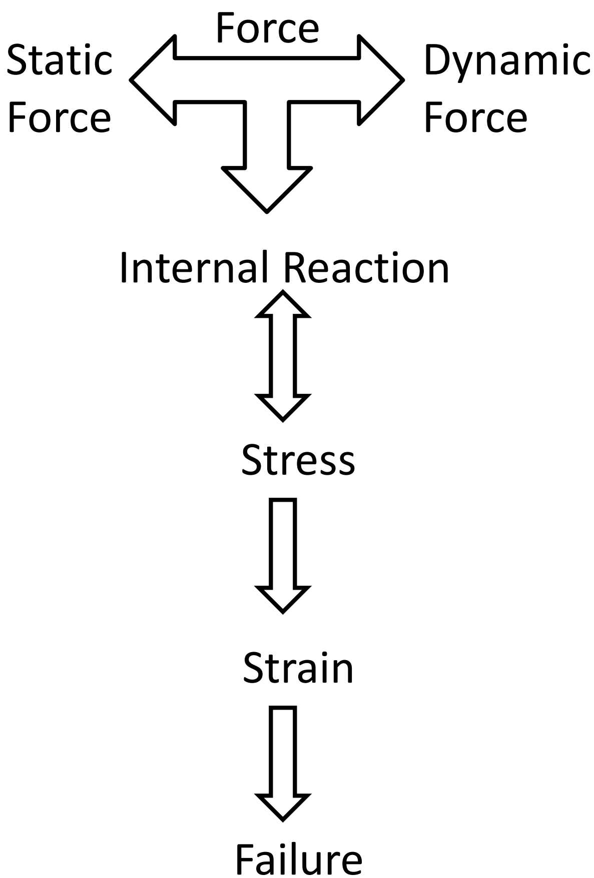
FORCE CONCEPT



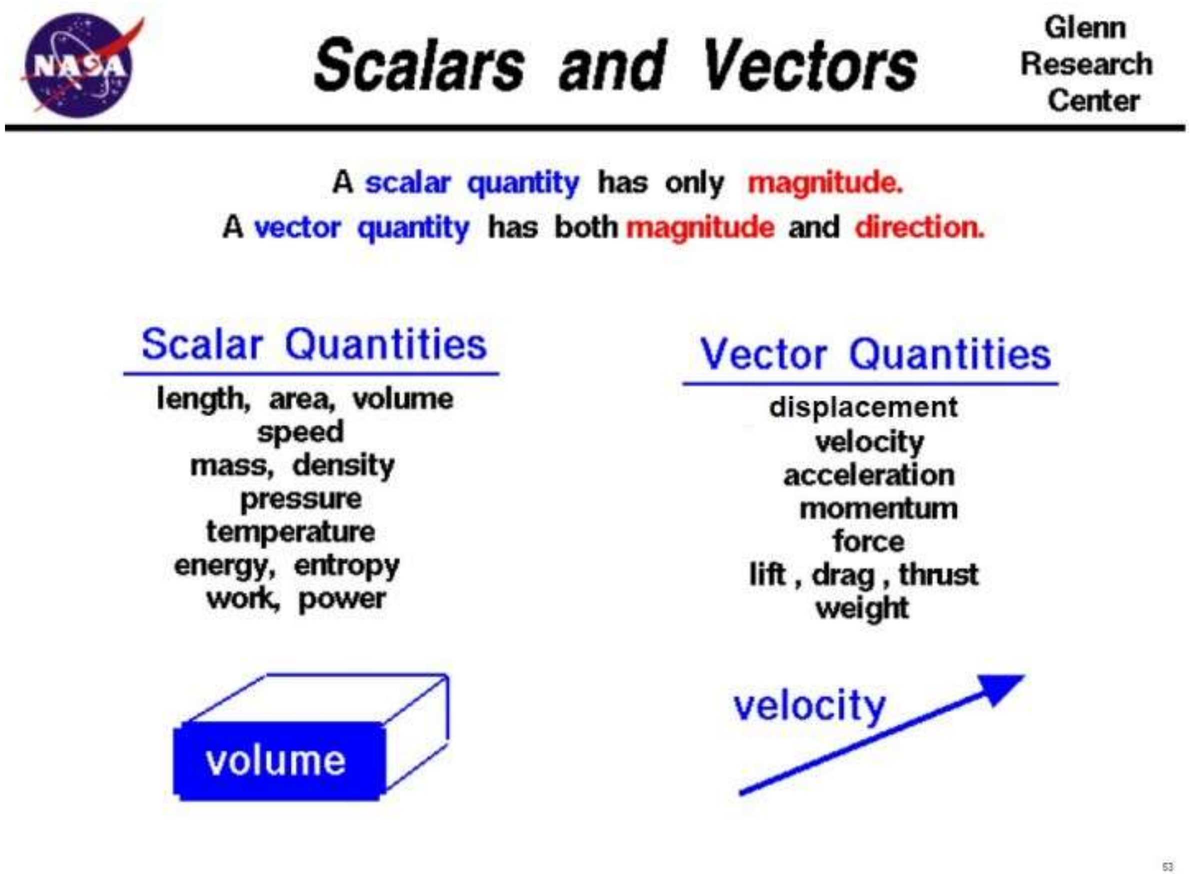
Basic flow chart of forces acting on deformable materials

