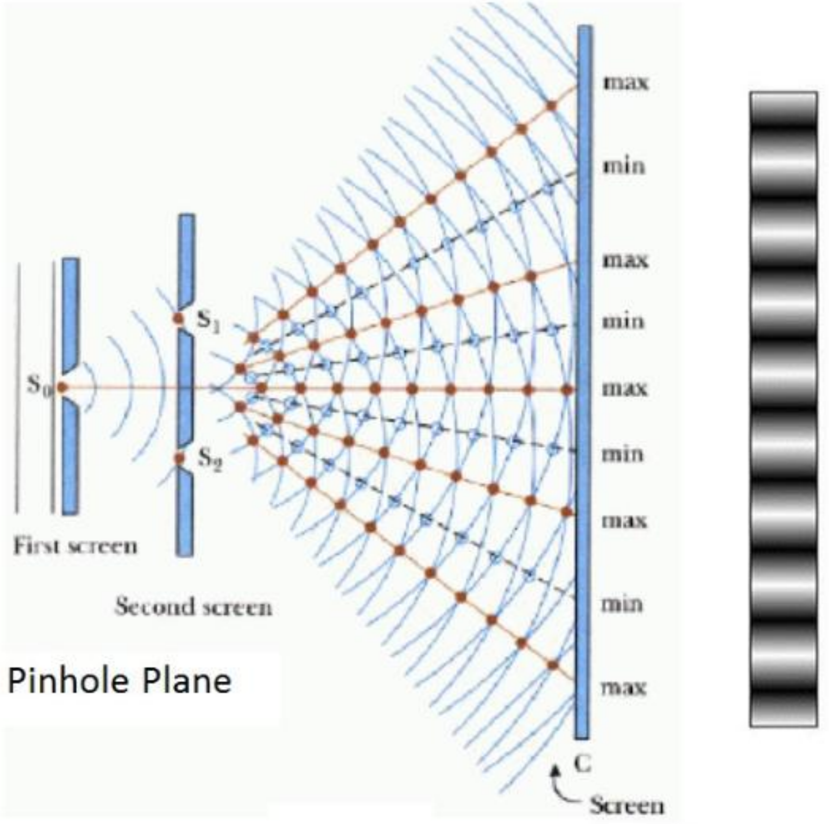
Interference

When two or more waves approach one another, it is called as interference. If light from a source is divided into two beams that are superimposed, an **Interference fringe** formed by bright or dark bands occurs.

This phenomenon was first experimentally performed by Thomas Young in 1801.

If two interfering light beams originate from the same source, fluctuations in the two beams are in general correlated and the beams are said to be completely or partially coherent

When beams from different sources are superimposed, no interference is observed

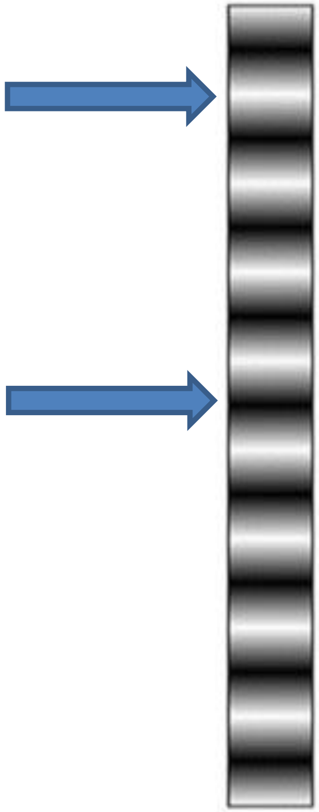


Observation Plane

Interference

When the two waves arriving at a point are in phase, the resultant intensity of light at that point is maximum, and that point appears bright. This type of interference is called a constructive interference.

When the two waves arriving at a point are out of phase, (i.e., the crest of one wave falls on the through of the other wave, and vice-versa), the resultant intensity of light at that point is minimum, and that point appears dark. This type of interference is called a destructive interference



