FORCE AND MOMENT IN SLENDER MEMBERS

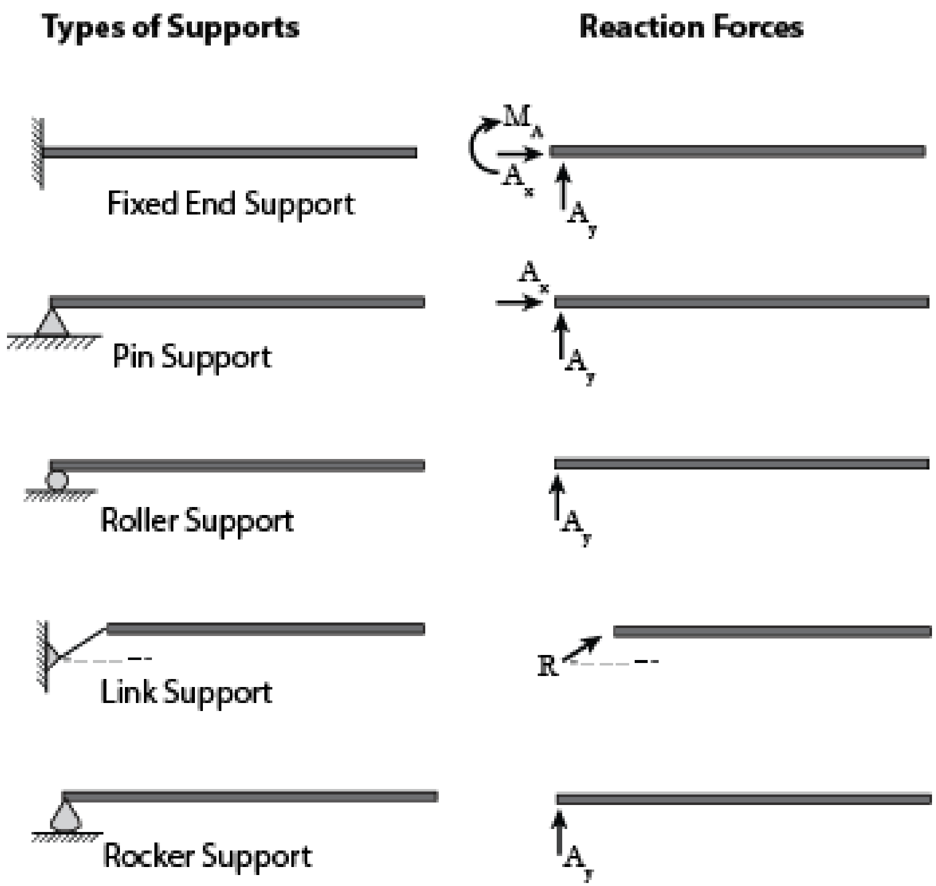
Slender Member: A structural component whose length is much more greater than its cross-section area.

Slender members i.e. beams, columns, shafts and struts transmit the loads as parts of buildings, bridges, etc.

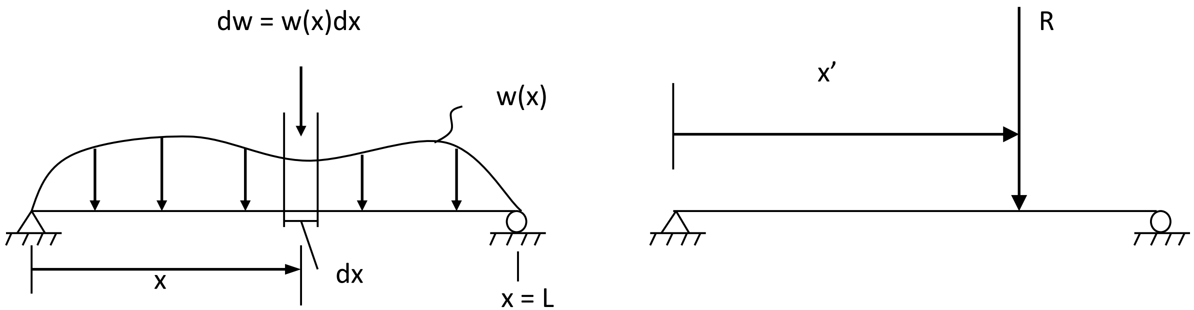
When subjected to loading; bending, compression, extension, etc. might happen. Due to external forces, the internal reactions of such members and possible strain must be considered during design.

