WEEK 12: MATHEMATICAL TERMS

MATHEMATICAL TERMS:

- **Derivatives:**

➢ Full $\dfrac{du}{dx}$

- The derivative of u w.r.t. x, where $u = u\,(x)$

➢ Partial $\dfrac{\partial u}{\partial x}$

- The derivative of u w.r.t. x, where $u = u\,(x, y, z, t)$

➢ Substantial $\dfrac{DN}{dt} = \dfrac{\partial N}{\partial x} + \vec{u}.\nabla N$

At this point it is possible to define an extensive property, N in terms of its corresponding intensive property, η as below:

$$N = \int_{m_s} \eta dm$$

where m_s is the mass of the system.

Since $dm = \rho d\forall$, then

$$N = \int_{\forall_s} \eta \rho d\forall$$

with $\forall_s$ being the volume of the system.

- **Vector differential operator**

➢ Del $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$

➢ The gradient: Consider $\xi(x,y,z,t)$ a scalar field

The gradient of ξ is defined as:

$$\nabla\xi = \frac{\partial\xi}{\partial x}i + \frac{\partial\xi}{\partial y}j + \frac{\partial\xi}{\partial z}k$$

which gives a vector field.

➢ The divergence: Consider $\vec{V}(x,y,z,t) = Ui + Vj + Wk$ a vector field

The divergence of $\vec{V}$ is therefore:

$$\nabla.\vec{V} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}$$

➢ The curl: The curl of $\vec{V}$ is

$$\nabla\, x \overleftarrow{V} = \left(\frac{\partial W}{\partial y} - \frac{\partial V}{\partial z}\right)i + \left(\frac{\partial U}{\partial z} - \frac{\partial W}{\partial x}\right)j + \left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right)k$$

- **Definitions - Rectangular coordinates:**

$$\nabla f = \frac{\partial f}{\partial x}i + \frac{\partial f}{\partial y}j + \frac{\partial f}{\partial z}k$$

$$\nabla.\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)i + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)j + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)k$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \mathbf{A} = \nabla^2 A_x i + \nabla^2 A_y j + \nabla^2 A_z k$$

- **Identities:**

➢ The curl: The curl of $\vec{V}$ is

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

$$\begin{aligned}(\mathbf{A} \cdot \nabla)\mathbf{B} = & \left[A_x \frac{\partial B_x}{\partial x} + A_y \frac{\partial B_x}{\partial y} + A_z \frac{\partial B_x}{\partial z}\right]i \\ & + \left[A_x \frac{\partial B_y}{\partial x} + A_y \frac{\partial B_y}{\partial y} + A_z \frac{\partial B_y}{\partial z}\right]j \\ & + \left[A_x \frac{\partial B_z}{\partial x} + A_y \frac{\partial B_z}{\partial y} + A_z \frac{\partial B_z}{\partial z}\right]k\end{aligned}$$

$$\int_\tau \nabla f \, d\tau = \int_S f \, \mathbf{da}$$

$$\int_\tau (\nabla \times \mathbf{A}) \, d\tau = -\int_S \mathbf{A} \times \mathbf{da}$$

where S is the surface which bounds volume τ