STRESS CONCEPT

Stress is one of the major elements of engineering mechanics. A single deformable body must withstand the external loads without deformation. When any type of lading happens, internal forces and moments take place in order to resist and keep the body in shape.

Moreover, each material has its own physical properties which can be experimentally determined. The origin of the material is not treated within laws of mechanics.

The mechanics of deformable bodies (materials) have mostly been developed in 19th century. Today, the laws extracted from such work are still being used with the scientist's names (i.e. Poisson, Lame, Boussinesq)

Basic Units

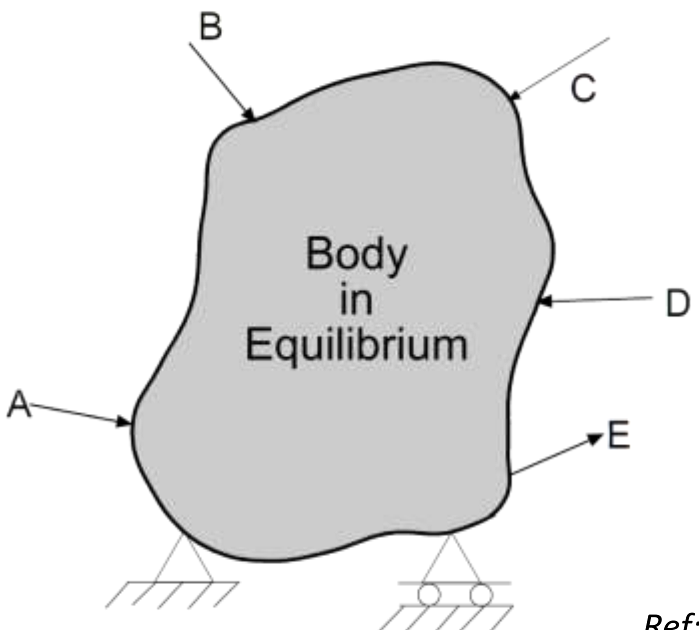
kg/cm^2

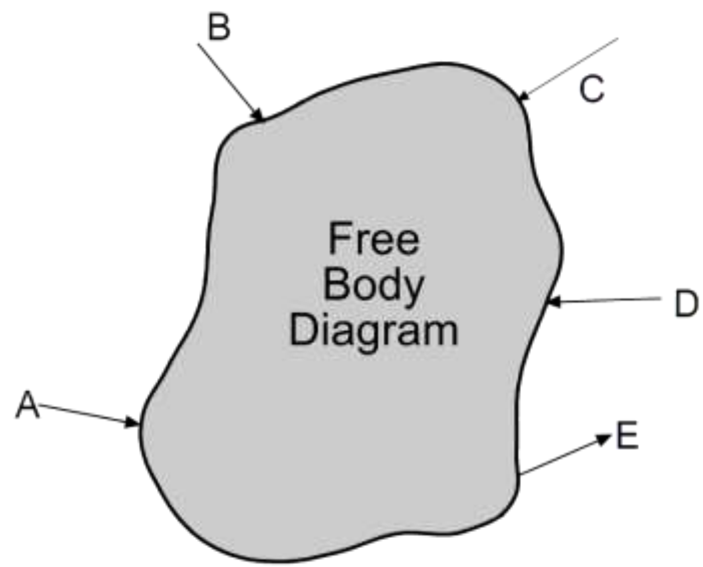
t/m^2

kN/m^2 (kPa)

METHOD OF SECTIONS

1. Free body diagram (FBD) of the body subjected to external loading is drawn. This will isolate the body from other external elements.





Ref: Introductory Mechanics of Deformable Bodies, 2008

2. The internal reaction forces due to external loading are indicated through an imaginary cross section plane. The balancing forces for equilibrium exist on such planes (Fig a).