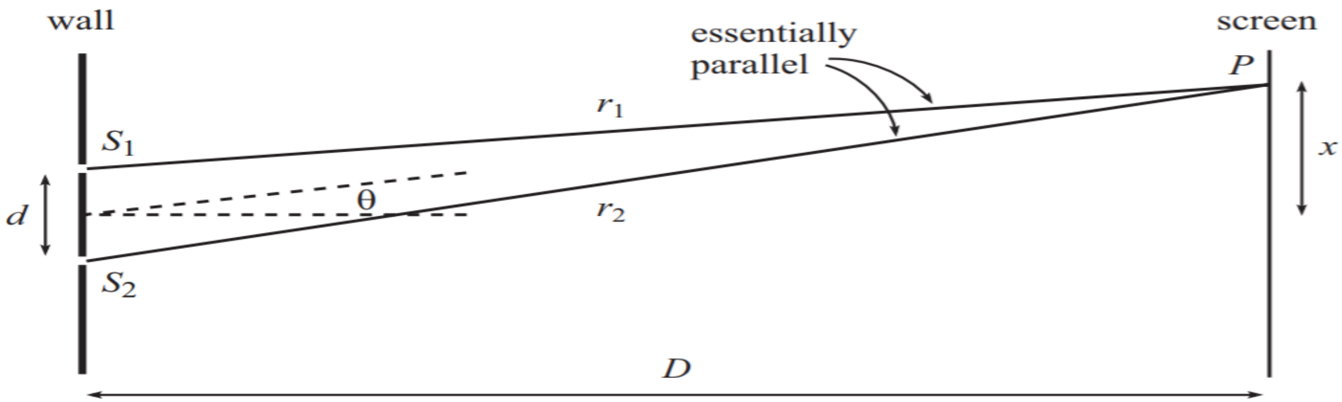
Observation Plane

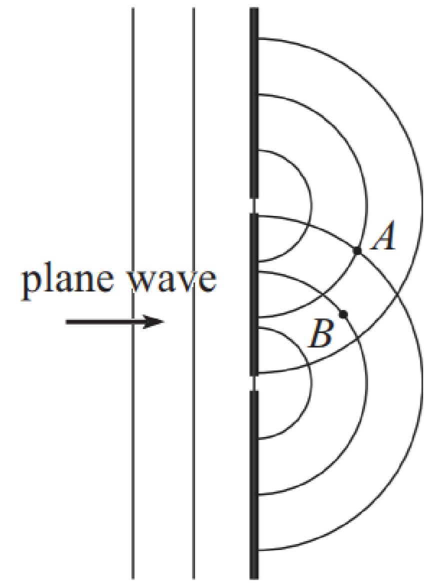
Conditions for steady interference pattern :

1) The two sources of light must be coherent : The two sources are said to be coherent if the waves produced by them are in phase or having constant phase difference.
2) The two sources of light must be monochromatic : A source of light is said to be monochromatic, if it emits light waves of only one wavelength.
3) The two sources must be equally bright, i.e., the waves emitted by the sources must have the same amplitude.
4) The two sources must be narrow.
5) The sources should be close to each other, their separation being of the order of the wavelength of light

Interference

Assume that a plane wave moving toward a wall, and its wavefronts are parallel to the wall.

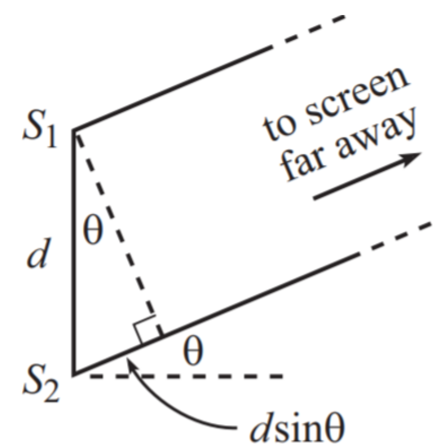




If D >>d so-called far-field, we can say that the two pathlengths r_1 and r_2 are essentially equal. Amplitude u_p at a point *P* a large distance, *D*, from the slits

$$u_p = \frac{u_o}{r_1} e^{i(kr_1 - \omega t)} + \frac{u_o}{r_2} e^{i(kr_2 - \omega t)}$$

We can write $r_1 \approx r_2$ for amplitudes. However the phase term can be calculated by



$$k(r_2 - r_1) = kd\sin\theta = \frac{2\pi}{\lambda} d\sin\theta$$

Interference

$$u_p = \frac{u_o}{r_1} e^{i(kr_1 - \omega t)} + \frac{u_o}{r_2} e^{i(kr_2 - \omega t)}$$

$$k(r_2 - r_1) = kd\sin\theta = \frac{2\pi}{\lambda} d\sin\theta$$

The total electric field at the P point is

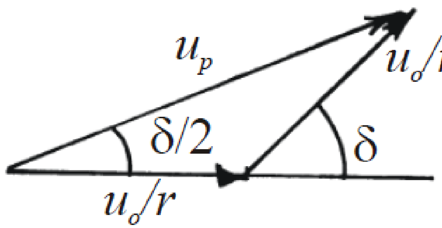
$$E_{\text{tot}}(P) = 2\frac{u_o}{r}\cos\left(\frac{kd\sin\theta}{2}\right) e^{i\left(kr - \omega t\right)}$$

The intensity is:

$$I_p = 4\left(\frac{u_o}{r}\right)^2 \cos^2\left(\frac{1}{2}kd\sin\theta\right)$$