Recall;

Statics deals with rigid bodies and no deformation

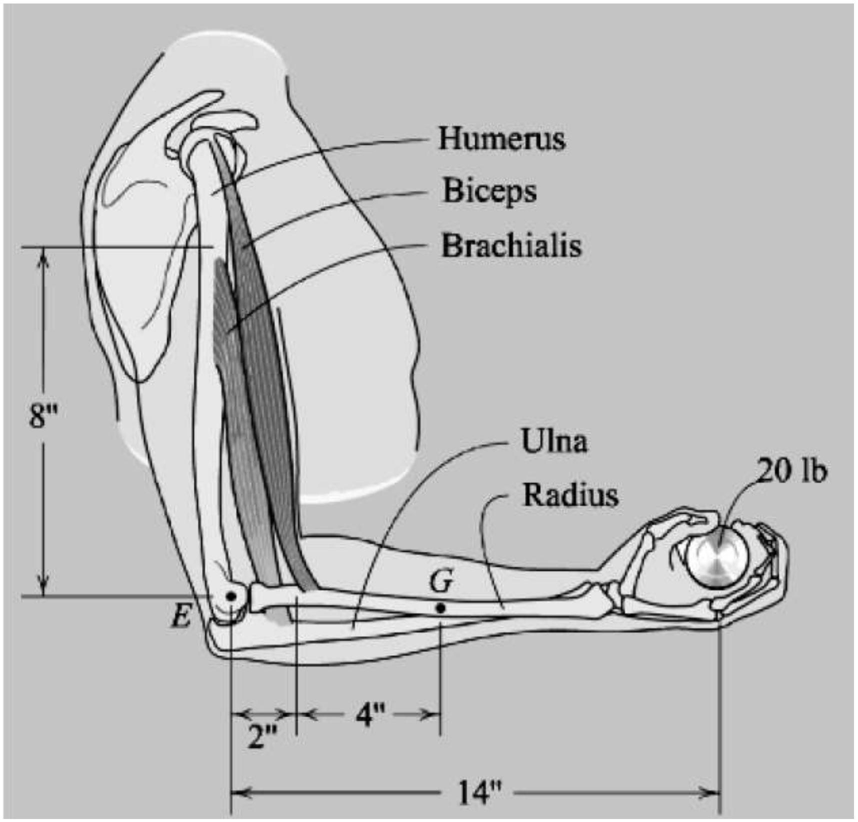
Material strength deals with deformable bodies under varying loading conditions

FORCE CONCEPT



A single force acting on materials is quantified by scalar and vectors. The main difference between both is the direction. Vectors have magnitudes over any direction in 2D and 3D space. Moreover, force vectors can be multiplied by scalars.

