# The frequency response[1-5]

## References:


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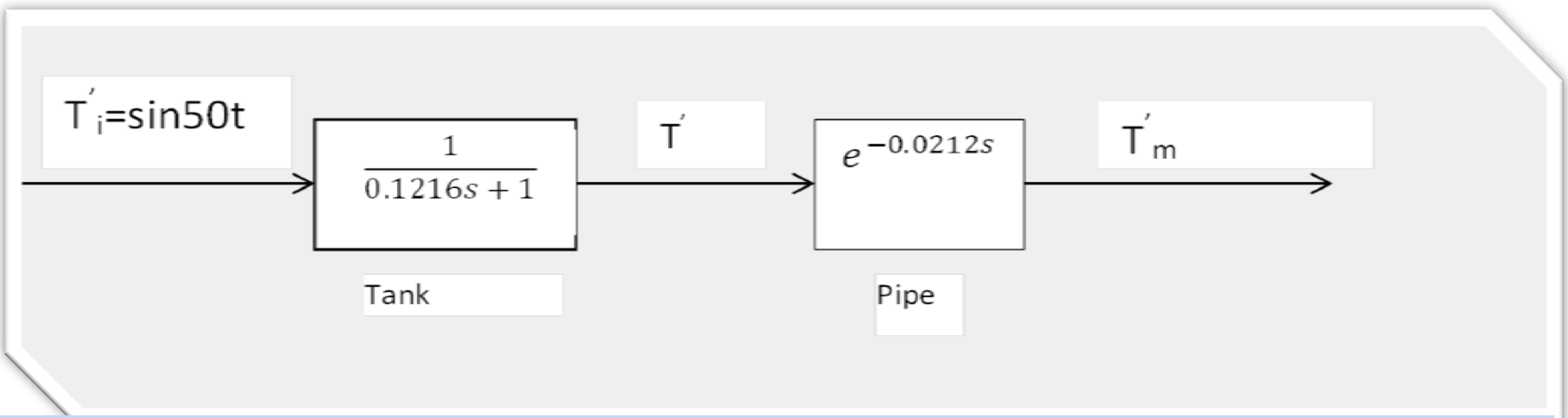
**3.** Seborg D.E., Mellichamp D. A., Edgar T.F, Doyle F.J., 2011, Process Dynamics and Control , John Wiley and Sons ISBN: 978-0-470-64610-6

**4.** Seborg D.E., Mellichamp D. A., Edgar T.F, Doyle F.J., ÇEVİRENLER: Tapan N.A., Erdoğan S. 3. baskıdan çeviriden 1.basım, 2012, Proses Dinamiği ve Kontrolu, Nobel Akademik Yayıncılık ISBN: 978-605-133-298-7

**5.** Alpbaz M.,Hapoğlu H.,Akay B., 2012, Proses Kontrol, Gazi Kitabevi Tic. Ltd. Şti. Ankara, ISBN:978-605-5543-64-8


$$T_i = \sin 50t = A \sin wt \quad (A = 1, w = 50)$$

**For the tank;**

$$G(s) = \frac{1}{\tau_1 s + 1}$$

$$s \to jw$$

$$G(jw) = \frac{1}{\tau_1 jw + 1} = \frac{-jw\tau_1}{\tau_1{}^2 w^2 + 1} + \frac{1}{\tau_1{}^2 w^2 + 1} = \underbrace{\frac{1}{\tau_1{}^2 w^2 + 1}}_{real\ part\ (a)} - j\underbrace{\frac{w\tau_1}{\tau_1{}^2 w^2 + 1}}_{imaginary\ part\ (b)}$$

$$Amplitude\ ratio: AR = \sqrt{a^2 + b^2} = \frac{1}{\sqrt{\tau_1{}^2 w^2 + 1}}$$

$$Phase\ lag: \phi = tan^{-1}\left(\frac{b}{a}\right) = tan^{-1}(-w\tau_1)$$

$$AR_1 = \frac{1}{\sqrt{(50^2)(0.1216^2) + 1}} = \frac{1}{6.16} = 0.162$$

$$\phi_1 = tan^{-1}(-(50)(0.1216)) = -80.66^o \sim -81^o$$

**For the pipe;**

$$G(s) = e^{-s\tau_2}$$

$$s \rightarrow jw$$

$$G(jw) = e^{-jw\tau_2} \quad \left(Polar\ form: Re^{j\phi}\right) \rightarrow \quad R = 1 \quad , \quad \phi = -w\tau_2$$

$$AR_2 = 1$$

$$\phi = -w\tau_2 = -(50)(0.0212) = -1.06\,rad = -60.76 \sim -61^o$$

Overall AR;

$$AR_{overall} = AR_1 * AR_2 = (0.162)(1) = 0.162$$

Overall $\phi$;

$$\phi_{overall} = \phi_1 + \phi_2 = (-81^o) + (-61^o) = -142^o$$

# the frequency response of $T_m$ to $T_i$

$$T_i' = T_i - 80 = \sin 50t$$

$$q_s = wC_p(T_{m_s} - T_{i_s})$$

$$T_{m_s} = 136.9^0 F$$

deviation variable $T_m$'