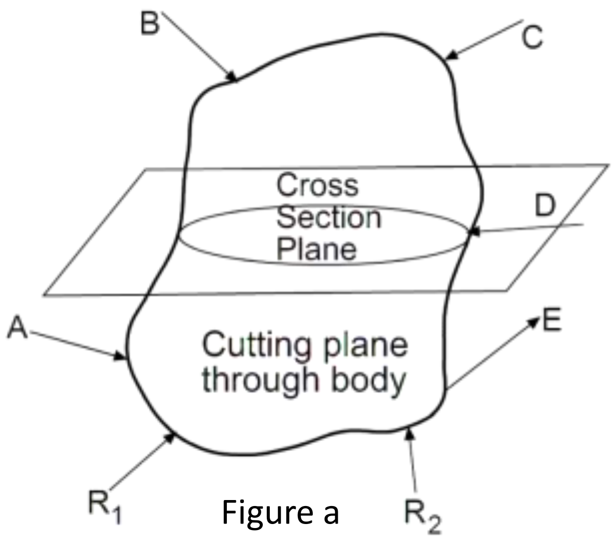


Figure a

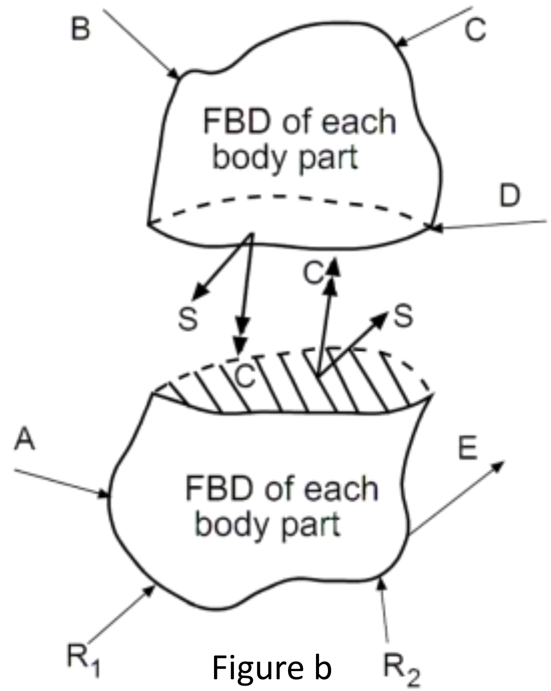
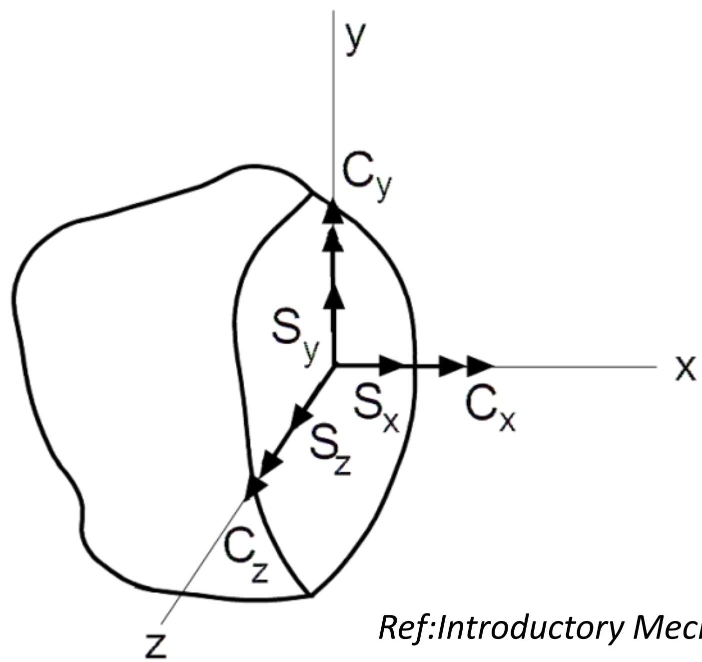


Figure b

Ref:Introductory Mechanics of Deformable Bodies, 2008

3. "S" is the Resultant of all forces and "C" is the Total moment or couple resultant as indicated on hatched area (Fig b)

4. The cartesian space is assigned to the plane. A total of six components of force and moments to be solved for equilibrium. The forces are called “Shear Force”. The force component on “x” plane which is perpendicular to the plane is **“Normal Force”.** Since moments occur, the moment components on “y” and “z” will be **“Bending Moments”** on the very same plane, while “y” component will cause twist-torque”. Based on the magnitude of loads, some of these components would be negligible.



