WEEK 7: Lattice Boltzmann Method (LBM)

- **What is LBM?**

➢ The Lattice Boltzmann Method (LBM) is steadily becoming a relevant method for solving the fluid flow equations.

➢ Its direct connection to the kinetic theory of gases allows for a deeper understanding and more detailed modeling of physical phenomena [5,6]

➢ A particle-based kinetic fluid flow solver based on the Lattice-Boltzmann Method (LBM) is utilized to computationally resolve the flow field and the heat transfer in the computational domain in which classic time consuming fluid-domain meshing is not employed, instead automatically generated lattice which is organized in an Octree structure is employed [7]

➢ There are many approaches to the LBM, and one can indeed think of it as group of methods aimed at solving systems of equations for nonlinear hyperbolic conservation laws. The common characteristic to all the LBM models (and Lattice Gas Automata)[8,9] is their time-stepping model, based on a propagate-collide scheme, on top of a lattice discretization

- **Equations**

The Boltzmann transport equation (mass and momentum equation) can be written in discrete velocities in a spatial environment as follows

$$\frac{\partial f_i}{\partial t} + e_i . \nabla f_i = \Omega_i , i = 1, ..., b,$$

where Ω_i is the collision operator that computes a post-collision state conserving mass and linear momentum. If it is assumed that Ω_i = 0, only a streaming operation is performed. This equation is discretized on the lattice as:

$$f_i(\mathbf{r}+\mathbf{e_i},t+dt)=f_i(\mathbf{r},t)+\Omega_i(f_1,f_2,....,f_b),i=1,....,b,$$

As in the continuum Boltzmann equation, the macroscopic variables can be derived from the statistical moment of the PDFs:

$$\rho=\sum_{i=1}^{b} f_i$$

$$\rho v=\sum_{i=1}^{b} f_i e_i$$

Moment of zero-order corresponds to the macroscopic density, and the moments of first-order provide the momentum in the three directions. Further moments of higher orders can be computed to obtain higher order quantities. Flow density *ρ*, macroscopic velocity, *u* and specific internal energy, *e*, can be found from the distribution function, *f* as follows:

$$\rho(x,t)=\int m\, f(x,v,t)\,dv$$
$$\rho(x,t)u(x,t)=\int mv\, f(x,v,t)\,dv$$
$$\rho(x,t)e(x,t)=\int m\,(v-u)^2/2\, f(x,v,t)\,dv$$

where *m* is the particle mass.

During the deformation of compressor flow at very high Reynolds numbers, very high normal stresses occur at the wall surfaces (rotor and chamber surfaces), which in turn result in intense turbulence fluctuations to be computationally modelled by a turbulence model. The approach used for turbulence modelling here is the large eddy simulation (LES) based wall adapting local eddy (WALE) viscosity model which provides a consistent local eddy-viscosity and the near-wall bahaviour. The actual implementation is formulated as follows:

$$\upsilon_{turbulent} = \Delta^2 \left| S \right|;$$
$$\Delta = C_s Vol^{1/3};$$
$$S = \sqrt{2 S_{\alpha\beta} S_{\alpha\beta}}$$

where Δ is the filter scale, *S* is the strain rate tensor of teh resolved scales and C_s is the Smagorinsky constant with a default value of 0.12.

References:

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[7] David M. Holman, Ruddy M. Brionnaud, and Zaki Abiza, Solution to industry benchmark problems with the lattice-Boltzmann code XFlow, ECCOMAS 2012

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