Amplitude from each slit on screen: u_o/r

Phasor diagram for two slit problem

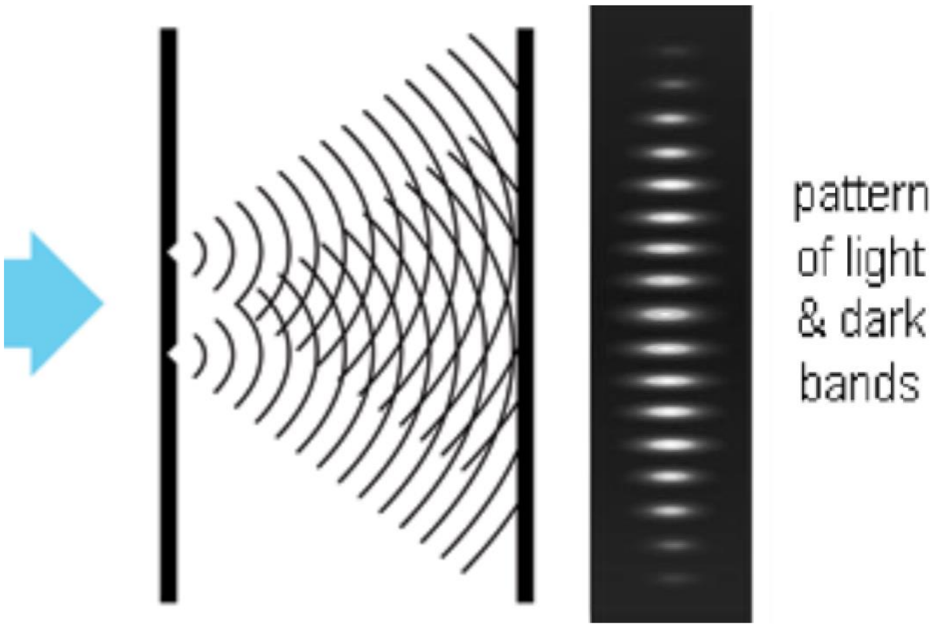


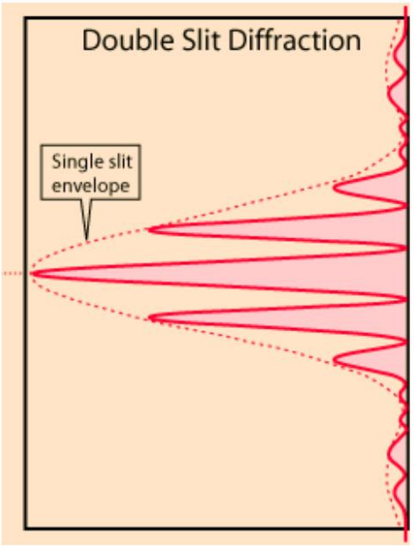
Resultant amplitude is then

$$u_p = 2\frac{u_o}{r}\cos(\delta/2)$$

Phase difference δ, owing to path difference $d\sin\theta$: $\delta = kd\sin\theta$

Interference





If the two waves interfering at a point are in opposite phase, that point appears dark. Suppose x_m and x_{m+1} be the distance of m_{th} and $(m + 1)_{th}$ dark bands from the centre of the interference pattern.

For m_{th} dark band,

$$\frac{x_m d}{D} = 2(m-1)\frac{\lambda}{2}$$

$$x_m = \frac{2(m-1)\lambda D}{2d}$$

Interference

Superposition of Two Plane Waves

Consider two monochromatic waves of the same frequency in a homogeneous medium, and consider only linearly polarized waves

$$\vec{E}_1(\vec{r},t) = \vec{E}_{01}\cos(\vec{k}_1\cdot\vec{r} - \omega t + \varphi_1)$$

$$\vec{E}_2(\vec{r},t) = \vec{E}_{02}\cos(\vec{k}_2\cdot\vec{r} - \omega t + \varphi_2)$$

The resultant wave is given by

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$E^2 = \vec{E}\cdot\vec{E} = (\vec{E}_1 + \vec{E}_2)\cdot(\vec{E}_1 + \vec{E}_2)$$
$$= E_1^2 + E_2^2 + 2\vec{E}_1\cdot\vec{E}_2$$

We know that $I = \langle \vec{E}^2 \rangle$

$$I = I_1 + I_2 + I_{12}$$

$$I_1 = \langle E_1^2 \rangle \quad I_2 = \langle E_2^2 \rangle \quad I_{12} = 2\langle \vec{E}_1\cdot\vec{E}_2 \rangle$$

$$I_{12} = 2\langle \vec{E}_1\cdot\vec{E}_2 \rangle = \vec{E}_{01}\cdot\vec{E}_{02}\cos(\vec{k}_1\cdot\vec{r} + \varphi_1 - \vec{k}_2\cdot\vec{r} - \varphi_2)$$