Ref:Introductory Mechanics of Deformable Bodies, 2008

Components of resultant force and couple on cartesian space

What is STRESS?

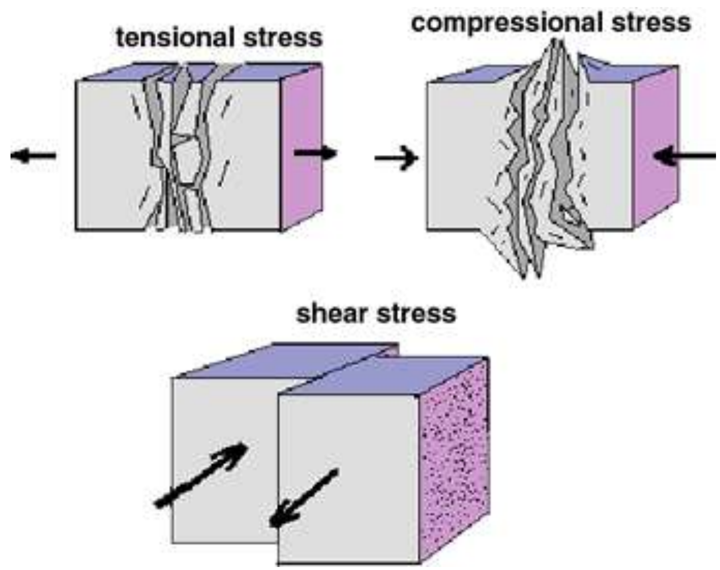
Stress is the intensity of an internal force over per unit area (Cauchy, 1822). Stresses may apply in varying directions over relevant unit area. Due, several types of stresses exist.

$$\sigma_x = \lim_{\Delta A \to \infty} \frac{\Delta S_x}{\Delta A} = \frac{dS_x}{dA} \Longrightarrow S_x = \int_A \sigma_x dA$$

Above equation introduces local average normal force per unit area.

Tensile Stress: Stress exerts a pulling effect away from the section (+)

Compressive Stress: Stress acts towards the section and exerts a thrust (-)



Shear Stress in at any point on "y" direction is:

$$\tau_{xy} = \lim_{\Delta A \to \infty} \frac{\Delta S_y}{\Delta A} = \frac{dS_y}{dA} \Longrightarrow S_y = \int_A \tau_{xy} dA$$

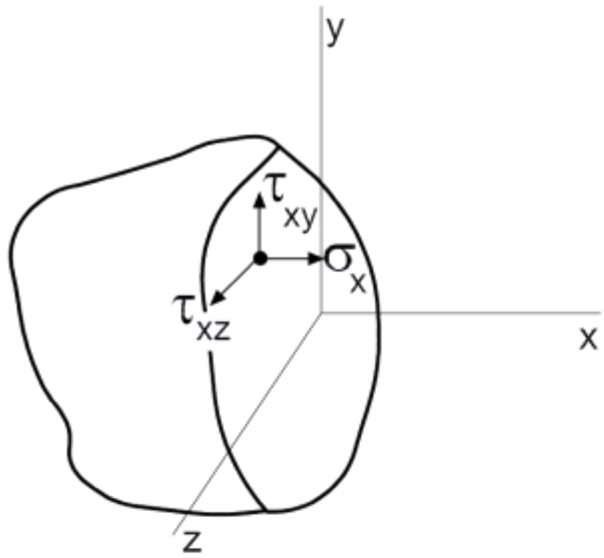
"x" indicates that; shear stress τ_{xy} acts in "x" direction
"y" is the direction of the stress itself

