Optoelectronics-I

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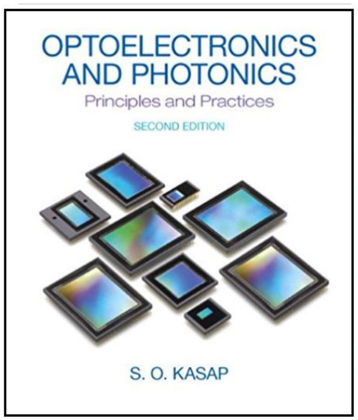
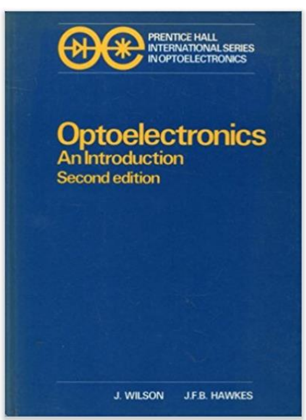
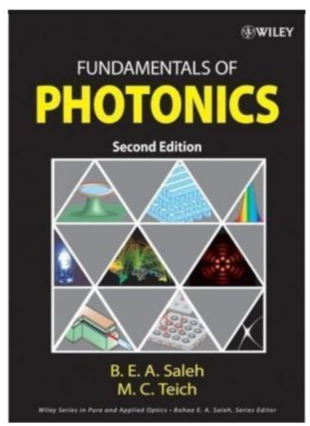
Lecture Notes - 2018

Recommended books







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Tutorial-3

Objectives

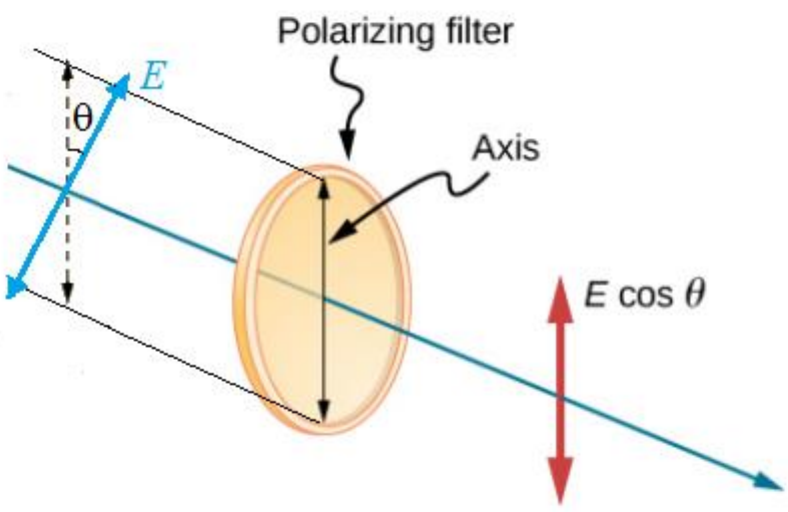
When you finish this lesson you will be able to:

- ✓ Q1. Polarization
- ✓ Q2. Reflection
- ✓ Q3. Anti Reflection Coating:
- ✓ Q4. Interference
- ✓ Q5. Thin film Interference:
- ✓ Q6. Planar Mirror Resonator
- ✓ Q7. Ring Resonator
- ✓ Q8. Photometry

Polarization:

Q1. What angle is needed between the direction of polarized light and the axis of a polarizing filter to reduce its intensity by 90.0% ?

Solution:



θ is the angle between the direction of polarized light and the axis of a polarizing filter.

$$E_{output} = E\cos(\theta)$$

$$I \propto |\vec{E}|^2$$

$$I_{input} = I \propto |\vec{E}|^2$$

$$I_{output} = I \propto |\vec{E}|^2 \cos^2(\theta)$$

$$\frac{I_{output}}{I_{input}} = \cos^2(\theta) = 1 - 0.9$$

$$\cos^2(\theta) = 0.1$$

$$\theta = 71.57°$$

Reflection:

Q2. Consider three dielectric media with flat and parallel boundaries with refractive indices n_1, n_2, and n_3. Show that for normal incidence the reflection coefficient between layers 1 and 2 is the same as that between layers 2 and 3 if $n2 = \sqrt{n_1 n_3}$. What is the significance of this?