EQUILIBRIUM of FORCES ACTING on BODIES



$$\mathbf{F}_R = \sum \mathbf{F} = 0$$

$$\mathbf{M}_{R,O} = \sum \mathbf{M}_{F,O} + \sum \mathbf{M} = 0$$

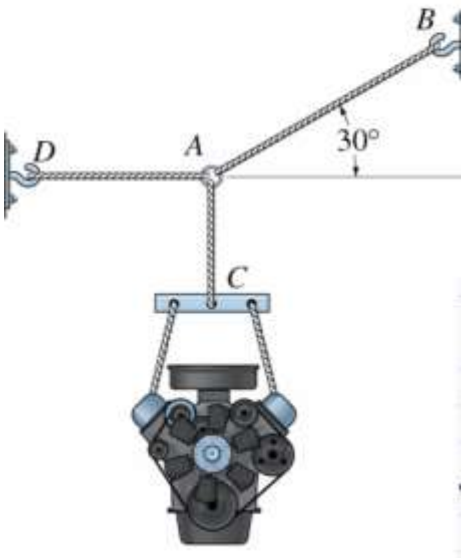
2-D problems:

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_O = 0 \end{cases}$$

3-D problems:

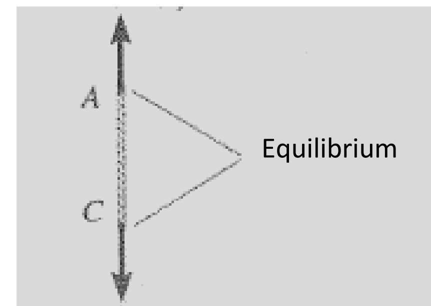
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases} \quad \begin{cases} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{cases}$$

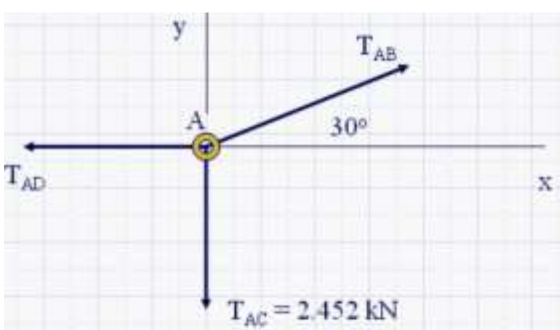
EQUILIBRIUM of FORCES



The engine has a mass of 250 kg. Determine the tensile forces on AB and AD wires to provide equilibrium

Solution: The forces at point "A" must be in equilibrium which are the tensile forces of wires and the engine itself. Weight of the engine is;
$W=250\ kg*9.81\ m/s^2 = 2.542\ kN$ due to gravity force. Recalling the 3rd Newton principle; CA wire applies the same force in an opposite direction.





$\sum F_x$ and $\sum F_y = 0$

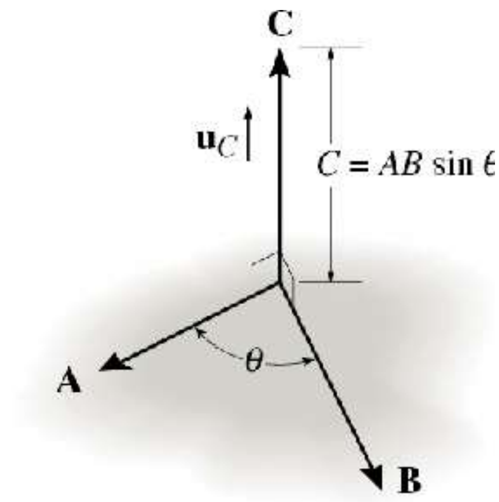
$T_{AB} \times \cos 30 - T_D = 0$
$T_{AB} \times \sin 30 - 2.452 kN = 0$