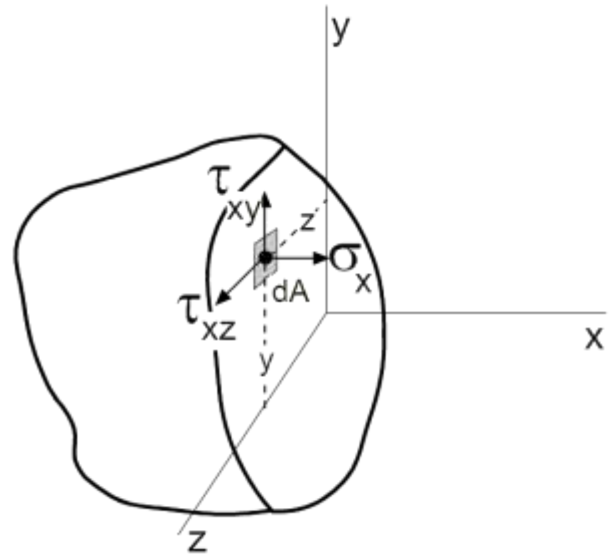
Local Average Shear Stress in "y" direction is: $\mathbf{\Delta S_y / \Delta A}$

Shear Stress in at any point on "z" direction is:

$$\tau_{xz} = \lim_{\Delta A \to \infty} \frac{\Delta S_z}{\Delta A} = \frac{dS_z}{dA} \Longrightarrow S_z = \int_A \tau_{xz} dA$$

Local Average Shear Stress in "z" direction is: $\mathbf{\Delta S_z / \Delta A}$





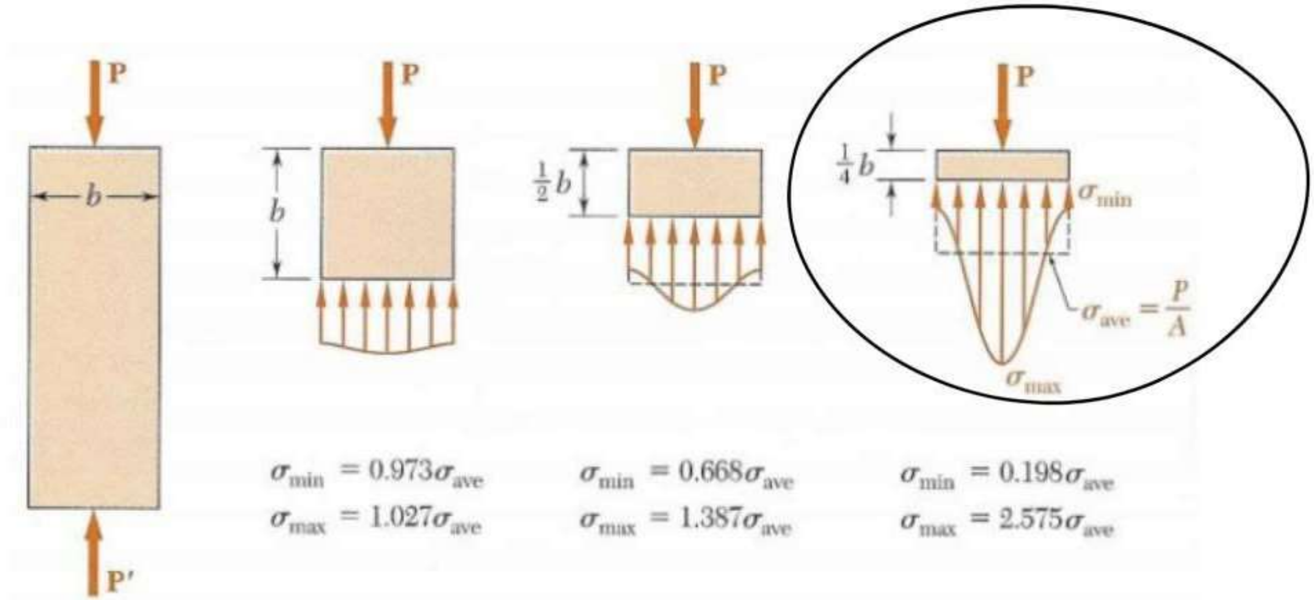
The moments about the x, y and z axes of the stresses. After integration of such stresses over the cross section;

$$C_x = \int_A (\tau_{xz} y - \tau_{xy} z) dA \qquad C_y = \int_A \sigma_x z dA \qquad C_z = -\int_A \sigma_x y dA$$

$$\sigma_x = \frac{S_x}{A} \qquad \tau_{xy} = \frac{S_y}{A} \qquad \tau_{xz} = \frac{S_z}{A}$$