Solution:

For light traveling in medium 1 when incident on the 1-2 interface at normal incidence the reflection coefficient is,

$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{n_1 - \sqrt{n_1 n_3}}{n_1 + \sqrt{n_1 n_3}} = \frac{1 - \sqrt{\dfrac{n_3}{n_1}}}{1 + \sqrt{\dfrac{n_3}{n_1}}}$$

For light traveling in medium 2 when incident on the 2-3 interface at normal incidence the reflection coefficient is,

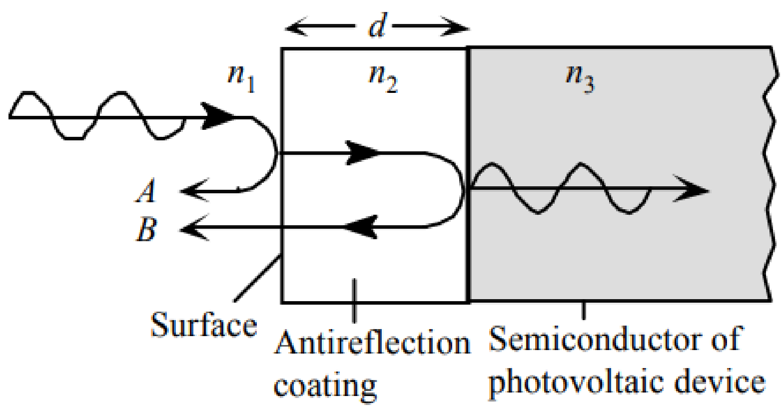
$$r_{23} = \frac{n_2 - n_3}{n_2 + n_3} = \frac{\sqrt{n_1 n_3} - n_3}{\sqrt{n_1 n_3} + n_3} = \frac{\sqrt{\dfrac{n_1}{n_3}} - 1}{\sqrt{\dfrac{n_1}{n_3}} + 1} = \frac{1 - \sqrt{\dfrac{n_3}{n_1}}}{1 + \sqrt{\dfrac{n_3}{n_1}}}$$

$$r_{23} = r_{12}$$

Reflection:

Solution: **Significance**

For an efficient antireflection effect, the waves A and B should interfere destructively and to obtain a good degree of destructive interference between waves A and B, the two amplitudes must be comparable. This can be achieved by $r_{12} = r_{23}$.



Thus, the layer 2 can act as an antireflection coating if its index if $n_2 = \sqrt{n_1 n_3}$.

Anti Reflection Coating:

Q3. Consider that light propagates at normal incidence from air, $n_1 = 1$, to semiconductor likes a photocell with a refractive index of $n_3=3.5$ as given in Fig-1(a).

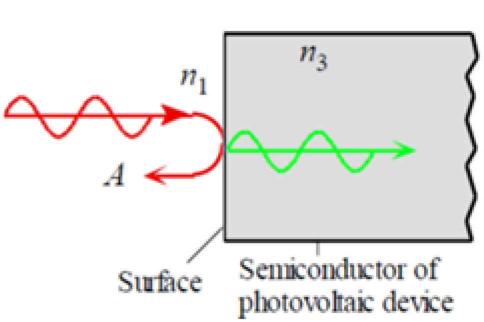


Fig-1(a)

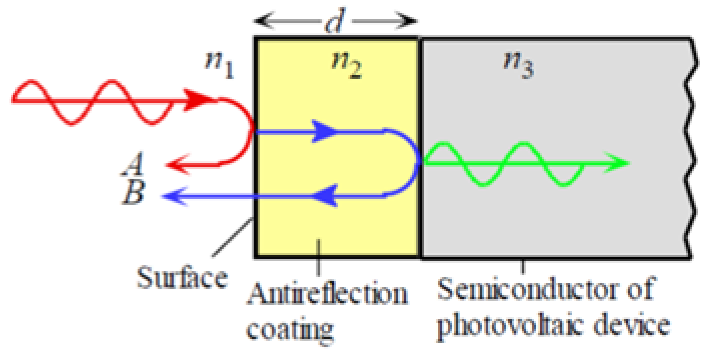


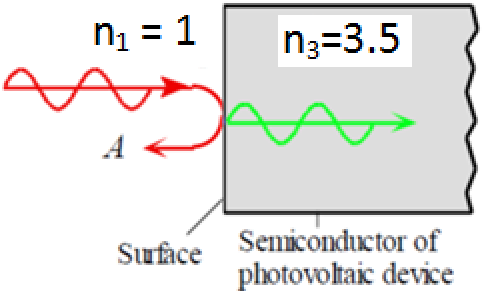
Fig-1(b)

a) What is the reflection coefficient (r) and the reflectance (R) with respect to the incident beam?
b) When the semiconductor material is coated with thin layer of electric material such as Si_3N_4 (silicon nitride) that has an intermediate refractive index of $n_2=1.9$ as given in Fig-1(b), the loss can be reduced. In this case, calculate the reflection coefficient (r) and the reflectance (R) and discuss the loss.
c) In this system, how can you explain the phase matching relation to thickness of antireflective layer?

