WEEK 5: THEORY OF HORIZONTAL AXIS WIND TURBINES (HAWTs)

- **Blade Element Momentum (BEM) Theory:**

➢ The blade element theory evaluates the aerodynamic forces on each section of the blade as a function of the geometric properties and the inflow angle. When the inflow angle is calculated the aerodynamic forces can be obtained from corresponding data of the airfoil section. The Blade Element Momentum theory is based on the following assumptions:

- The flow is steady, incompressible and 2D
- The flow is uniform, homogeneous and non turbulent
- There is no aerodynamic interaction between the elements (so no flow in radial direction)
- The forces of the blades are only determined by lift and drag characteristics
- Free-stream flow is perpendicular to the plane of rotation (so no yaw)
- The blades are assumed to be rigid.

➢ For each section along the radius the angles and velocities can be defined as shown in Figure 4.

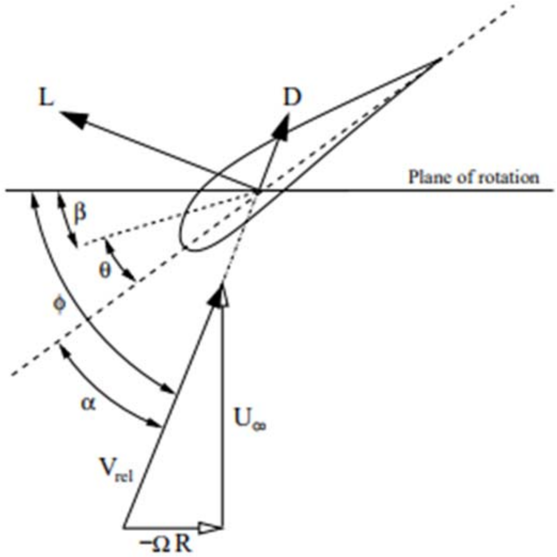

Figure 4: Definition of the different angles [3]

- The geometric angle is defined as the pitch angle increased by the local twist angle of the section. The difference between this angle and the relative inflow angle can be defined as the local angle of incidence:

$$\alpha = \varphi - (\theta + \beta) \tag{10}$$

where α is the sectional angle of incidence [deg], θ the geometric twist angle [deg] and β the pitch angle of the blade [deg].

- When considering the aerodynamic forces with the blade element theory and the aid of Figure 4, the next equations can be derived:

$$dT(r) = B\tfrac{1}{2}\rho U_{rel}^{2}(C_l(cos\varphi) + C_d(sin\varphi))cdr$$
$$dQ(r) = B\tfrac{1}{2}\rho U_{rel}^{2}(C_l(sin\varphi) - C_d(cos\varphi))crdr \tag{11}$$

where dT is the contribution to the thrust in [N] per section dr, dQ the contribution to the torque in [Nm], B the number of blades, φ the relative inflow angle [deg], U_{rel} the relative velocity in [m/s], ρ density in [kg/m^3], C_l the lift coefficient, C_d the drag coefficient, “c” the chord of the local section in [m] and “r” the radial position of the section in [m]. The relative velocity in the above equations depends on the induced axial and rotational velocity. With the aid of Figure 4 and the use of the induced velocities the relative velocity and the relative inflow angle can be defined by the following equations:

$$U_{rel} = \sqrt{(U_{\infty}(1-a))^2 + (\Omega r(1+a'))^2}$$

$$\varphi = arctan\left(\frac{U_{\infty}(1-a)}{\Omega r(1+a')}\right) \qquad (12)$$

where the relative velocity is defined in [m/s], U_{∞}represent the free stream wind velocity in [m/s], a the axial induction factor, φ the relative inflow angle in [deg], a' the angular induction factor, Ω the rotational velocity in [rad/s] and r the radial position at the blade in [m]. Since above equations are dependent on the radius r the relative velocity and inflow angle differs for every section. Using Equation 12 and the definition of the speed ratio makes it possible to derive an equation for the relation between the two induction factors:

$$tan(\varphi) = \frac{1-a}{\lambda_r(1+a')} \qquad (13)$$

$$a = 1 - tan(\varphi)\lambda_r(1+a') \qquad (14)$$

where λ_r is defined as the speed ratio:

$$\lambda_r = \frac{\Omega r}{U_{\infty}} \qquad (15)$$

The definitions for the relative velocity, introduced in Equations 12 and 13, can be combined to:

$$U_{rel} = \frac{U_\infty(1-a)}{sin(\varphi)} \tag{16}$$

The performance of a wind turbine is often expressed in terms of power. To eliminate the direct influence of the wind speed, the dimensionless power coefficient is computed, which is the ratio between the extracted power and the power available in the flow. The power coefficient can be defined as:

$$C_p = \frac{P}{\frac{1}{2}\rho U_\infty A} \tag{17}$$

where P is the power extracted by the wind turbine in [W] defined as $P = T\Omega$, A the surface area of the rotation plane of the blades in [m^2] and U_∞ the free stream wind velocity in [m/s]. To see what the contribution of each section dr is to the total power coefficient the sectional power coefficient can be computed by:

$$C_p = \lambda_r{}^2 4a'(r)(1 - a(r)) \tag{18}$$

where Equation 9 is substituted for the torque, $2\pi rdr$ is substituted for A, since this is the surface area of the annular rings and the velocity ratio is replaced by $\lambda_r{}^2$. Equation 18 suggests that a higher speed ratio and a low axial induction factor should result in a higher power coefficient. But since the speed ratio and the induction factors are related to each other is it difficult to see directly how the power coefficient will behave by varying these parameters. But as is known from the theory of Betz, the maximum power is produced for an axial induction factor of 0.333 [3].

References:

[3] BEM theory and CFD for Wind Turbine Aerodynamics Internship Report. JUR MOURITS. 13-01-2014 University of Twente & University of Liverpool. https://essay.utwente.nl/69276/1/Report_Internship_Jur_Mourits_final.pdf

[4] J. G. Schepers. Engineering Models in Wind Energy Aerodynamics. PhD thesis, Delft University of Technology, The Netherlands, 2012.