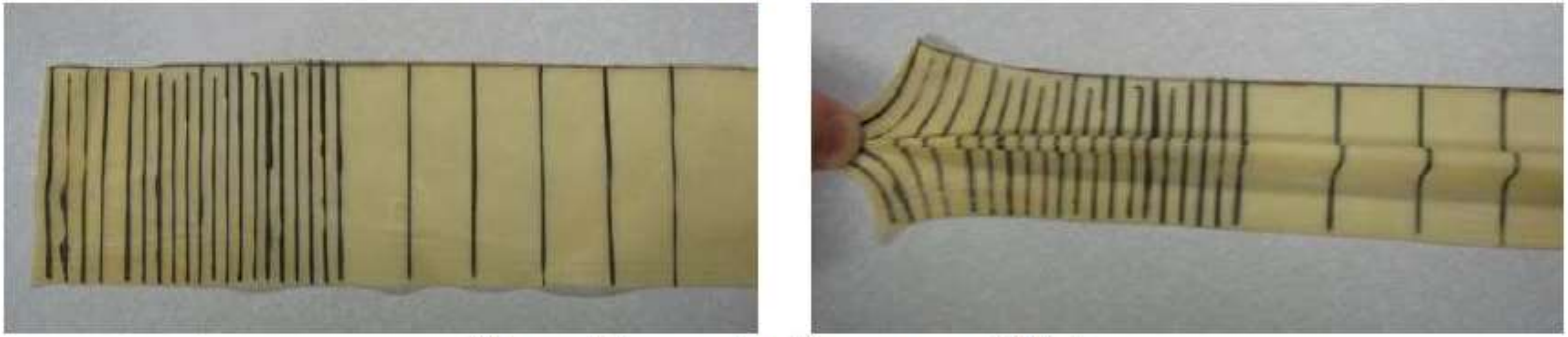
SAINT-VENANT'S PRINCIPLE

If the forces acting on a small portion of the surface of an elastic body are replaced by another statically equivalent system of forces acting on the same portion of the surface, this redistribution of loading produces substantial changes in the stresses locally but has a negligible effect on the stresses at distances which are large in comparison with the linear dimensions of the surface on which the forces are changed (Barre de Saint-Venant in Mem.savants etrangers, vol.14; 1855)