These will lead to determine the internal forces, moment and equilibrium for several types of structural elements and loading type.

Support: 3D Structural members are generally justified to 2D planes to simplify the analysis. Some common support types are used for their separate resistance to forces. Types of supports are considered for use of their resisting force in any directions.



<u>Distributed Load:</u> A simply supported beam has a distributed load applied over its entire length. The distributed load w(x) varies in intensity (height) with position x. The load intensity w(x) has units of force/length (lb/ft or kN/m).



To find R, the original load is broken into strips of width **dx** with a small force **dw = w(x)dx** centered on each strip. Equivalent force R is the sum of all small dw’s. As dx , there are more and more dw’s to add up and the sum becomes an integral. The equation to find R is then;

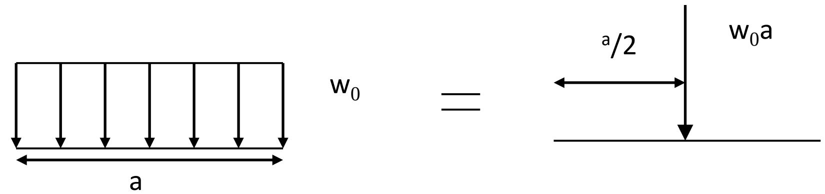
$$R = \int_0^L dw = \int_0^L w(x)dx$$

The location x’ is found based on the principle of moments. Each small dw has a moment about some point (say x = 0). The total moment of all the dw’s about this point must equal the moment of R about the same point. x’ is;

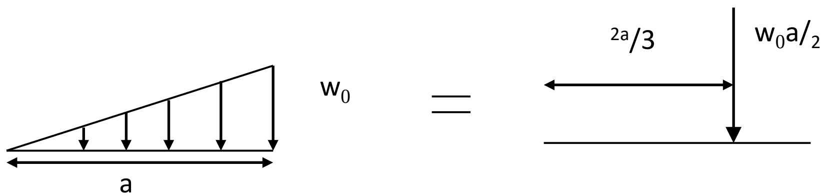
$$x'R = \int_0^L x\,dw = \int_0^L x\,w(x)dx$$

<u>The equivalent force R is equal to the area under the distributed load curve. The location of the force (given by the distance x’) is at the centroid of the distributed load area.</u>

