

Bode diagrams [1-5]

References:

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Bode diagram for the system having the transfer function $G_p(s) = \frac{1}{s+1}$.

Amplitude ratio and phase angle for first order system:

$$AR = \frac{K_p}{(\tau^2 \omega^2 + 1)^{0.5}} = \frac{1}{(\omega^2 + 1)^{0.5}} \quad \phi = \tan^{-1}(-\tau\omega) = \tan^{-1}(-\omega)$$

Taking the logarithm of amplitude ratio,

$$\log(AR) = \log\left(\frac{1}{(\omega^2 + 1)^{0.5}}\right)$$

Low frequency asymptote;

$$\omega \rightarrow 0, \log(AR) = \log\left(\frac{1}{(\omega^2 + 1)^{0.5}}\right) \rightarrow \log(AR) = 0 \rightarrow AR = 1$$
$$\phi = \tan^{-1}(-1 \times 0) = 0^\circ$$

High frequency asymptote;

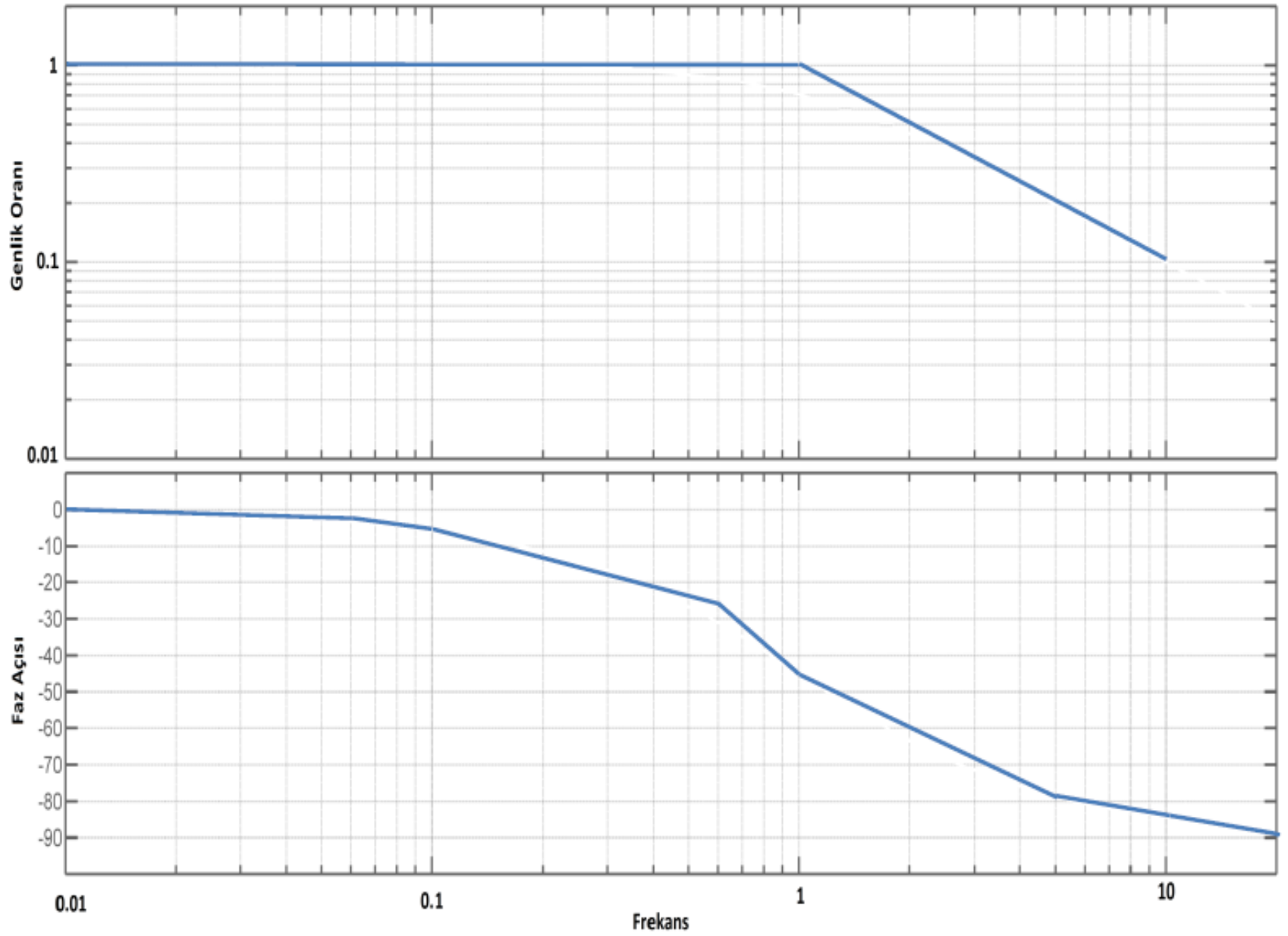
$$\omega \rightarrow \infty, AR = \frac{1}{(\omega^2 + 1)^{0.5}} \rightarrow AR = \frac{1}{\omega}$$

$$\log(AR) = \log\frac{1}{\omega} = -\log(\omega) \rightarrow \text{eğim} = -1$$
$$\phi = \tan^{-1}(-\infty) = -90^\circ$$