Interference

Superposition of Two Plane Waves

$$\delta = (\vec{k}_1 \cdot \vec{r} + \varphi_1 - \vec{k}_2 \cdot \vec{r} - \varphi_2)$$

$$I_{12} = \vec{E}_{01} \cdot \vec{E}_{02} \cos\delta$$

$$I_{12} = 2\sqrt{I_1 I_2} \cos\delta$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\delta$$

If $\vec{E}_1 \perp \vec{E}_2$, then $I_{12} = 0$ $\quad I = I_1 + I_2$

If $\vec{E}_1 || \vec{E}_2$, then $I_{12} = E_{01} E_{02} \cos\delta$

A maximum I is obtained when $\cos\delta = 1$: $\quad \delta = 0, \ \pm 2\pi, \ \pm 4\pi, \cdots$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

Total constructive interference

Interference

Superposition of Two Plane Waves

A minimum I is obtained when $\cos\delta = -1$: $\delta = \pm\pi, \ \pm 3\pi, \ \pm 5\pi, \cdots$

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

Total destructive interference

If $I_1 = I_2 = I_0$,

$$I = 2I_0(1 + \cos\delta) = 4I_0 \cos^2\frac{\delta}{2}$$

Under this condition

$$I_{\min} = 0$$

$$I_{\max} = 4I_0$$

Interference

Conditions for Interference

1. Polarization: same polarization
2. Form a stable phase difference

Frequency and wavelength should be the same or very close

The contrast of interference pattern:

$$\eta = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

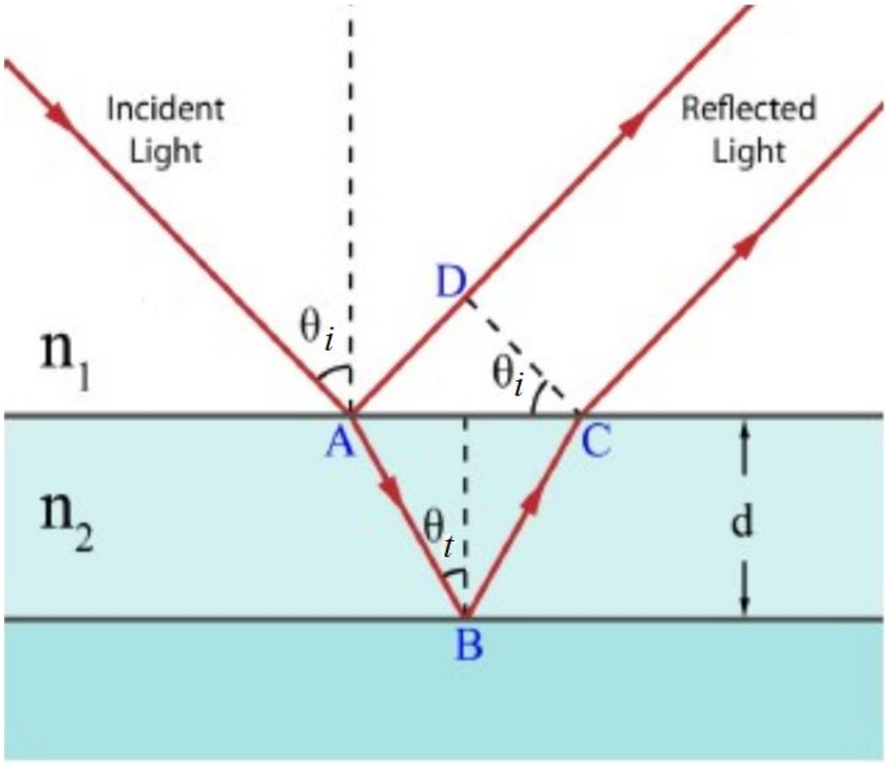
When $\eta = 1$, one obtain the most clear interference pattern.