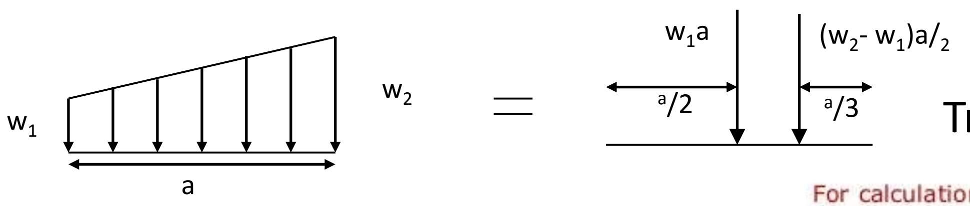
Common Types of Distributed Load



Uniform



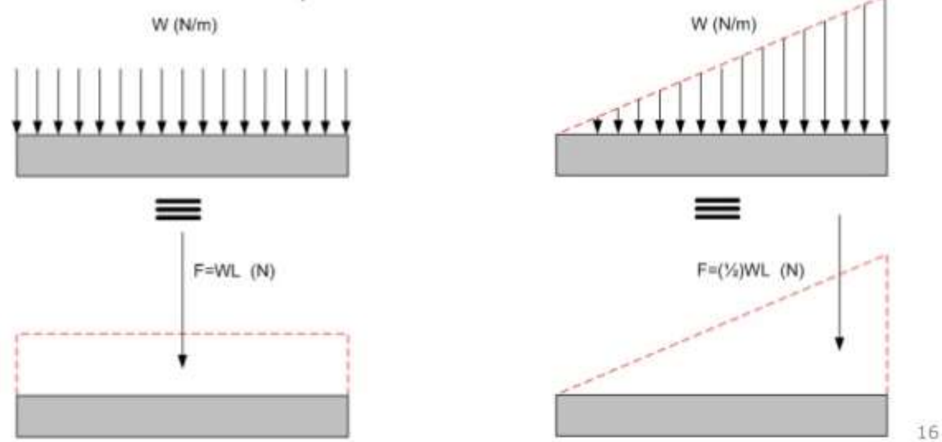
Triangular



Trapezoid

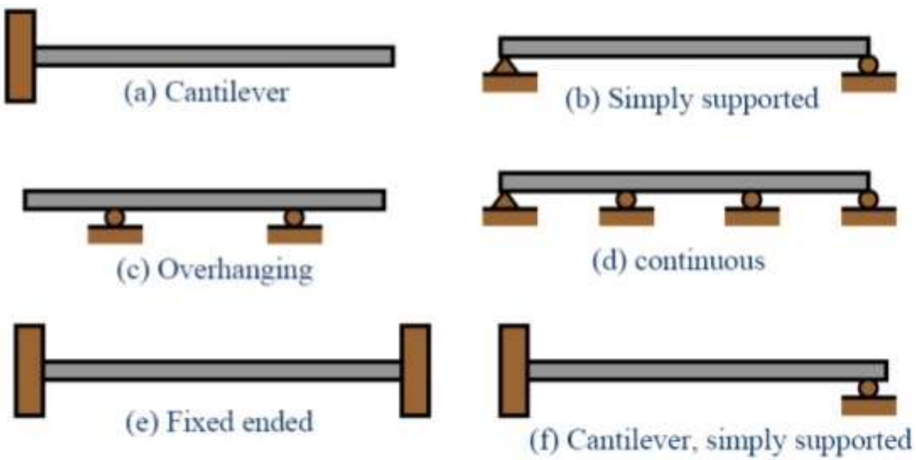
For calculation purposes, distributed load can be represented as a single load acting on the center point of the distributed area.

Total force = area of distributed load (W : height and L: length)
Point of action: center point of the area

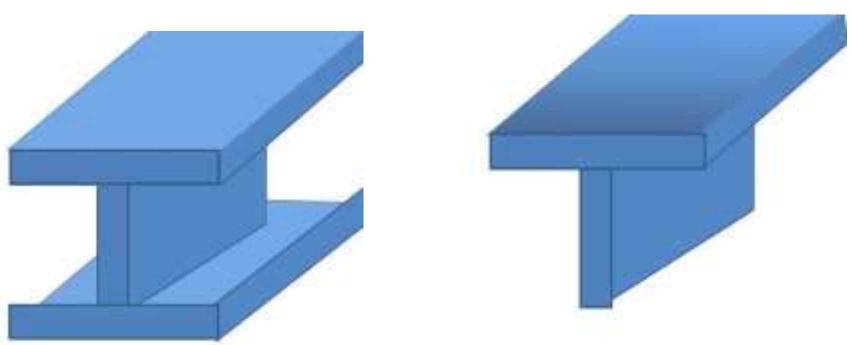


Types of Beams

<u>Beam:</u> Horizontal bar which undergoes lateral load or couple which tends to bend the bar or a horizontal bar undergoes bending stress known as beam. Beams might be classified based on their use of support, cross section shape or statically determinate/indeterminate conditions.



