PID controller parameters calculation [1-5]

References:

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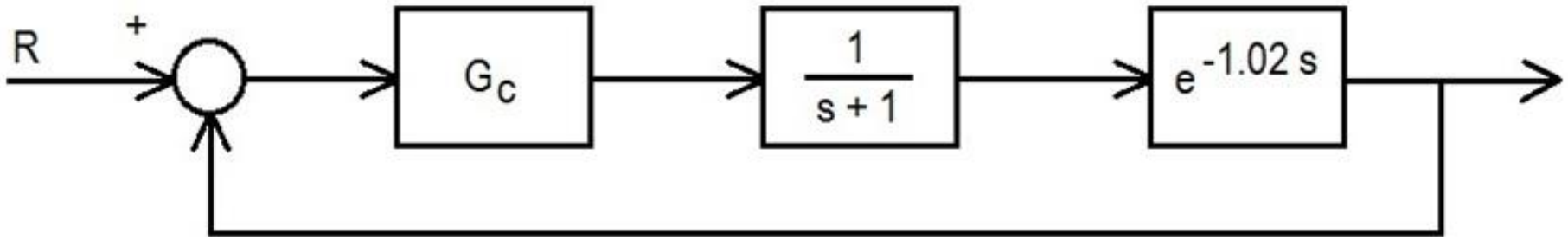
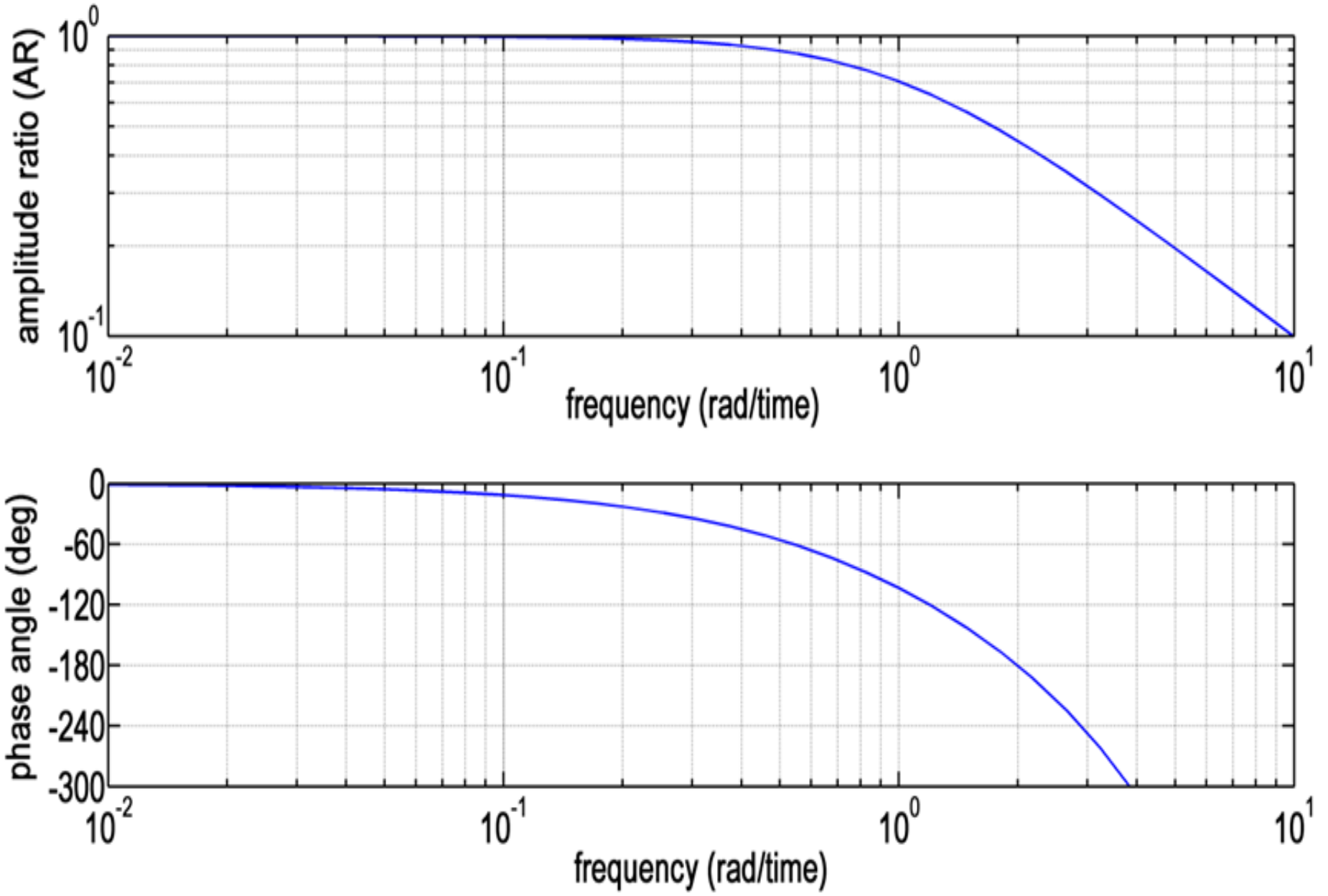
the Ziegler –Nichols rules