At this condition, $I_1 = I_{2.}$

Interference

Thin Film Interference

Optical path length difference=ΔL



$$\Delta L = n_2(\overline{AB}+\overline{BC}) - n_1\overline{AD}$$

$$= \frac{2n_2 d}{\cos\theta_t} - n_1\overline{AC}\sin\theta_i$$

$$= \frac{2n_2 d}{\cos\theta_t} - n_1 \times 2d\tan\theta_t\sin\theta_i$$

$$\approx \frac{2n_2 d}{\cos\theta_t} - \frac{2n_2 d\sin^2\theta_t}{\cos\theta_t}$$

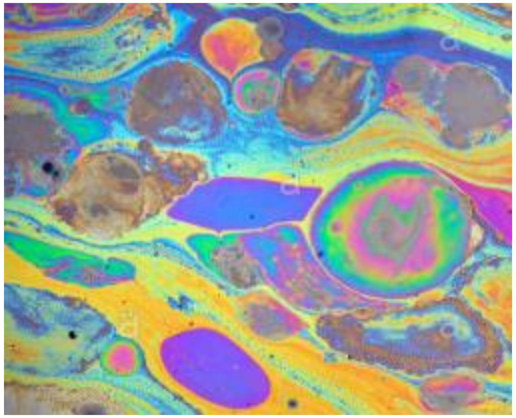
$$= 2n_2 d\cos\theta_t$$

Diffraction

Diffraction occurs when a wave encounters an obstacle or a slit. It is defined as the bending of waves around the corners of an obstacle or aperture into the region of geometrical shadow of the obstacle.

The patterns resulting from diffraction and interference are similar. Interference occurs when two or more waves are at the same location at the same time. However, only one wave passing near the edge of an obstacle is sufficient to produce any diffraction pattern.

Diffraction, and interference are phenomena observed with all waves.



Interference on a puddle of oily water

Soap bubble

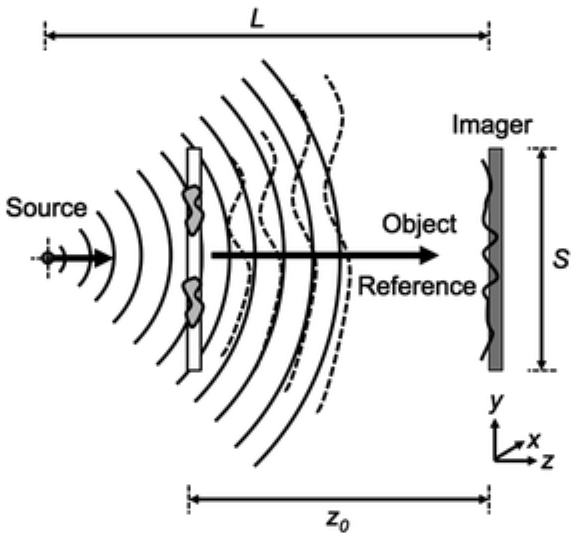
Back face of CD / DVD

Diffraction

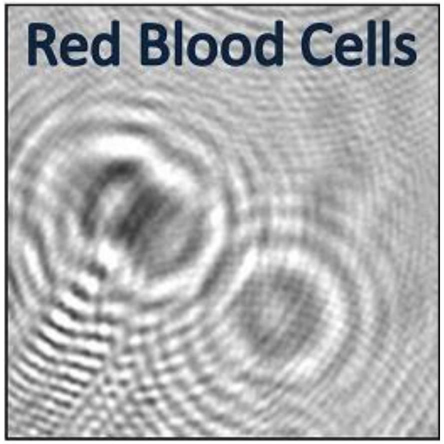
Light does not always travel in a straight line. It tends to bend around objects. This tendency is called **diffraction**.

Diffraction is used in many optical applications.

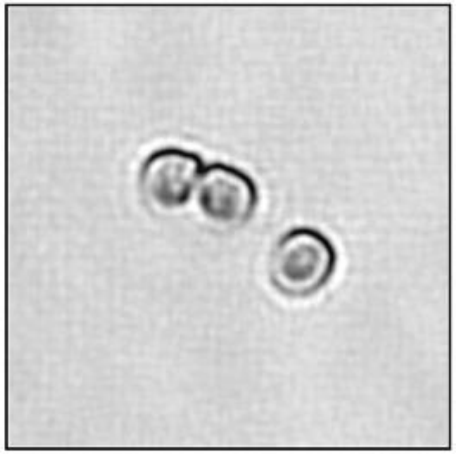
- Spectroscopy,
- Communication and detection systems (fibre optics, lasers, radars),
- Holography, optical holographic imaging
- Structural analysis (X-ray),
- High spatial resolution, imaging and positioning systems