→ Load resistance



I Beam

T Beam

→ X-Sec

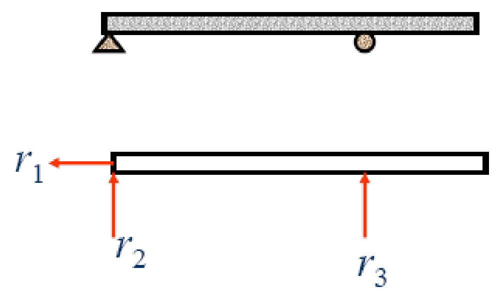
Determinacy of Beams

In order to provide equilibrium, if there exist total of “n” parts and “r” reactions;

r=3n; statically determinate

r>3n; statically indeterminate

Case 1 :

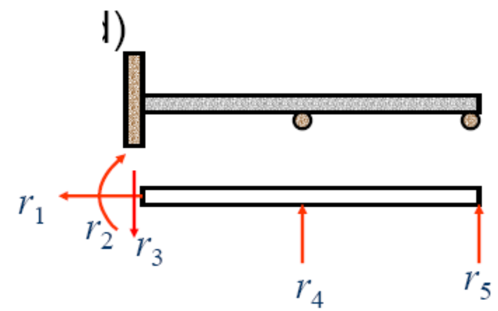


Applying Eq. 30,

$r = 3,\ n = 1$, therefore,

$r = 3n, \Rightarrow 3 = [3(1) = 3] \Rightarrow$ statically determinate

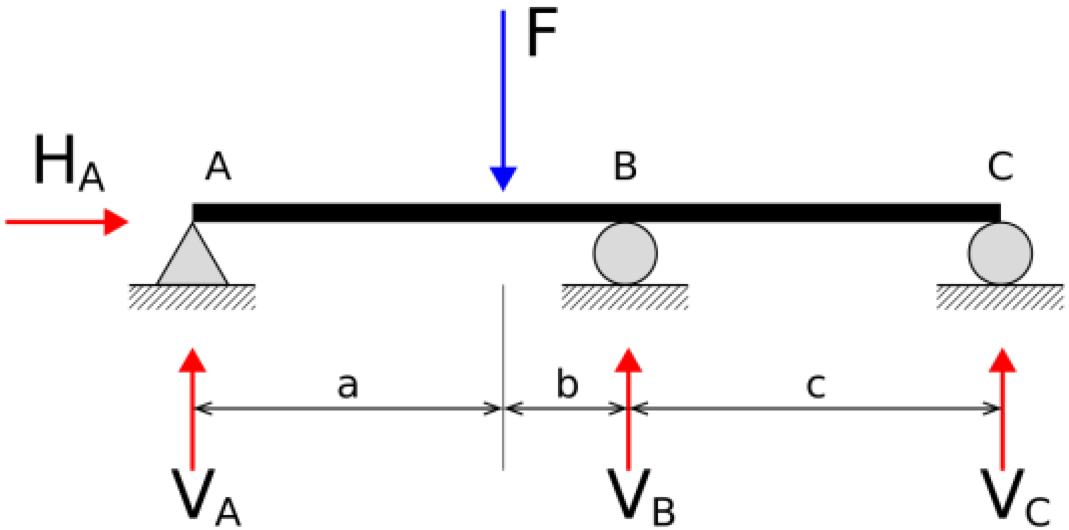
Case 2 :



Applying Eq. 30,

$r = 5,\ n = 1$, therefore,

$r > 3n, \Rightarrow 5 > [3(1) > 3] \Rightarrow$ statically indeterminate to second degree



In statics, a structure is **statically indeterminate** (or **hyperstatic**) when the static equilibrium equations are insufficient for determining the internal forces and reactions on that structure.

- $\sum \vec{F} = 0$: the vectorial sum of the forces acting on the body equals zero. This translates to

  Σ $H = 0$: the sum of the horizontal components of the forces equals zero;

  Σ $V = 0$: the sum of the vertical components of forces equals zero;

- $\sum \vec{M} = 0$: the sum of the moments (about an arbitrary point) of all forces equals zero.