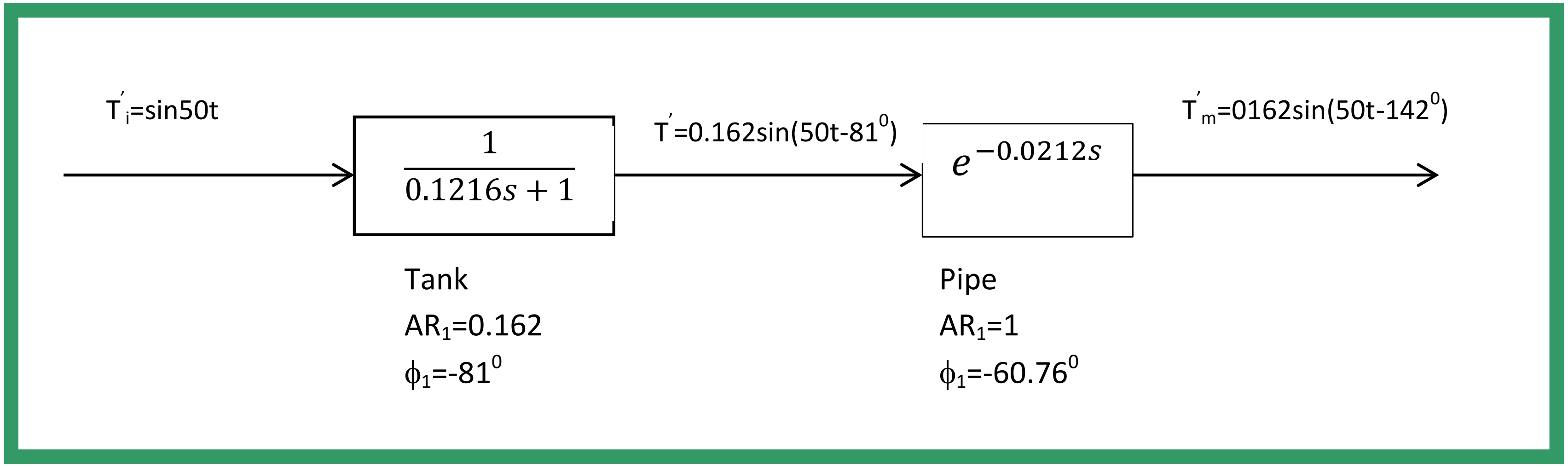
$$T_m' = T_m - 136.9 \qquad T_m = T_m' + 136.9$$