$T_{AB} = 4.91$ kN
$T_D = 4.25$ kN

FORCE SYSTEM RESULTANTS
CROSS-PRODUCT

- The *Cross product* of two vectors

$$\bar{C} = \bar{A} \times \bar{B}$$

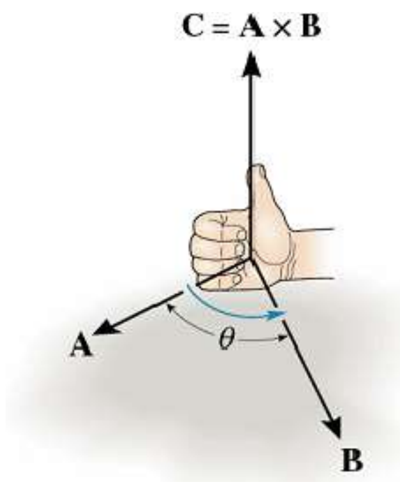


Magnitude

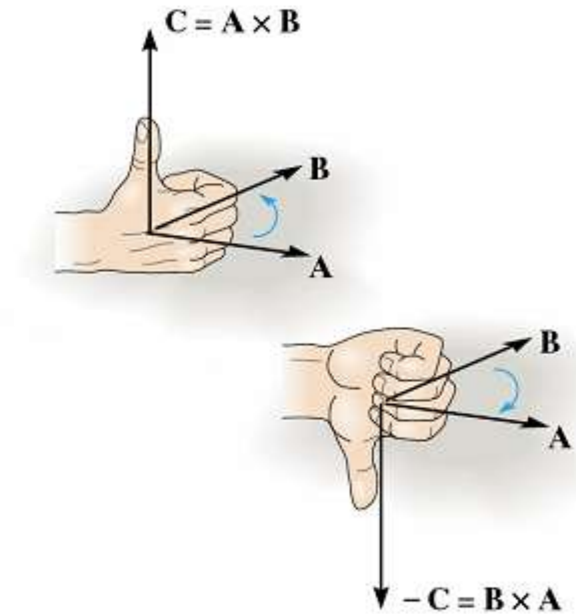
C=*AB* sinθ

Direction

C is perpendicular to both A and B



CROSS-PRODUCT BASICS



Commutative law is not valid

$$\bar{A}\times\bar{B}\neq\bar{B}\times\bar{A}$$

In fact

$$\bar{A}\times\bar{B}=-\bar{B}\times\bar{A}$$

Scalar Multiplication

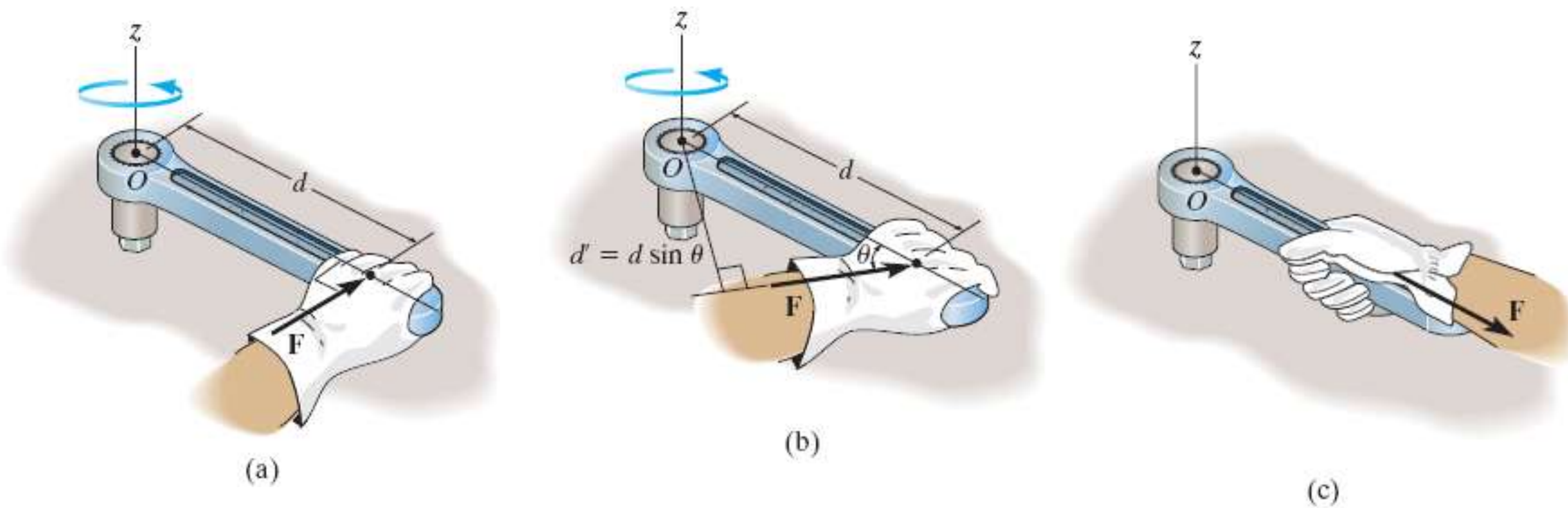
$$\alpha\left(\bar{A}\times\bar{B}\right)=\left(\alpha\bar{A}\right)\times\bar{B}=\bar{A}\times\left(\alpha\bar{B}\right)=\left(\bar{A}\times\bar{B}\right)\alpha$$