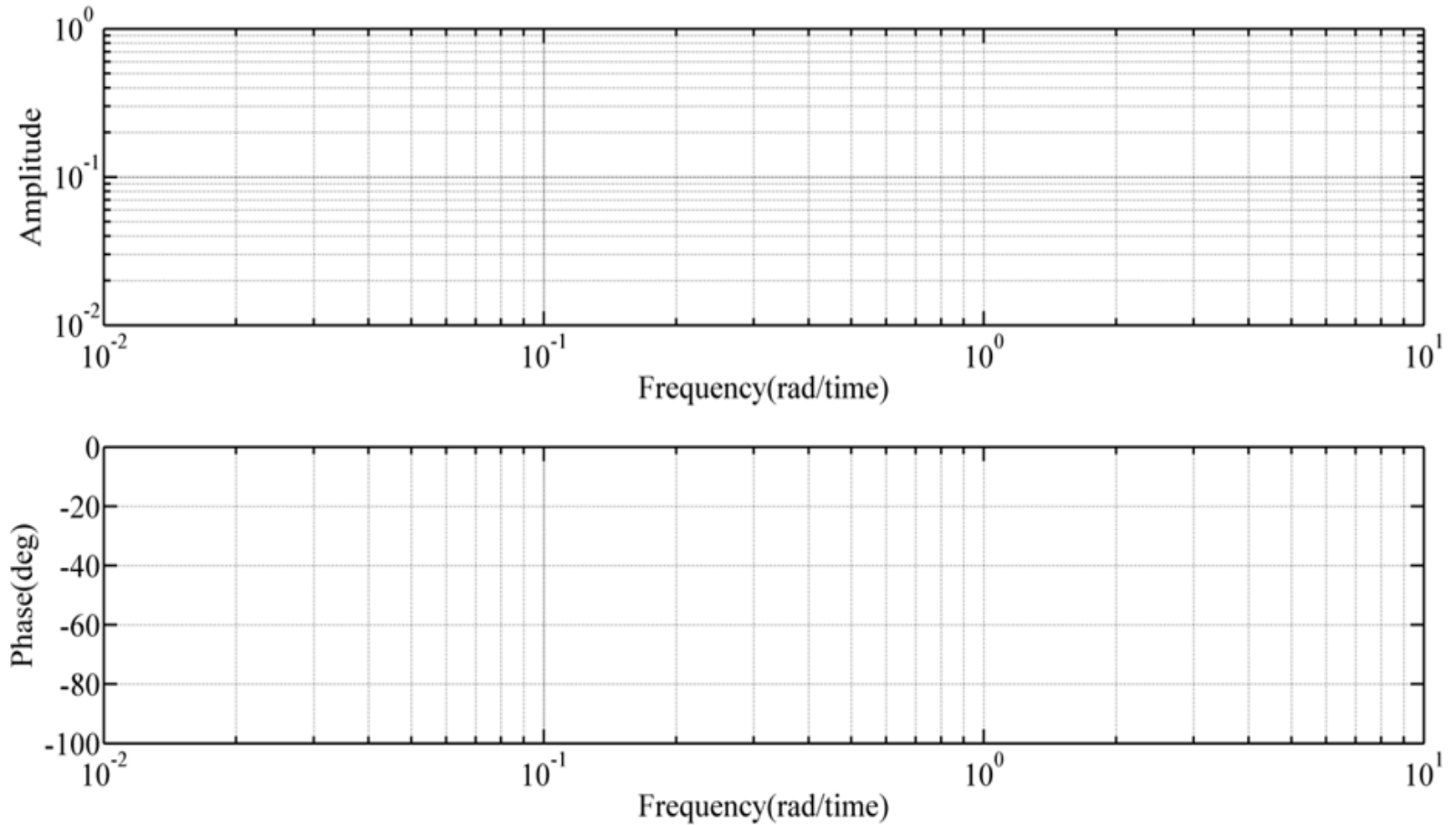
Intercept of two asymptote;

$$w_c = \frac{1}{\tau} = 1$$

Frequency	Phase angle
0	0
0.05	-2.86
0.1	-5.71
0.5	-26.6
1	-45
5	-78.69
10	-84.28



Bode diagram for the system having the transfer function $G_p(s) = \frac{1}{5s + 1}$



Amplitude ratio and phase angle for first order system:

$$AR = \frac{K_p}{\sqrt{(\tau^2 \omega^2 + 1)}} = \frac{1}{\sqrt{(25\omega^2 + 1)}} \quad \phi = \tan^{-1}(-\tau\omega) = \tan^{-1}(-5\omega)$$