Anti Reflection Coating:

Solution:

a)

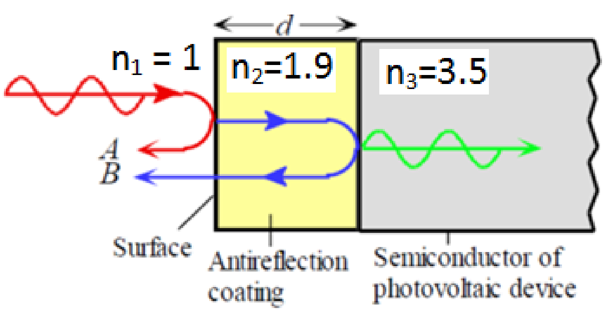


the reflectance of the silicon surface in air

$$R = \left(\frac{1-3.5}{1+3.5}\right)^2 = 0.309 \text{ or } 30.9\%$$

This means there is a 30.9% loss in efficiency even before the light enters the silicon solar cell

b)



$$R_A = \left(\frac{1-1.9}{1+1.9}\right)^2 = 0.0963 \text{ or } 9.63\%$$

$$R_B = \left(\frac{1.9-3.5}{1.9+3.5}\right)^2 = 0.0878 \text{ or } 8.78\%$$

If one coats the solar cell with a thin layer of electric material such as Si_3N_4 (silicon nitride) that has an intermediate refractive index of 1.9, then we can reduce the loss

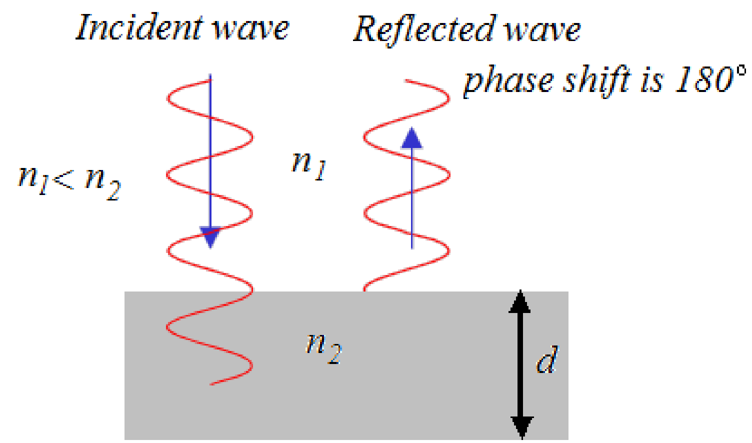
The total loss %=[0.0963+(1-0.0963)*0.0878]*100 =17.56%

Anti Reflection Coating:

Solution:

c) For a good degree of destructive interference, the amplitudes of the A and B waves must be comparable. Thus, we need $n_2 = \sqrt{n_1 n_3} = 1.87$ which is close to that of Si_3N_4 (n_2=1.9).

On the other hand, the reflected light off of all normal interfaces is 180° out of phase with incident light.



Because each interface is at a transition from a lower to a higher refractive index, there is 180° phase shift at each reflection (for 1 and 2). So they undergo the ***same phase shift.*** **(1<1.9<3.5)**

We need to do is calculate the phase difference between rays 1 and 2 due to the path length difference.

The phase difference in the two waves (A and B) is equivalent to $k_2(2d)$ where $k_2 = n_2 k = 2\pi n_2 /\lambda$. So**, Phase matching occurs when $k_2(2d) = m\pi$**

$$\left(\frac{2\pi n_2}{\lambda}\right)2d = m\pi \rightarrow d = m\left(\frac{\lambda}{4n_2}\right)$$

Interference:

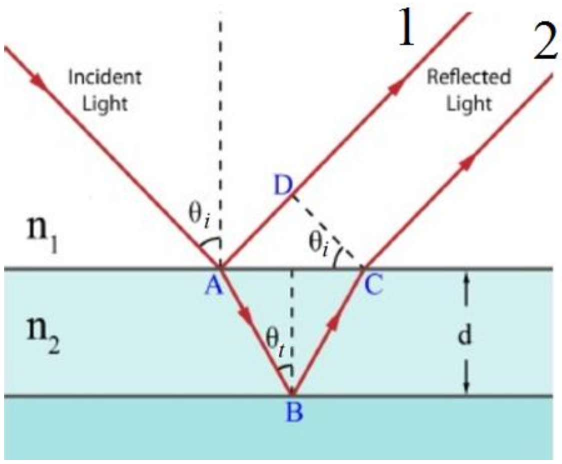
Q4. A thin film of a material (e.g. liquid oil) is floating on water (n = 1.33). When the material has a refractive index of n = 1.20, the film looks bright in reflected light as its thickness approaches zero. But when the material has a refractive index of n = 1.45, the film looks black in reflected light as the thickness approaches zero. Explain these observations in terms of constructive and destructive interference and the phase changes that occur when light waves undergo reflection.