Distributive Law

$$\bar{A}\times\left(\bar{B}+\bar{D}\right)=\bar{A}\times\bar{B}+\bar{A}\times\bar{D}$$

MOMENT of FORCES and FORCE SYSTEMS

- ***Moment*** of a force about a point or axis : Measure of the tendency of the force to cause a body to rotate about the **point or axis**
- Torque – tendency of rotation caused by $\mathbf{F}_x$ or simple moment $(\mathbf{M}_o)_z$



MOMENT of FORCES and FORCE SYSTEMS

Magnitude

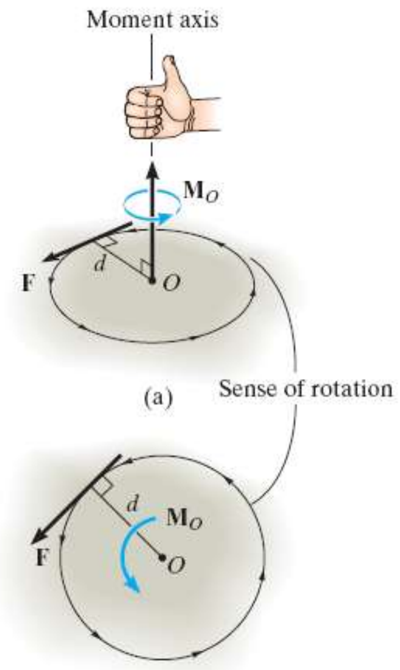
- For magnitude of $\mathbf{M}_O$,

$$\mathbf{M}_O = Fd\ (Nm)$$

where d = perpendicular distance from O to its line of action of force

Direction

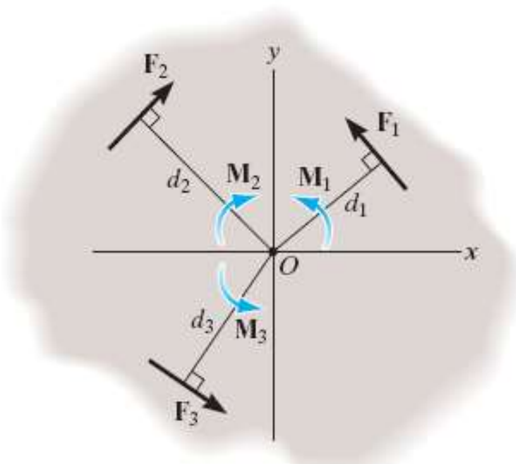
- Direction using "right hand rule"



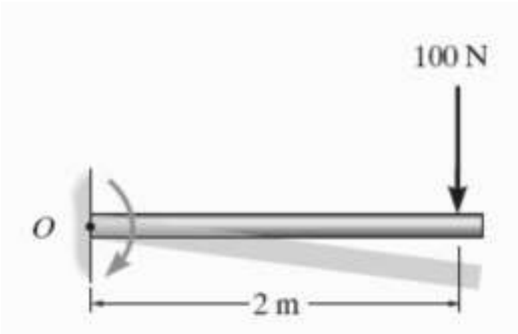
Resultant Moment

- Resultant moment, $\mathbf{M}_{Ro}$ = moments of all the forces

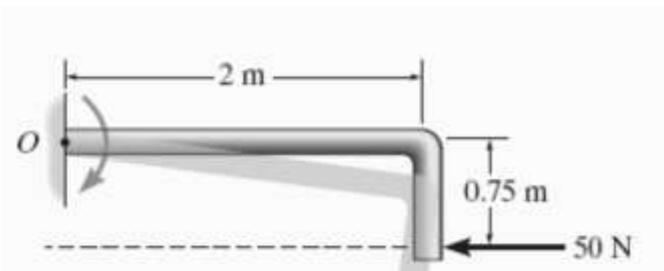
$$\mathbf{M}_{Ro} = \sum Fd$$



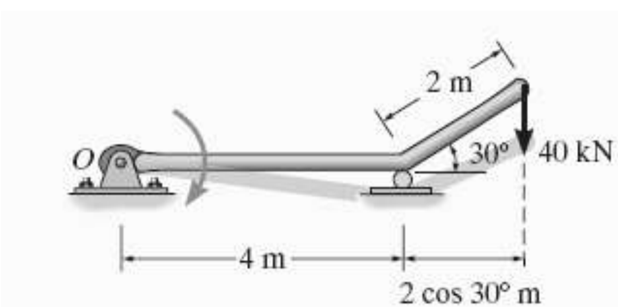
Sample Question: For each case, determine the moment of the force about point "**O**"