amplitude ratio (AR)
10^{0}
10^{-1}
10^{-2}
10^{-1}
10^{0}
10^{1}
frequency (rad/time)
phase angle (deg)
0
-60
-120
-180
-240
-300
10^{-2}
10^{-1}
10^{0}
10^{1}
frequency (rad/time)

$$K_u = \frac{1}{AR_{\omega_{co}}} \qquad P_u = \frac{2\pi}{\omega_{co}}$$

Ziegler-Nichols controller settings

Type of control	K_c	τ_I	τ_D
P	$0.5\,K_u$		
PI	$0.45\,K_u$	$P_u / 1.2$	
PID	$0.6\,K_u$	$P_u / 2$	$P_u / 8$

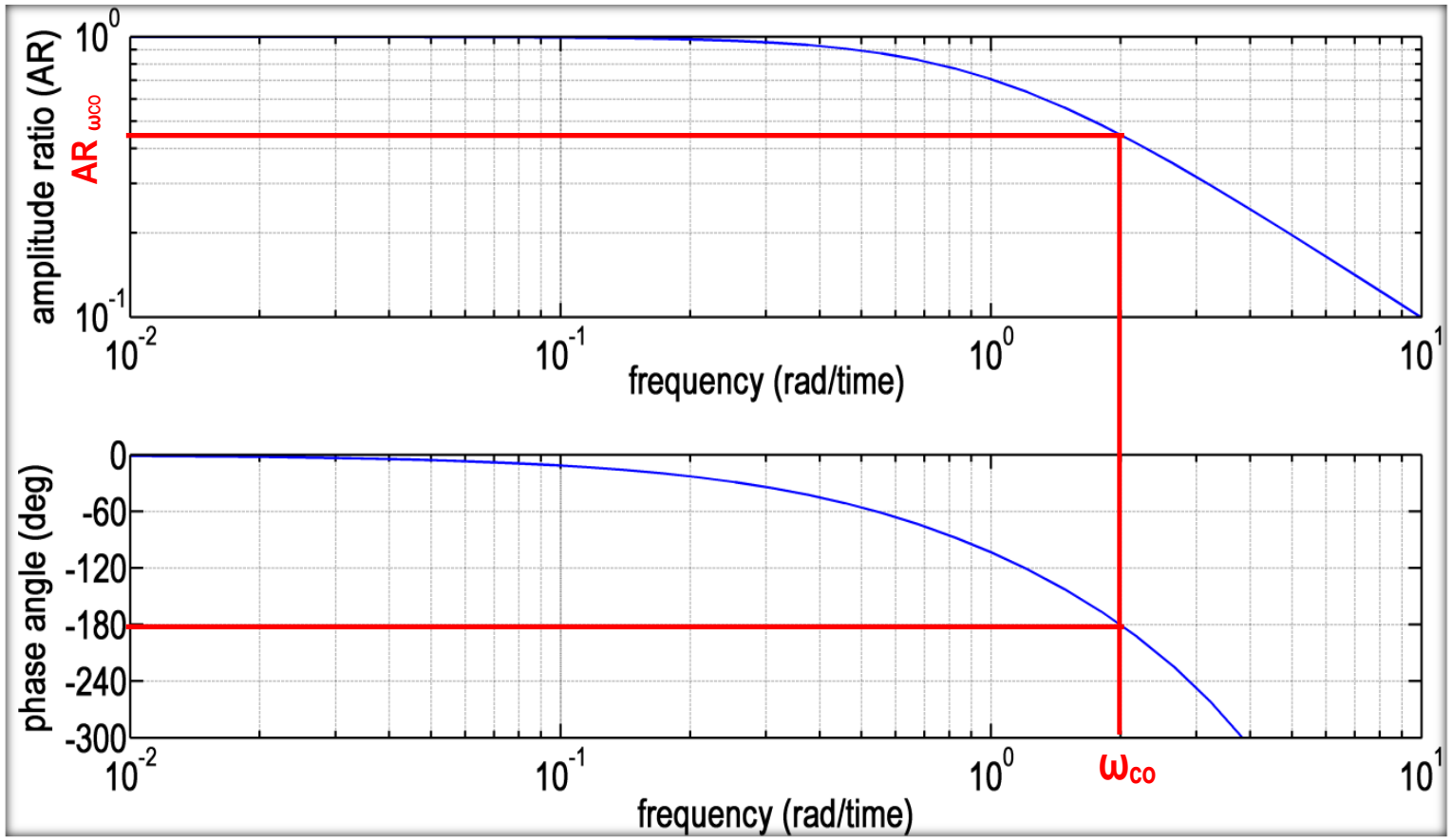
the crossover frequency and amplitude ratio from the Bode diagram for $e^{-1.02s}/(s+1)$