holographic imaging system



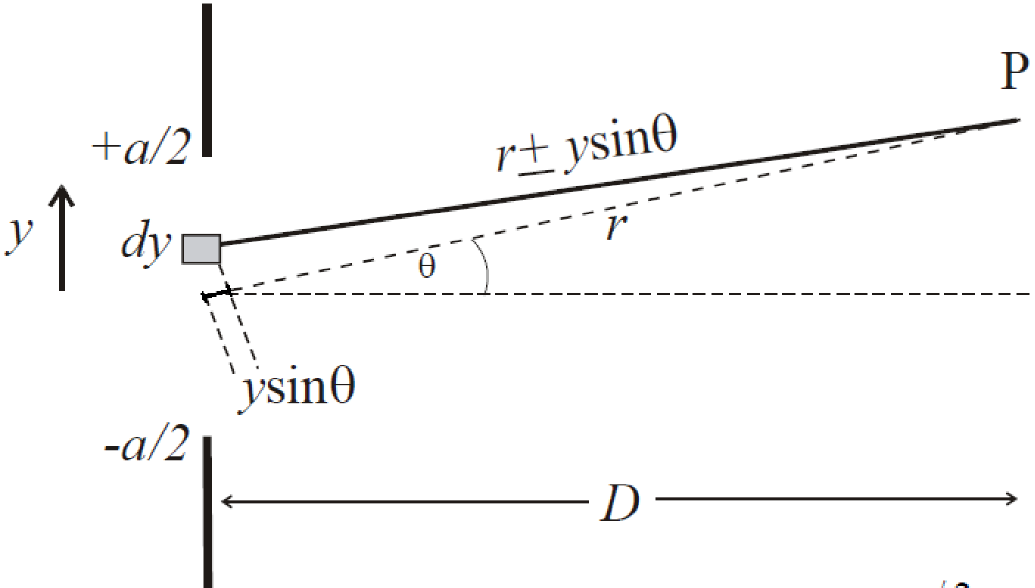
raw holographic image


reconstructed image

Diffraction

Diffraction from a finite slit

Consider that a monochromatic plane wave passing through an aperture of width "a" is observed in a plane at large distance D from aperture.



The amplitude in the plane of aperture is u_o per unit length.

An infinitesimal element of length dy at position y,

$$\frac{u_o dy}{r} e^{i\delta(y)}$$

The phase factor $\delta(y) = k(r \pm y\sin\theta)$

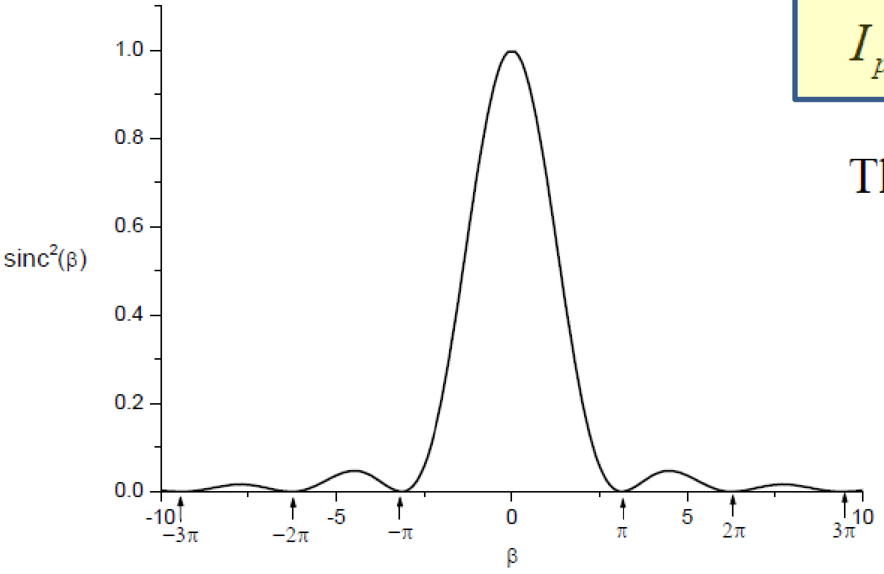
The total amplitude at P, $$u_p = \frac{u_o}{r} e^{ikr} \int_{-a/2}^{a/2} e^{ik\sin\theta . y} dy$$