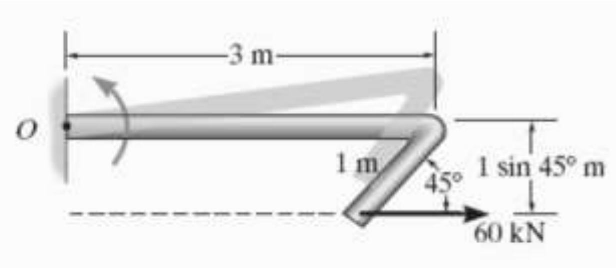
$$M_o = (100N)(2m) = 200N.m(CW)$$



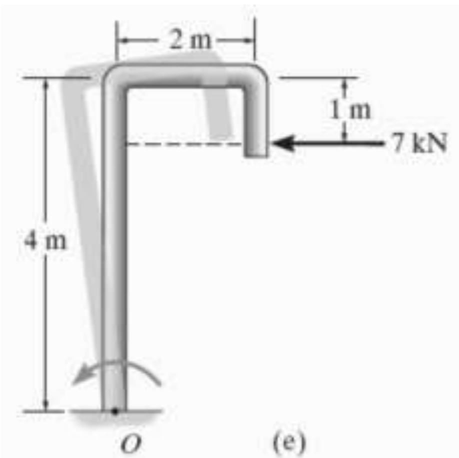
$$M_o = (50N)(0.75m) = 37.5N.m(CW)$$



$$M_o = (40N)(4m + 2\cos 30^\circ m) = 229N.m(CW)$$



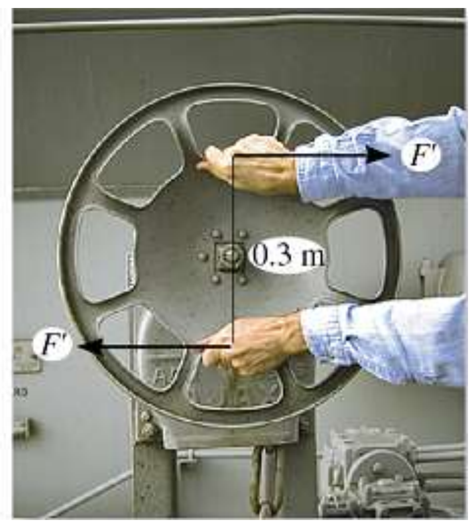
$$M_o = (60N)(1\sin 45^{\circ} m) = 42.4N.m(CCW)$$



$$M_o = (7kN)(4m - 1m) = 21.0kN.m(CCW)$$

MOMENT DUE to FORCE COUPLES

- Couple: Two Forces having the same magnitude, parallel lines of action, and opposite sense separated by a perpendicular distance. Sum of the forces in each direction is zero, so a couple does not affect the sum of forces equations
- A force couple will however tend to rotate the body it is acting on
- By multiplying the magnitude of one Force by the distance between the Forces in the Couple, the moment due to the couple can be calculated.



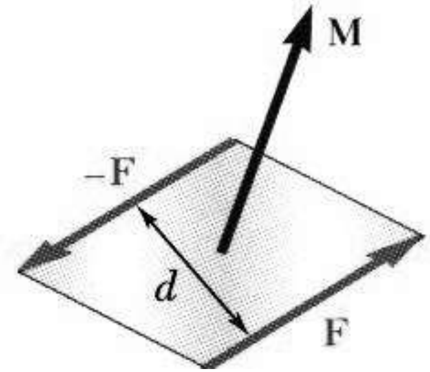
$$M_R = M_1 + M_2 = (r \times F)$$

Two couples will have equal moments if

- $F_1 d_1 = F_2 d_2$

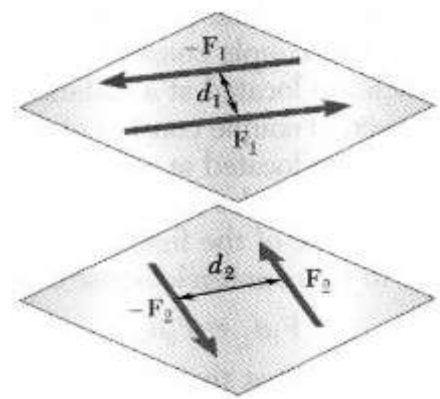
- the two couples lie in parallel planes, and

- the two couples have the same sense or the tendency to cause rotation in the same direction.