ω_{co} = 2 rad /min

$AR_{\omega co}$ = 0.45

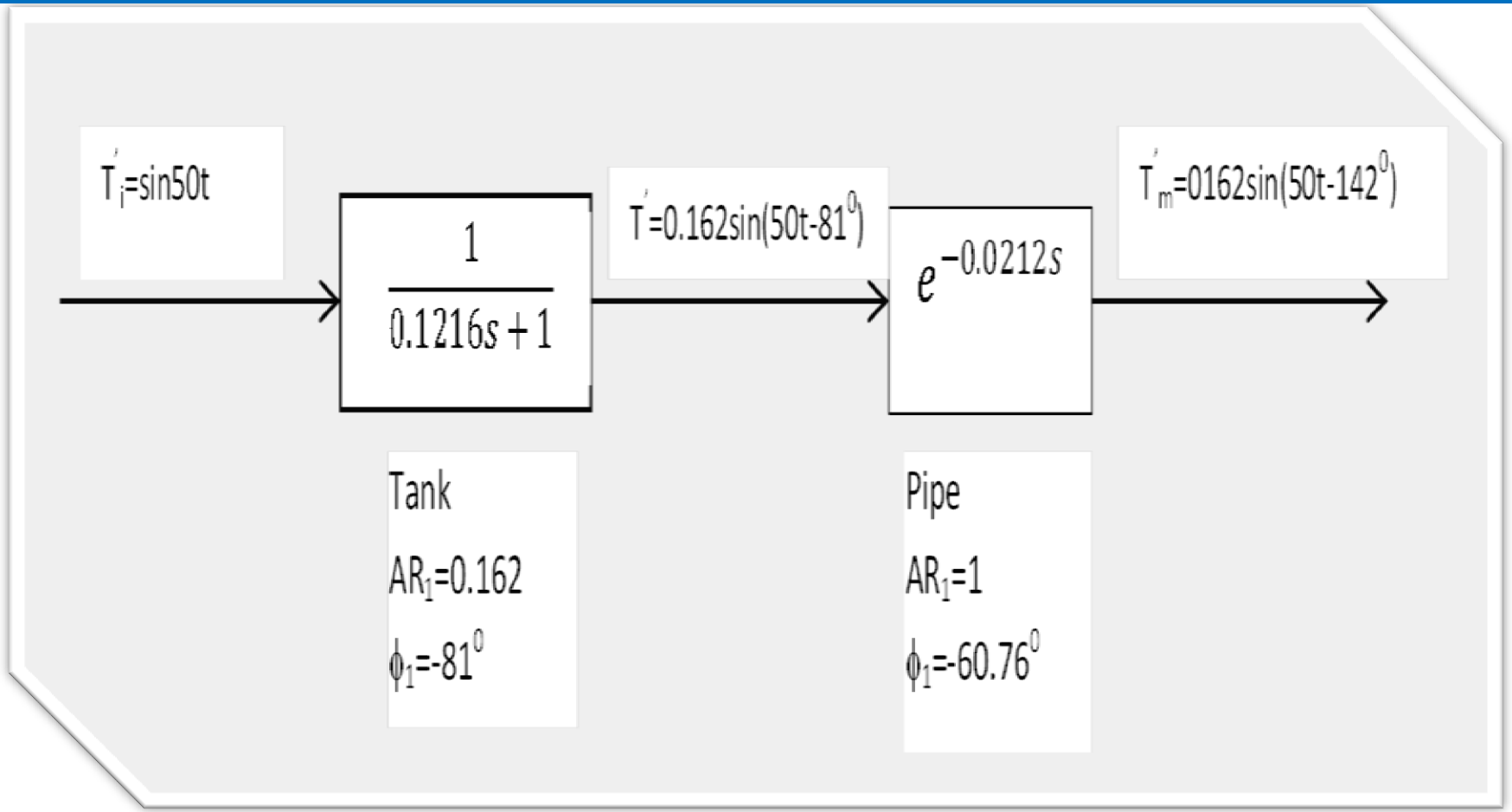
$$K_u = \frac{1}{AR_{\omega_{CO}}} = \frac{1}{0.45} = 2.22 \qquad P_u = \frac{2\pi}{\omega_{CO}} = \frac{2\pi}{2} = \pi \text{ sec/ } cycle$$