In the beam construction on the right, the four unknown reactions are V_A, V_B, V_C and H_A. The equilibrium equations are:

Σ $V = \mathbf{0}$:

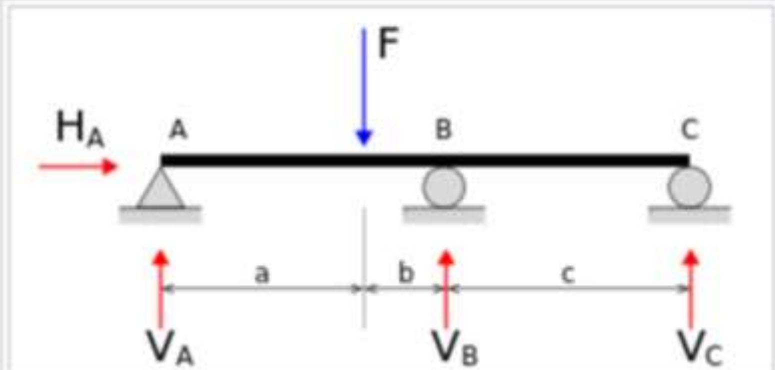
$$V_A - F_v + V_B + V_C = 0$$

Σ $H = \mathbf{0}$:

$$H_A - F_h = 0$$

Σ $M_A = 0$:

$$F_v \cdot a - V_B \cdot (a + b) - V_C \cdot (a + b + c) = 0.$$

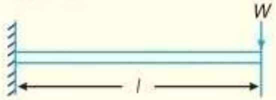
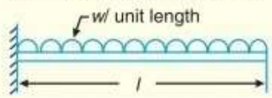
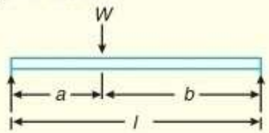
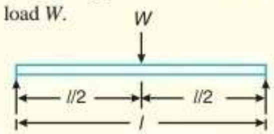
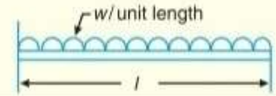
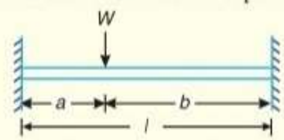
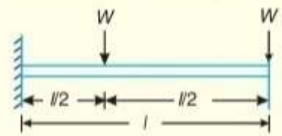
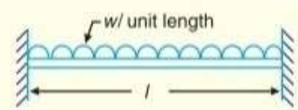


Free body diagram of a statically indeterminate beam.

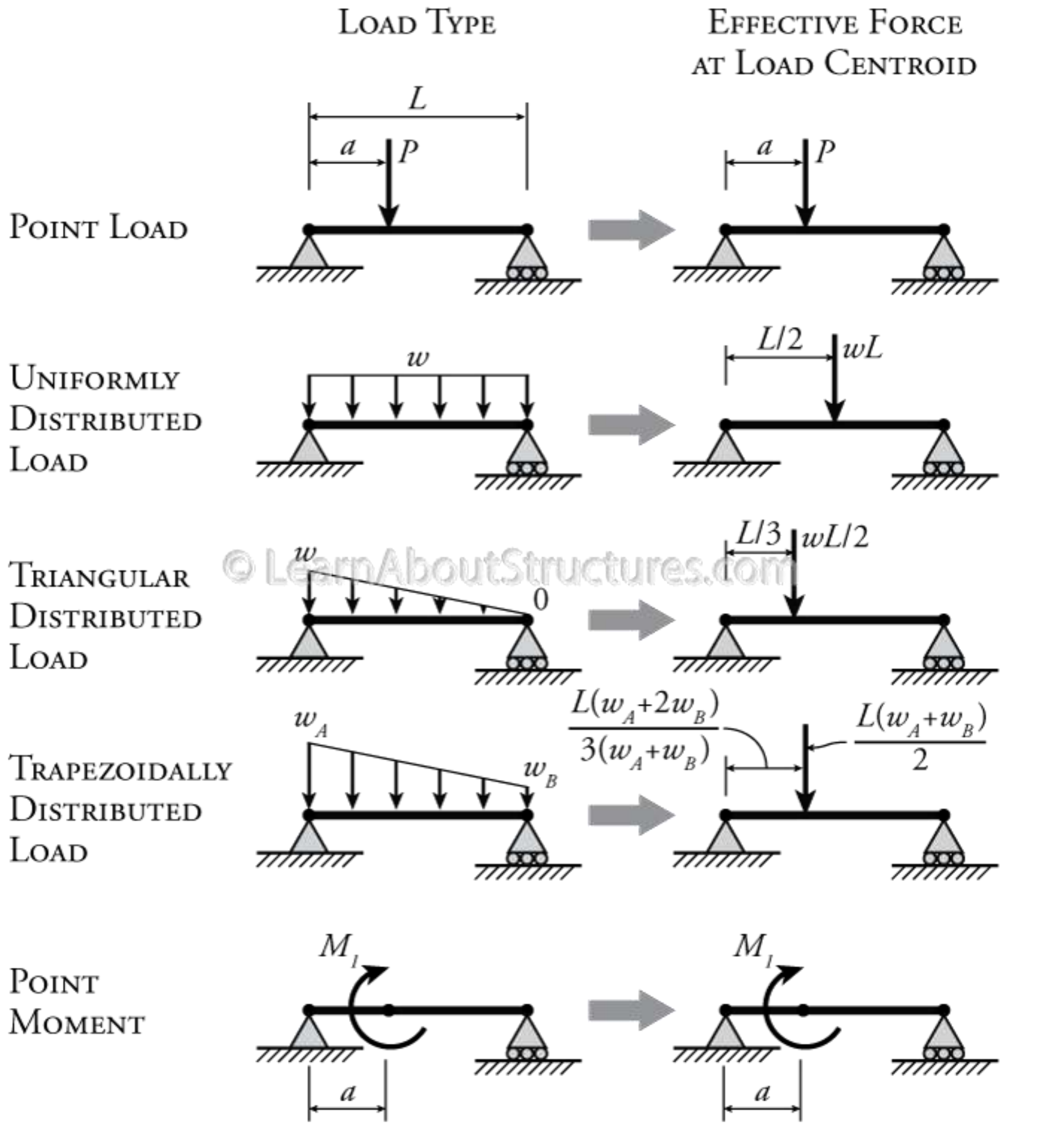
Since there are four unknown forces (or *variables*) (V_A, V_B, V_C and H_A) but only three equilibrium equations, this system of simultaneous equations does not have a unique solution. The structure is therefore classified as *statically indeterminate*. Considerations in the material properties and compatibility in deformations are taken to solve statically indeterminate systems or structures.