Solution:

If the refractive index of the film is greater than that of the water below, the phase shift of the ray traveling through the film and reflecting off the interface between the film and the water is zero.
If the refractive index of the film is less than that of water the phase shift of the ray traveling through the film and reflecting off the interface between the film and the water is π. Both reflected rays now have the same phase. As the film thickness approaches zero, they are in phase and the material will look bright.
The colorful patterns that you see when light reflects off a compact disk are produced by thin film interference

Thin film Interference:

Q5. A thin soap bubble film has thickness t = 250 nm and has air above and below the film. The index of refraction of the film is n = 1.36. White light shines onto the film from a nearly perpendicular incidence.



a) What is the phase change upon reflection from the top surface (ray 1)?
b) What is the phase change upon reflection from the bottom surface (ray 2)?
c) What equation gives the path length difference between rays 1 and 2 for maximum reflection?
d) Which colours appear strong in the reflected light?

Solution: Perpendicular incidence → $\theta t \approx 0$

a) There will be a 180º phase change on reflection from the top surface (ray-1) because of n1<n2, r1=(n1-n2)/(n1+n2)<0

b) no phase change in reflected ray-2 because of r2>0

Thin film Interference:

Solution:

c) Path length difference: $\Delta L = 2n_2 d\cos\theta_t$

Ray-1 already has a 180 degree phase change. If $k\Delta L$ is equal to 180°, the phase difference between Ray-1 and Ray2 is reduced to zero, so constructive interference occurs (maximum reflection)

$k\Delta L = (2m+1)\pi$ m=0,1,2…

$(2\pi/\lambda)\, 2n_2 d\cos\theta_t = (2\pi/\lambda)\, 2n_2 d = (2m+1)\pi$ ⇨ $2d = (\lambda/n_2)(m+1/2)$

d) $\lambda = 4dn_2/(2m+1)$ m=0,1,2,3…

λ=4x250x1.36 =1360nm for m=0 (infrared region is invisible)

λ=4x250x1.36/3 =453nm for m=1 (blue-violet).

λ=4x250x1.36/5 =272nm for m=1 (Ultra-violet region is invisible).

why do we see different colors in the soap bubble balloon?

Ring Mirror Resonators

Q6. Consider a 50 μm long resonator filled with air (n=1).
a) Calculate the mode frequencies corresponding first five modes.
b) Calculate the resonance wavelengths for these modes.
c) Calculate the free spectral range.

Solution:

a) $v_m = m\frac{c}{2L}$

$v_1 = 6x10^{12}$ Hz for m=1.

$v_2 = 12x10^{12}$ Hz for m=2.

$v_3 = 18x10^{12}$ Hz for m=3.

$v_4 = 24x10^{12}$ Hz for m=4.

$v_5 = 3x10^{13}$ Hz for m=5.

b) $\lambda_m = \frac{c}{v_m}$

$\lambda_1 = 50$ μm for m=1.

$\lambda_2 = 25$ μm for m=2.

$\lambda_3 = 16.67$ μm for m=3.

$\lambda_4 = 12.5$ μm for m=2.

$\lambda_5 = 10$ μm for m=5.

c)

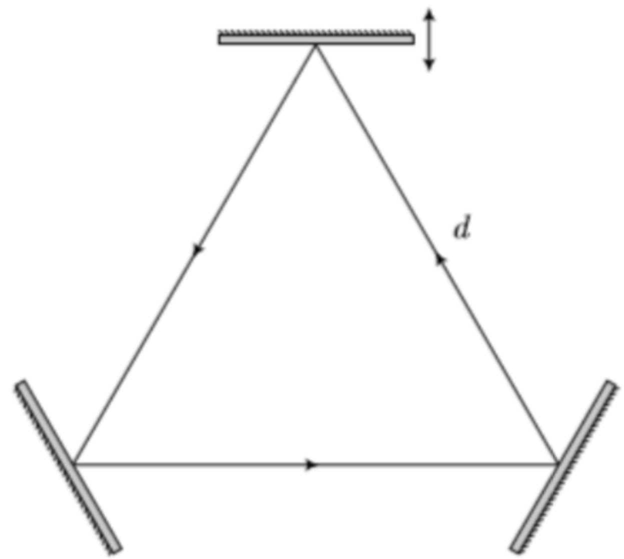
$v_f = \frac{c}{2L}$

$v_f = 6x10^{12}$ Hz

Ring Resonator

Q7. Consider a ring resonator consisting of three planar mirrors, arranged at the vertices of an equilateral triangle of side length d as shown in the diagram (a) What is the free spectral range of the cavity?