Taking the logarithm of amplitude ratio,

$$\log(AR) = \log\left(\frac{1}{\sqrt{(25\omega^2 + 1)}}\right)$$

Low frequency asymptote;

$$\omega \rightarrow 0, \log(AR) = \log\left(\frac{1}{\sqrt{(25\omega^2 + 1)}}\right) \rightarrow \log(AR) = 0 \rightarrow AR = 1$$

$$\phi = \tan^{-1}(-5 \times 0) = 0^\circ$$

High frequency asymptote;

$$\omega \rightarrow \infty, AR = \frac{1}{\sqrt{(25\omega^2 + 1)}} \rightarrow AR = \frac{1}{5\omega}$$

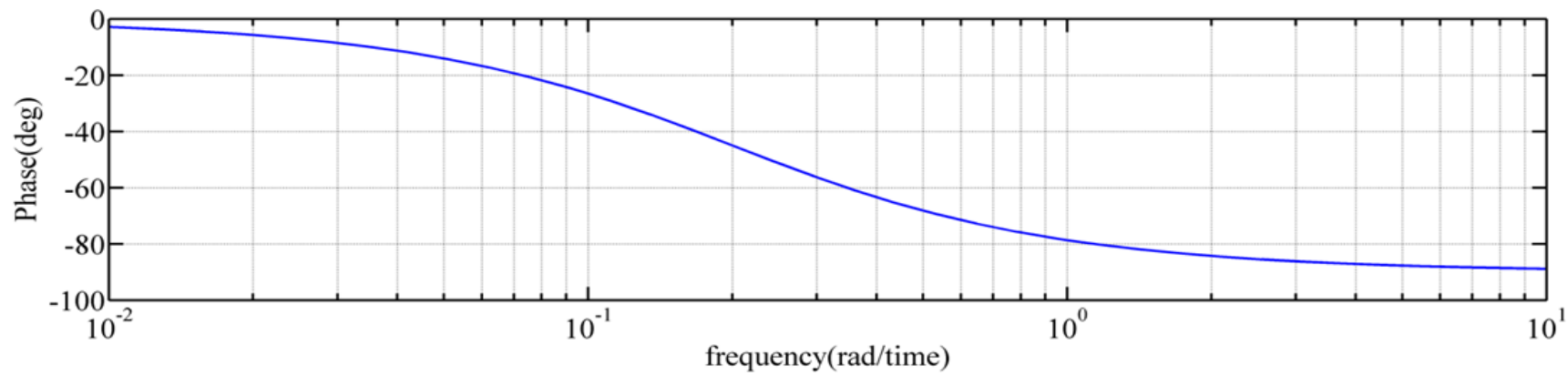
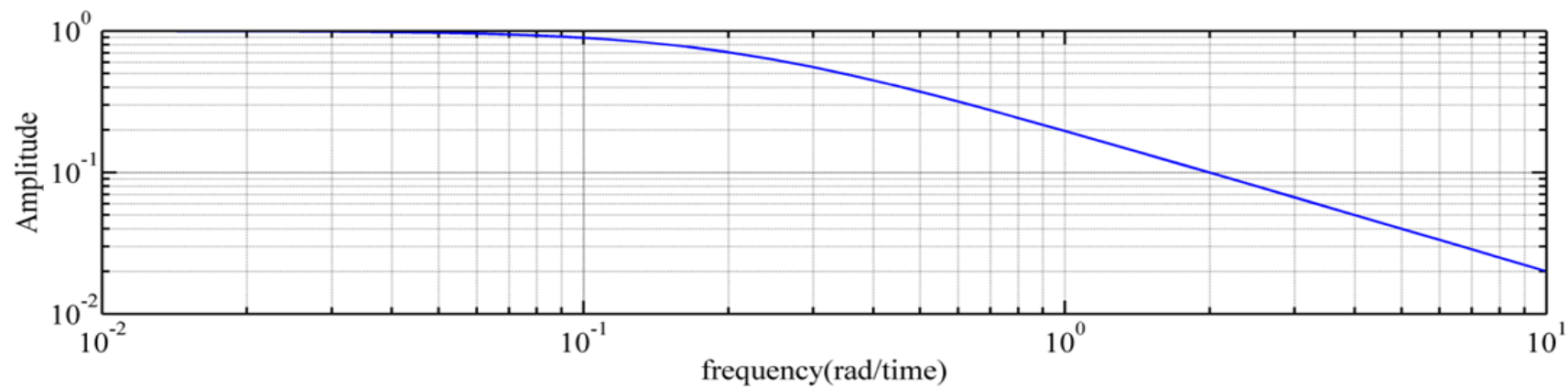
$$\log(AR) = \log \frac{1}{5\omega} = -\log(5\omega) \rightarrow \text{slope} = -1$$

$$\phi = \tan^{-1}(-\infty) = -90^\circ$$

Intercept of two asymptote;

$$\omega_c = \frac{1}{\tau} = \frac{1}{5} = 0.2$$

Frequency	Phase angle	Amplitude ratio
0.001	-2.86	0.99
0.005	-14.04	0.97
0.1	-26.57	0.89
0.5	-68.20	0.37
1	-78.69	0.20
5	-87.71	0.04
10	-88.85	0.02



THE BODE CRITERION:

$AR_{w_{co}} \leq 1$ Stable

$AR_{w_{co}} > 1$ Unstable