$$I_p = I(0)\,\mathrm{sinc}^2\beta$$

$$\beta = \frac{1}{2} ka\sin\theta$$

Diffraction

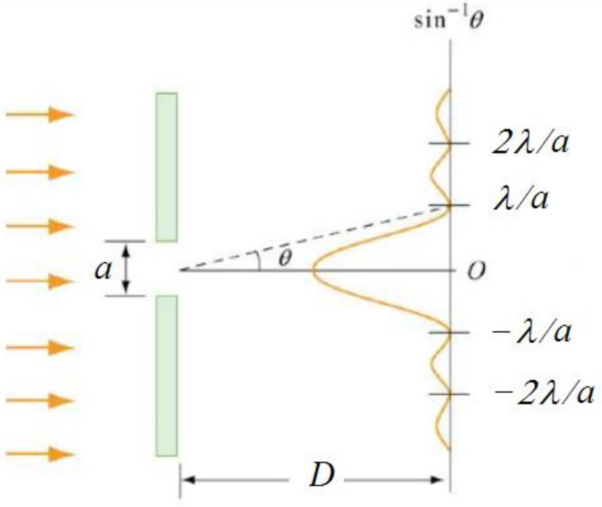
Diffraction from a finite slit

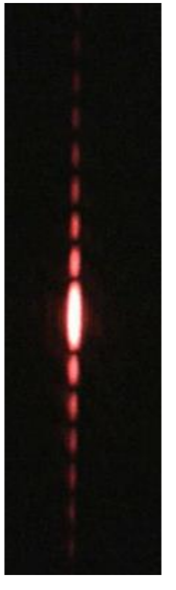


$$I_p = I(0)\,\mathrm{sinc}^2\beta \qquad \beta = \frac{1}{2}ka\sin\theta$$

The first minimum is at $\beta = \pi$,

$$2\pi = \frac{2\pi}{\lambda}a\sin\theta$$

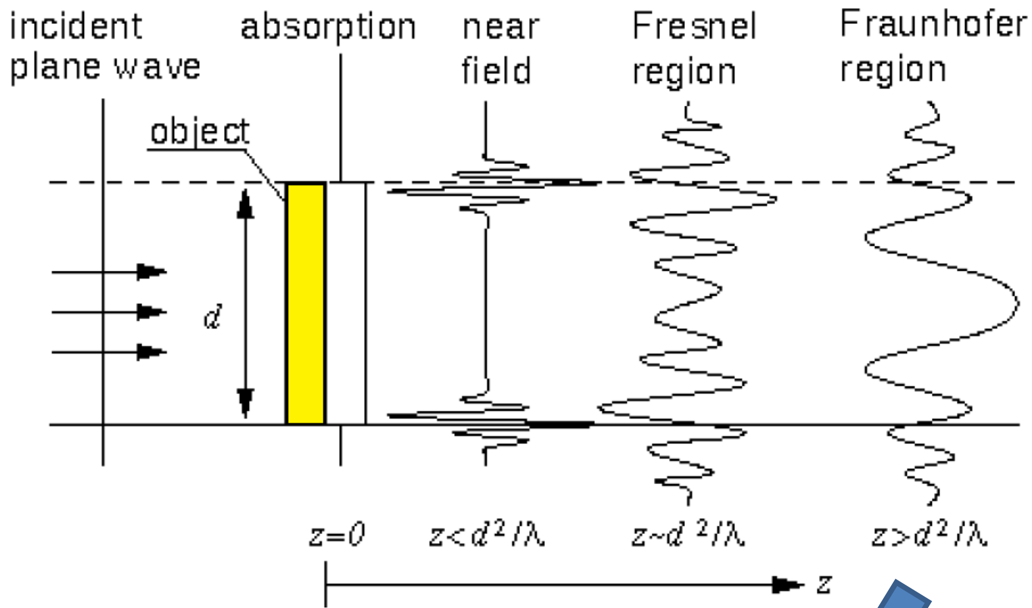




Diffraction

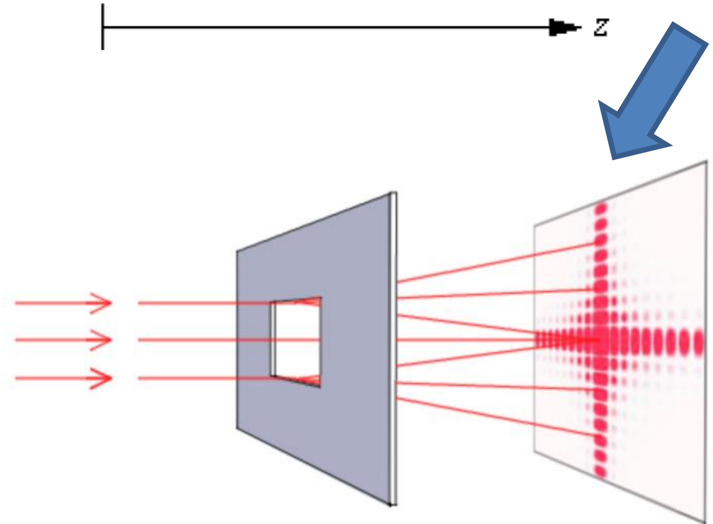
Diffraction from *2-D aperture in x,y plane*

Diffraction is classified as Fresnel and Fraunhofer diffraction.



In order to Fresnel diffraction, the observation plane and aperture are located close together.