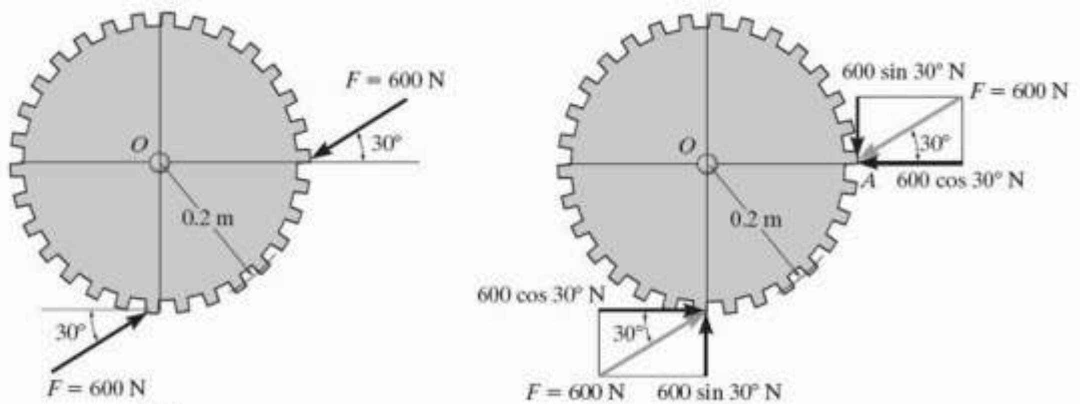
NOTE: Couples are Free Vectors

- The point of action of a Couple does not matter
- The plane that the Couple is acting in does not matter
- All that matters is the orientation of the plane the Couple is acting in
- Therefore, a Force Couple is said to be a Free Vector and can be applied at any point on the body it is acting



SAMPLE QUESTION

Determine the magnitude and direction of the couple moment acting on the gear in Fig. 4–31*a*.



(a) (b)

The easiest solution requires resolving each force into its components as shown in Fig. 4–31*b*. The couple moment can be determined by summing the moments of these force components about any point, for example, the center *O* of the gear or point *A*. If we consider counterclockwise moments as positive, we have

$$\circlearrowleft + M = \Sigma M_O;\ M = (600 \cos 30^\circ \text{ N})(0.2 \text{ m}) - (600 \sin 30^\circ \text{ N})(0.2 \text{ m})$$

$$= 43.9 \text{ N}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$

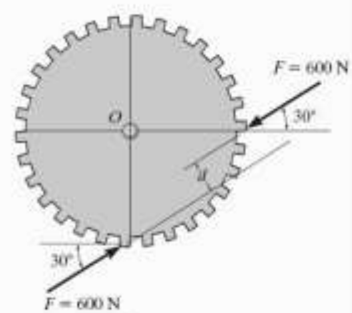
or

$$\circlearrowleft + M = \Sigma M_A;\ M = (600 \cos 30^\circ \text{ N})(0.2 \text{ m}) - (600 \sin 30^\circ \text{ N})(0.2 \text{ m})$$

$$= 43.9 \text{ N}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$

This positive result indicates that **M** has a counterclockwise rotational sense, so it is directed outward, perpendicular to the page.

NOTE: The same result can also be obtained using $M = Fd$, where d is the perpendicular distance between the lines of action of the couple forces, Fig. 4–31*c*. However, the computation for d is more involved. Realize that the couple moment is a free vector and can act at any point on the gear and produce the same turning effect about point *O*.



(c)

Fig. 4–31

SUMMARY

- ✓ Force is the backbone of both statics and strength problems
- ✓Force might be signified by scalars or vectors
- ✓Force vectors have magnitude and direction
- ✓Forces acting on materials cause moment
- ✓Force couples with the same magnitude and opposite direction cause movement
- ✓ Force and moment concepts here are given as reminders before getting into stress and strain chapters