Type of control	K_c	τ_I	τ_D
PI	$0.45\,K_u = 0.45 * 2.22 = 0.99$	$P_u/1.2 = \pi/1.2 = 2.62$	-

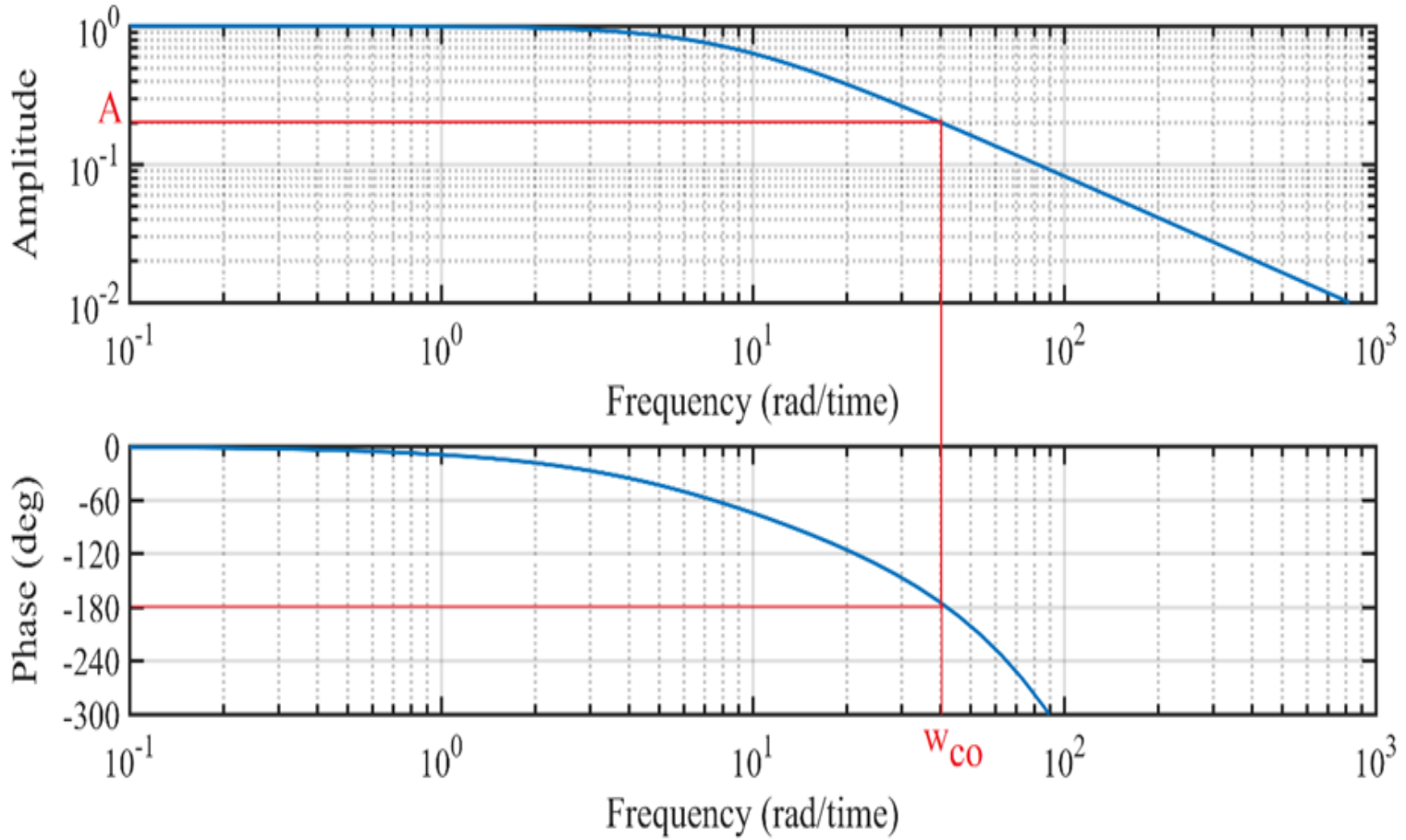

amplitude ratio (AR)
10^0
10^{-1}
AR ω_{co}
10^{-2}
10^{-1}
10^0
10^1
frequency (rad/time)
phase angle (deg)
0
-60
-120
-180
-240
-300
ω_{co}
frequency (rad/time)

Type of control	K_c	τ_I	τ_D
P	$0.5\ K_u = 0.5 * 2.22 =$ **1.11**	-	-
PI	$0.45\ K_u = 0.45 * 2.22 =$ **0.99**	$P_u / 1.2 = \pi / 1.2 =$ **2.62**	-
PID	$0.6\ K_u = 0.6 * 2.22 =$ **1.33**	$P_u / 2 = \pi / 2 =$ **1.57**	$P_u / 8 = \pi / 8 =$ **0.39**

the two systems are in series



$$\frac{T'(s)}{T_i'(s)} * \frac{T_m'(s)}{T'(s)} = \frac{T_m'(s)}{T_i'(s)} = \frac{e^{-0.0212\,s}}{0.1216\,s + 1}$$



$$At\ \phi = -180^o \quad \rightarrow \quad w_{co} = 40\ \frac{rad}{\min}$$

$$At\ w_{co} = 40\frac{rad}{\min} \quad \rightarrow \quad A = 0.2$$

$$K_u = \frac{1}{A} = \frac{1}{0.2} = 5$$

$$P_u = \frac{2\pi}{w_{co}} = \frac{(2)(3.14)}{40} = 0.157\frac{min}{cycle}$$

For PID controller;

$$K_c = 0.6 * K_u = (0.6)(5) = 3$$

$$\tau_I = \frac{P_u}{2} = \frac{0.157}{2} = 0.0785\ \text{min}$$

$$\tau_D = \frac{P_u}{8} = \frac{0.157}{8} = 0.0196\ \text{min}$$

Ziegler-Nichols controller settings:

Type of control	K_c	τ_I	τ_D
PID	$0.6\,K_u$	$P_u/2$	$P_u/8$