(b) Suppose that the top mirror is moved vertically as shown in the diagram. If the cavity is resonant with light of wavelength λ, by how much should the mirror move to go through one complete interference fringe?



Solution:

$$\Delta v_m = v_{m+1} - v_m = v_f$$

$$v_f = \frac{c}{2L}$$ for two parallel mirror

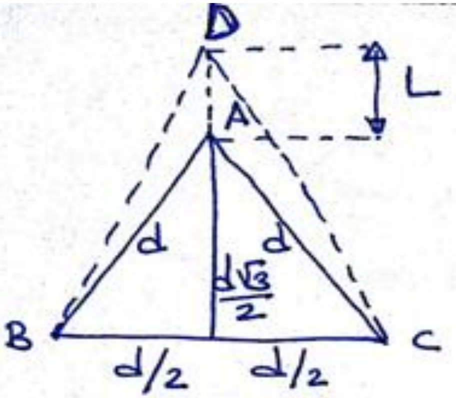
$$v_f = \frac{c}{3d}$$ for the ring resonator shown in the figure

in terms of wavelength

$$\Delta v = \frac{c\Delta\lambda}{\lambda^2} \Rightarrow \Delta\lambda = \frac{c}{3d}\frac{\lambda^2}{c} = \frac{\lambda^2}{3d}$$

Ring Resonator

b)



Assume that the top mirror is displaced by an amount L in the vertical direction.

Due to displacement of mirror, DBC is the new geometry.

$$CD = \sqrt{\left(\frac{d}{2}\right)^2 + \left(L + \frac{d\sqrt{3}}{2}\right)^2} \Rightarrow CD = \sqrt{L^2 + d^2 + dL\sqrt{3}}$$

Change in cavity length is $\Delta L_{cav} = 2\sqrt{L^2+d^2+dL\sqrt{3}} + d - 3d$

$$\Delta L_{cav} = 2\left[\sqrt{L^2+d^2+dL\sqrt{3}} - d\right]$$

For mirror to go through 1 complete fringe shift

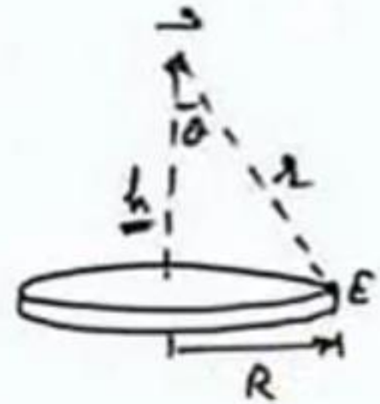
$$\Delta\lambda = \lambda \Rightarrow \lambda = \Delta L_{cav}$$

$$\frac{\lambda}{2} = \sqrt{L^2+d^2+dL\sqrt{3}} - d = d\sqrt{\left(\frac{L}{d}\right)^2 + 1 + \frac{L}{d}\sqrt{3}} - d$$

Assuming $L \ll d \Rightarrow \frac{\lambda}{2d} + 1 = \sqrt{1 + \frac{L\sqrt{3}}{d}}$

$$\Rightarrow \boxed{L = \frac{\lambda^2 + 4d\lambda}{4\sqrt{3}\,d}}$$

Q8. A point source of light is placed at a height h above the centre of a horizontal circular disc of radius R. What must be the value of h so that illumination is maximum on edge of disc.



Sol^n: If L be the luminous intensity of source the Illuminance at point E will be-

$$I = \frac{L\cos\theta}{r^2} = \frac{L}{(R^2+h^2)} \times \frac{h}{\sqrt{R^2+h^2}}$$

$$r = \sqrt{R^2+h^2}$$

$$I_E = \frac{Lh}{(R^2+h^2)^{3/2}}$$

for I_E to be max $\frac{dI_E}{dh} = 0$

$$\frac{(h^2+R^2)^{3/2} - \frac{3}{2}(R^2+h^2)^{1/2}\cdot 2h\cdot h}{(R^2+h^2)^3} = 0$$

$$h^2 + R^2 - 3h^2 = 0 \Rightarrow h = R/\sqrt{2} \text{ Ans.}$$

https://www.youtube.com/watch?v=09HaJjXUUtw