ENE 505 – Applied Computational Fluid Dynamics in Renewable Energy Technologies

WEEK 4: GOVERNING EQUATIONS

GOVERNING EQUATIONS:

- **The governing equations include laws of physics as:**

  - Conservation of mass
  - Conservation of momentum: Newton's second law
  - Conservation of energy: The first law of thermodynamics

- **The macroscopic properties:**

  - Velocity, $\vec{V}$
  - Pressure, p
  - Density, ρ
  - Temperature, T
  - Energy, E

- **The mass balance:**

The rate of increase in a infinitesimal fluid element equals to the new rate of flow of mass within this fluid element

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

In vector notation.

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{V}) = 0$$

For incompressible fluid, $\partial \rho / \partial t = 0$.

The equation becomes

$$div(\rho \vec{V}) = 0$$

Alternative way to write this equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

and

$$\frac{\partial u_i}{\partial x_i}=0$$

For a fluid element for an arbitrary conserved property, ϕ:

$$\frac{\partial(\rho\varphi)}{\partial x_i}+div\left(\rho\varphi\vec{V}\right)=0$$

- **The momentum conservation:**

  - Newton's second law: rate of change of momentum equals sum of forces.

  - Rate of increase of *x*-, *y*-, and *z*-momentum:

$$\rho\frac{Du}{Dt} \qquad \rho\frac{Dv}{Dt} \qquad \rho\frac{Dw}{Dt}$$

  - Forces on fluid particles are classified in two groups:

  - Surface forces such as pressure and viscous forces.

  - Body forces, which act on a volume, such as gravity, centrifugal, Coriolis, and electromagnetic forces.

  - The rate of change of x-momentum for a fluid particle D*u*/D*t* equal to:

$$\rho\frac{Du}{Dt}=\frac{\partial(-p+\tau_{xx})}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}+S_{Mx}$$

  - The rate of change of y-momentum for a fluid particle D*v*/D*t* equal to:

$$\rho \frac{Dv}{Dt} = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial(-p+\tau_{yy})}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + S_{My}$$

- The rate of change of y-momentum for a fluid particle D*w*/D*t* equal to:

$$\rho \frac{Dw}{Dt} = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial(-p+\tau_{zz})}{\partial z} + S_{Mz}$$

- **The energy conservation:**

  - The total derivative for the energy in a fluid particle equal to the derived work and energy flux terms, results in the following energy equation

$$\rho \frac{DE}{Dt} = -\,div(p\vec{V}) + \left[\frac{\partial(u\tau_{xx})}{\partial x} + \frac{\partial(u\tau_{yx})}{\partial y} + \frac{\partial(u\tau_{zx})}{\partial z} + \frac{\partial(v\tau_{xy})}{\partial x} + \frac{\partial(v\tau_{yy})}{\partial y} + \frac{\partial(v\tau_{zy})}{\partial z} + \frac{\partial(w\tau_{xz})}{\partial x} + \frac{\partial(w\tau_{yz})}{\partial y} + \frac{\partial(u\tau_{zz})}{\partial z} \right] + div(k\ grad\ T) + S_E$$

  - The added a source term S_E that includes sources (potential energy, sources due to heat production from chemical reactions, etc.).

  - The internal energy equation is on the other hand:

$$\rho \frac{Di}{Dt} = -\,p\ div\ \vec{V} + \left[\tau_{xx}\frac{\partial u}{\partial x} + \tau_{yx}\frac{\partial u}{\partial y} + \tau_{zx}\frac{\partial u}{\partial z} + \tau_{xy}\frac{\partial v}{\partial x} + \tau_{yy}\frac{\partial v}{\partial y} + \tau_{zy}\frac{\partial v}{\partial z} + \tau_{xz}\frac{\partial w}{\partial x} + \tau_{yz}\frac{\partial w}{\partial y} + \tau_{zz}\frac{\partial u}{\partial z} \right] + div(k\ grad\ T) + S_i$$

References:

1. Aksel, M.H., 2016, “Notes on Fluids Mechanics”, Vol. 1, METU Publications
2. Versteeg H.K., and W. Malalasekera V., 1995, “Computational Fluid Dynamics: The Finite Volume Method", Longman Scientific & Technical, ISBN 0-582-21884-5