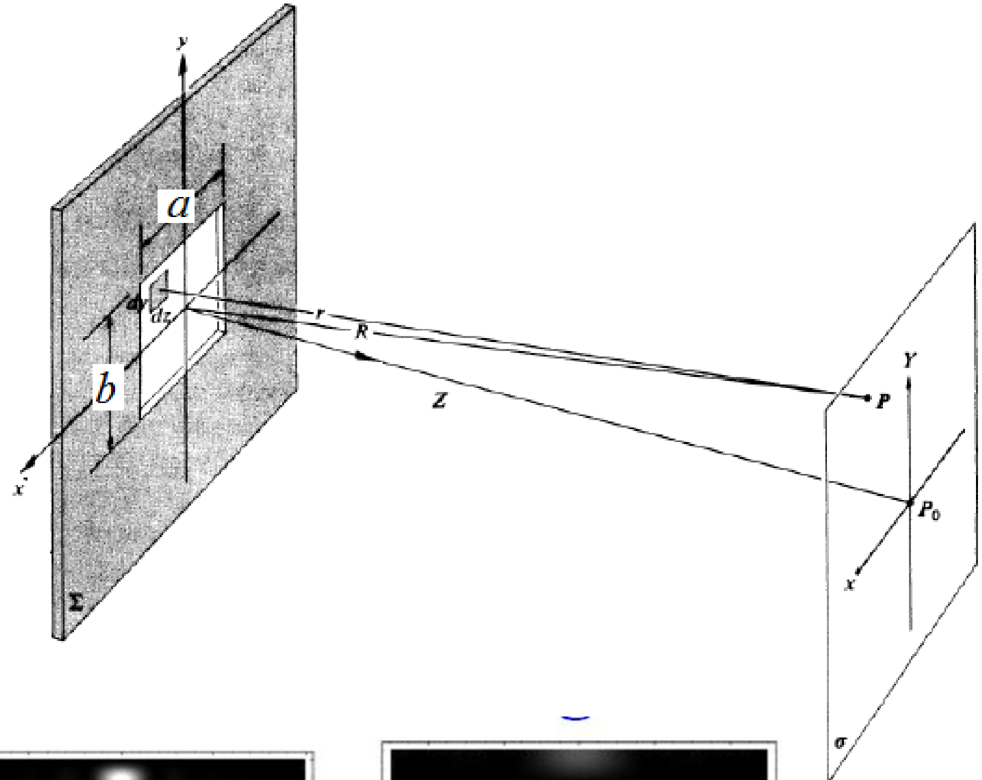
In the Fraunhofer diffraction, the distance between the observation plane and the aperture is too high. Therefore, this diffraction is called as *far field diffraction*.



Diffraction

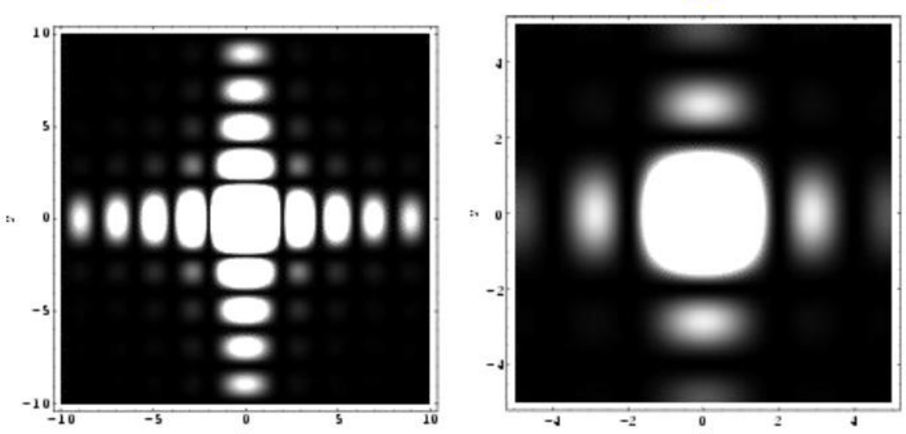
Fraunhofer diffraction

$$I = I_0 \left(\frac{\sin \alpha'}{\alpha'} \right)^2 \left(\frac{\sin \beta'}{\beta'} \right)^2$$



$$\alpha' = \frac{1}{2} kbx/R$$

$$\beta' = \frac{1}{2} kay/R$$



Diffraction

Fraunhofer diffraction

EXERCISE

Fraunhofer Diffraction from a Rectangular Aperture. Verify that the Fraunhofer diffraction pattern from a rectangular aperture, of height and width D_x and D_y respectively, observed at a distance d is

$$I(x, y) = I_o \operatorname{sinc}^2 \frac{D_x x}{\lambda d} \operatorname{sinc}^2 \frac{D_y y}{\lambda d},$$

Here, $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$

Consider a rectangular aperture with a D_x=2μ and D_y=5μ. Calculate the diffraction pattern on x-y plain using the Matlab programming and sketch the diffraction pattern.