

THEOREMS[1-5]

References:

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For application of **Final value theorem**,
check whether the function final value available

$$t \rightarrow \infty$$

To find the value of function $x(t)$ at time, $t \rightarrow \infty$
in terms of the final value theorem
and Laplace Transform $x(s)$

$$X(s) = \frac{s^3 + 3s^2 + 3s + 1}{s^6 + 5s^5 + 10s^4 + 10s^3 + 5s^2 + s}$$

By applying **final value theorem**

$$\lim_{s \rightarrow 0} sX(s) = \lim_{s \rightarrow 0} \frac{(s+1)^3}{(s+1)^5} = 1$$

The case :

Final Value of the following function is not exist

$$x(s) = \frac{3s + 1}{(s - 8)(s + 1)}$$

The roots:

$$s=8, s=-1$$

$$t \rightarrow \infty$$

$$x(t) \rightarrow \infty$$

In this case, the denominator of the function $x(s)$ have positive roots
“**Final Value Theorem**” is not valid.

Initial Value Theorem

$$X(s) = \frac{s^3 + 3s^2 + 3s + 1}{s^6 + 5s^5 + 10s^4 + 10s^3 + 5s^2 + s}$$

$$\lim_{s \rightarrow \infty} sX(s) = \lim_{s \rightarrow \infty} \frac{(s+1)^3}{(s+1)^5} = 0$$

$$X(s) = \frac{1}{s(s^4 + 4s^3 + 6s^2 + 4s + 1)} \quad x(t)|_{t \rightarrow 0} = ?$$

$$\lim_{t \rightarrow 0} x(t) = \lim_{s \rightarrow \infty} [s * X(s)]$$

$$\lim_{t \rightarrow 0} x(t) = \lim_{s \rightarrow \infty} \left[\frac{s}{s(s^4 + 4s^3 + 6s^2 + 4s + 1)} \right] = \lim_{s \rightarrow \infty} \left[\frac{s}{s(s+1)^4} \right] = \lim_{s \rightarrow \infty} \left[\frac{1}{(s+1)^4} \right] = 0$$

$u(t)$ is the unit step function , $u(t)=1$.

(1):the value of the first step effect is $(1/h)*u(t)$.

(2):the value of the second step effect is $-(1/h)*u(t-h)$.

The laplace transform of a pulse effect: $L\{(1)+(2)\}$

$$\frac{Y(s)}{X(s)} = \frac{1}{5.2s + 1}$$

$$X(t) = 0.7[u(t) - u(t - 10)]$$

$$X(s) = \frac{0.7}{s} - \frac{0.7}{s} e^{-10s} = \frac{0.7}{s} (1 - e^{-10s})$$

$$Y(s) = X(s) \left(\frac{1}{5.2s + 1} \right)$$

$$Y(s) = \frac{0.7}{s} (1 - e^{-10s}) \left(\frac{1}{5.2s + 1} \right)$$

$$Y(s) = 0.7 \left(\frac{1}{s(5.2s + 1)} - \frac{e^{-10s}}{s(5.2s + 1)} \right)$$