If the support at *B* is removed, the reaction V_B cannot occur, and the system becomes **statically determinate** (or **isostatic**). If, the support at *"A"* is changed to a roller support, the number of reactions are reduced to three (without H_A), but the beam can now be moved horizontally; the system becomes *unstable* or *partially constrained*—a mechanism rather than a structure. Statical indeterminacy is the existence of a non-trivial (non-zero) solution to the homogeneous system of equilibrium equations. It indicates the possibility of self-stress (stress in the absence of an external load) that may be induced by mechanical or thermal action.

$$H_A = F_h$$
$$V_C = \frac{F_v \cdot a}{a+b+c}$$
$$V_A = F_v - V_C$$

S.No.	Type of beam	Deflection (δ)
1.	Cantilever beam with a point load W at the free end. 	$\delta = \frac{Wl^3}{3EI}$ (at the free end)
2.	Cantilever beam with a uniformly distributed load of w per unit length. 	$\delta = \frac{wl^4}{8EI}$ (at the free end)
3.	Simply supported beam with an eccentric point load W. 	$\delta = \frac{Wa^2b^2}{3EIl}$ (at the point load)
4.	Simply supported beam with a central point load W. 	$\delta = \frac{Wl^3}{48EI}$ (at the centre)
5.	Simply supported beam with a uniformly distributed load of w per unit length. 	$\delta = \frac{5}{384} \times \frac{wl^4}{EI}$ (at the centre)
6.	Fixed beam with an eccentric point load W. 	$\delta = \frac{Wa^3b^3}{3EIl}$ (at the point load)
7.	Fixed beam with a central point load W. 	$\delta = \frac{Wl^3}{192EI}$ (at the centre)
8.	Fixed beam with a uniformly distributed load of w per unit length. 	$\delta = \frac{wl^4}{384EI}$ (at the centre)

Possible bending and deflections of beams based on several types of lading



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SUMMARY

✓Slender member is the structural component whose length is much more greater than its cross-section area

✓Beams might be loaded by point (concentrated) or distributed loads

✓Problems are statically determinate or indeterminate