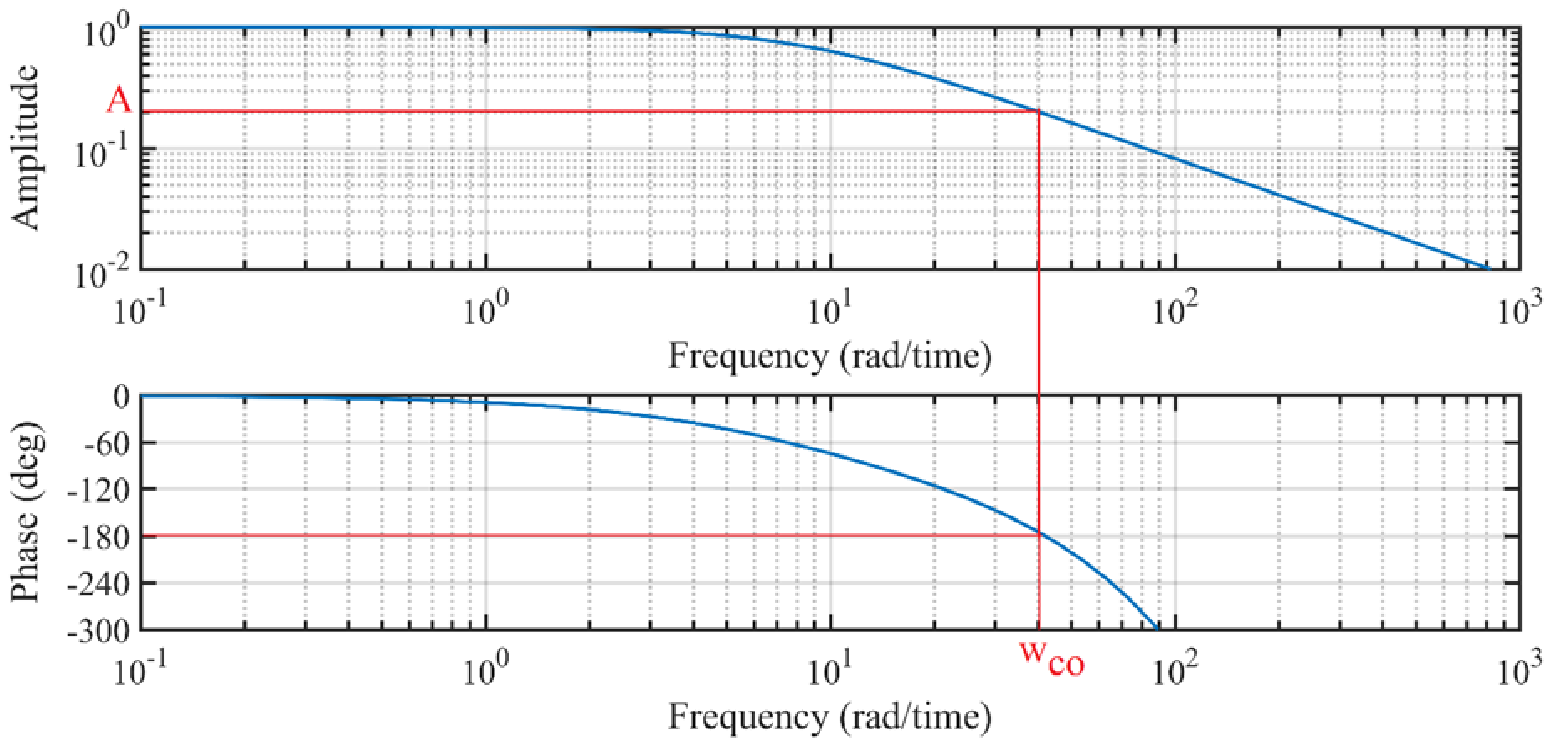
$$T_m' = \frac{A}{\sqrt{\tau^2 w^2 + 1}} \sin(wt + \phi)$$

$$T_m' = A * AR_{overall} * \sin(wt + \phi_{overall})$$

$$T_m' = (1) * (0.162) * \sin(50t - 142^o)$$

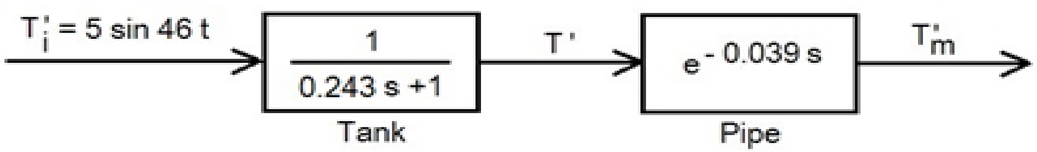
$$T_m = 136.9 + 0.162 * \sin(50t - 142^o)$$


T'i=sin50t
1/(0.1216s + 1)
T'=0.162sin(50t-81⁰)
e^(−0.0212s)
T'm=0162sin(50t-142⁰)
Tank
AR1=0.162
ϕ1=-81⁰
Pipe
AR1=1
ϕ1=-60.76⁰


Amplitude
A
Frequency (rad/time)
Phase (deg)
wco
Frequency (rad/time)

# the two systems are in series

$$\frac{T_m'}{T_i'} = \frac{e^{-\tau_2 s}}{\tau_1 s + 1} = \frac{e^{-0.039s}}{0.243s + 1}$$




For the tank,

$$AR = \frac{1}{\sqrt{\omega^2 \tau_1^{\,2} + 1}} = \frac{1}{\sqrt{(46 * 0.243)^2 + 1}} = \frac{1}{11.223} = 0.089$$

$$Phase\,angle = \tan^{-1}(-\omega\tau_1) = \tan^{-1}\left[(-46)(0.243)\right] = -85^o$$

For the pipe

AR is unity

$$Phase\,angle = -\omega\tau_2 = (-46)(0.039) = -1.794\,rad = -103^o$$

The overall phase lag from **$T_i$' to $T_m$'**

$$\measuredangle \frac{T_m'}{T_i'} = -85 - 103 = -188^o$$

$$T_m = 76.7 + 5(0.089)\sin(46t - 188^o) = 76.7 + 0.445\sin(46t - 188^o)$$

T' = tank temperature – 76.7 °C