WEEK 4: THEORY OF HORIZONTAL AXIS WIND TURBINES (HAWTs)

- **Axial Momentum Theory**

➢ The axial momentum theory applies the conservation laws on a 1D stream tube [3].

➢ The rotor of the wind turbine is considered as a uniform actuator disc that introduces a pressure discontinuity. The reason that an actuator disc is considered as a rotor with a infinite number of blades is that a uniform flow is assumed, which is not possible with a finite number of blades.The situation can be sketched as presented in Figure 2.

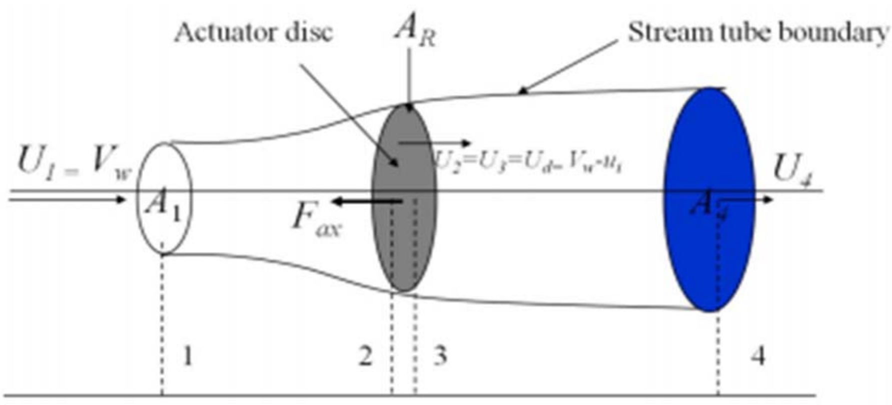

Figure 2: Stream tube with indicated the velocities [4]

The axial momentum theory is valid for the following assumptions:

- Steady, incompressible and 1D flow
- Uniform homogeneous and non turbulent flow
- No frictional drag
- No heat transfer
- The rotor disc can be considered as a rotor with an infinite number of blades

- Applying the conservation laws to the stream tube sketched in Figure 2 results in the following equations from respectively the conservation of mass, momentum and energy

$$\dot{m} = \rho U_1 A_1 = \rho U_d A_d = \rho U_4 A_4 \qquad (1)$$

$$T = \dot{m}(U_4 - U_1) \qquad (2)$$

$$P = TU_d = \frac{1}{2}\dot{m}({U_1}^2 - {U_4}^2) \qquad (3)$$

Where $\dot{m}$ is the mass flux [kg/s], ρ the density in [kg/m^3], P the power extracted by the actuator disc [W], T the thrust force in axial direction in [N], A_1, A_4, A_d the sectional surface area of the stream tube in [m^2] as displayed in Figure 1 and U_1, U_4, U_d the velocities indicated in Figure 2 in [m/s]. Combining Equation 2 and 3 leads to a relation for the velocity U_d at the disc:

$$U_d = \frac{P}{T} = \frac{1}{2}(U_1 + U_4) \qquad (4)$$

- Defining the axial induction factor as the fractional decrease of the wind velocity at the rotor plane with respect to the free stream wind velocity:

$$a = \frac{U_\infty + U_d}{U_\infty} \qquad (5)$$

Where U_α is the free stream velocity which is equal to the velocity U_1 in Figure 2. Rewriting this will result in an expression for the velocity at the rotor plane as a function of the induction factor.

$$U_d = U_\infty(1 - a) \qquad (6)$$

- Substituting Equation 6 into Equation 4 leads to an expression for the downstream velocity U_4.

$$U_4 = U_\infty(1 - 2a) \qquad (7)$$

- Since U_1 and U_4 can both be described by a function of U_α and a the axial thrust can be expressed as a function of these variables, by using Equation 2

$$T = \dot{m}(U_4 - U_1) = \rho {U_\infty}^2 2a(1 - a)A_r \quad (8)$$

- where A_r the surface area of the rotor plane. In order to express the axial thrust force as a function of the radius the rotor surface can be divided into annular rings as is shown in Figure 3, where dF_{ax} represents the thrust and V_w the free stream velocity. The axial thrust force for each ring can then be determined by:

$$dT(r) = \rho {U_\infty}^2 4a(1 - a)\pi r dr \quad (9)$$

Where the surface area of the annular ring is determined by $A_r = 2\pi r dr$.

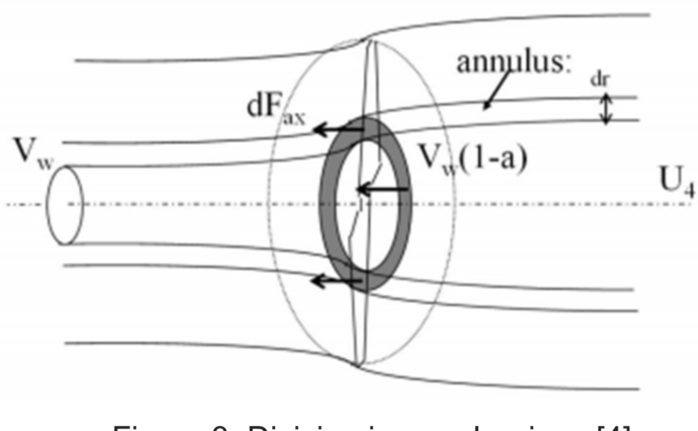

Figure 3: Division in annular rings [4]

References:

[3] BEM theory and CFD for Wind Turbine Aerodynamics Internship Report. JUR MOURITS. 13-01-2014 University of Twente & University of Liverpool. https://essay.utwente.nl/69276/1/Report_Internship_Jur_Mourits_final.pdf

[4] J. G. Schepers. Engineering Models in Wind Energy Aerodynamics. PhD thesis, Delft University of Technology, The Netherlands, 2012.