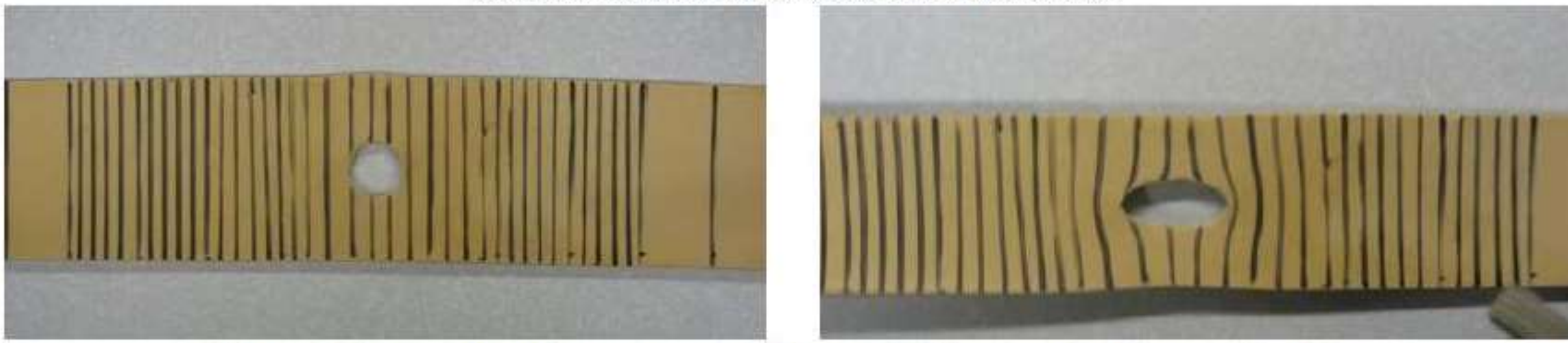


STRESSES NEAR THE POINT OF APPLICATION OF CONCENTRATED LOADS ARE MUCH HIGHER THAN THE AVERAGE VALUE OF

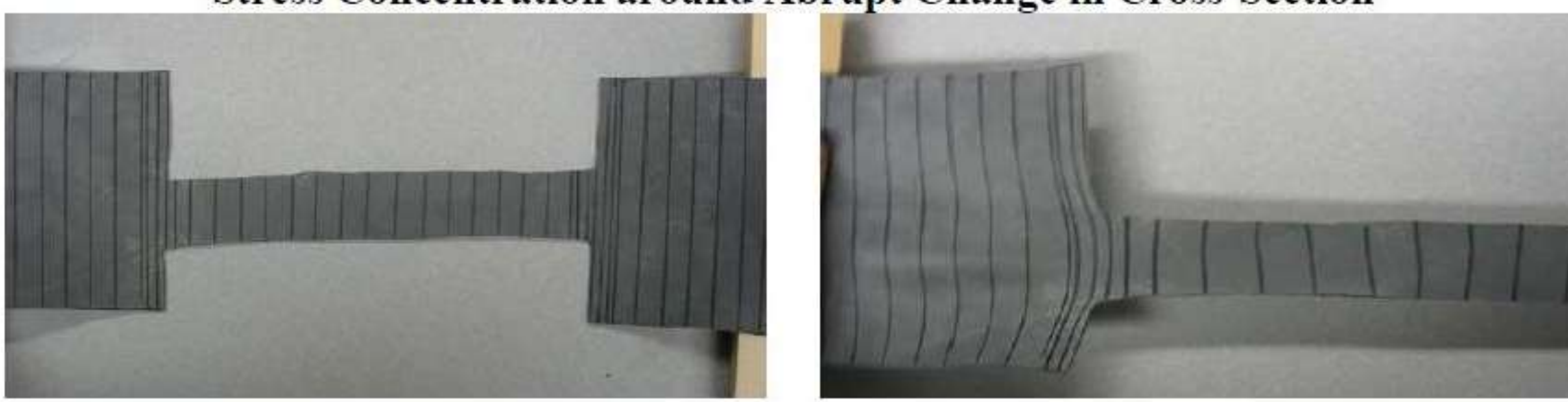
Stress Concentration at Point Load



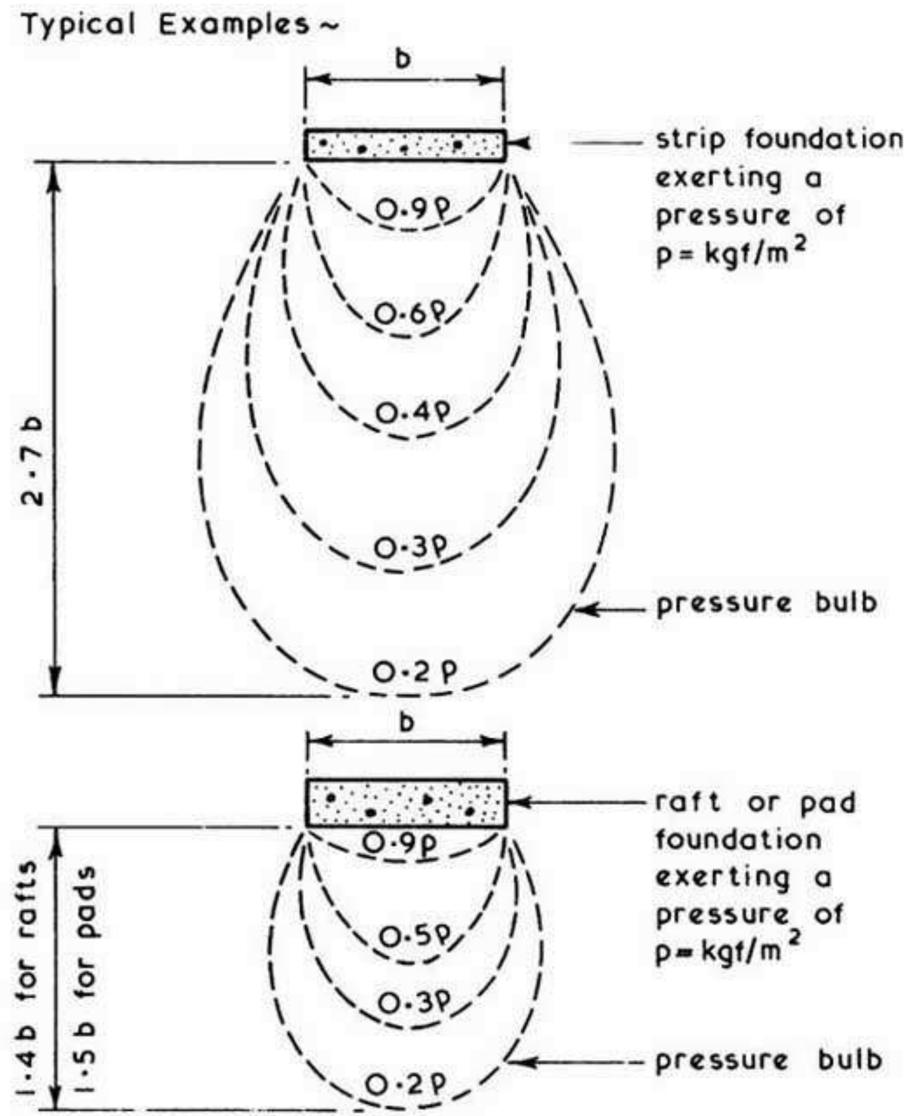
Stress Concentration around Hole



Stress Concentration around Abrupt Change in Cross-Section



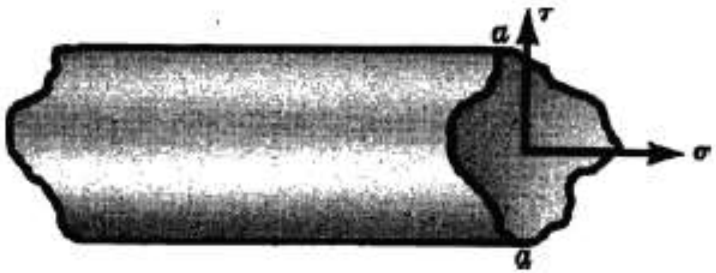
Application of Saint-Venant's Principle (*http://vlab.amrita.edu/?sub=77&brch=299&sim=1674&cnt=1)*



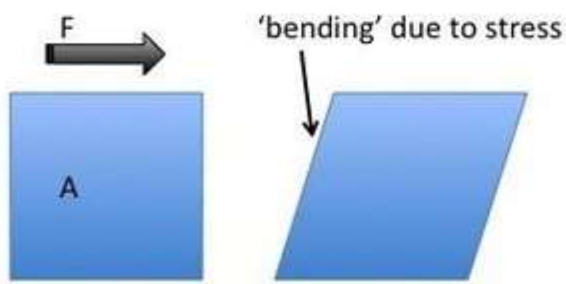
Pressure bulb under strip and raft foundations, notice the effect of surficial loads with depth

STRESSES CAUSED BY AXIAL LOADING

τ: Shear Stress



σ: Normal Stress



A simple illustration of stress is given by considering a cylindrical test specimen, with uniform Section of radius r, subjected to an axial compressive force F as shown in Figure 1.1(a). Assuming the force acts uniformly across the Section of the specimen, the stress σ_{n0} on a plane PQ perpendicular to the direction of the force, as shown in Figure 1.1(a), is given by

$$\sigma_{n0} = \frac{F}{A} \qquad (1.1)$$

where A is the cross-sectional area of the specimen. As this is the only stress acting across the plane, and it is perpendicular to the plane, σ_{n0} is a principal stress.

Consider now a plane such as PR in Figure 1.1(b), inclined at an angle θ to the radial planes on which σ_{n0} acts. The force F has components N acting normal (perpendicular) to the plane and T acting along the plane, in the direction of maximum inclination θ. Thus

$$N = F\cos\theta \qquad (1.2a)$$

$$T = F\sin\theta \qquad (1.2b)$$

As the inclined plane is an ellipse with area $A/\cos\theta$, the direct stress $\sigma_{n\theta}$ normal to the plane and shear stress τ_θ along the plane, in the direction of maximum inclination, are given by:

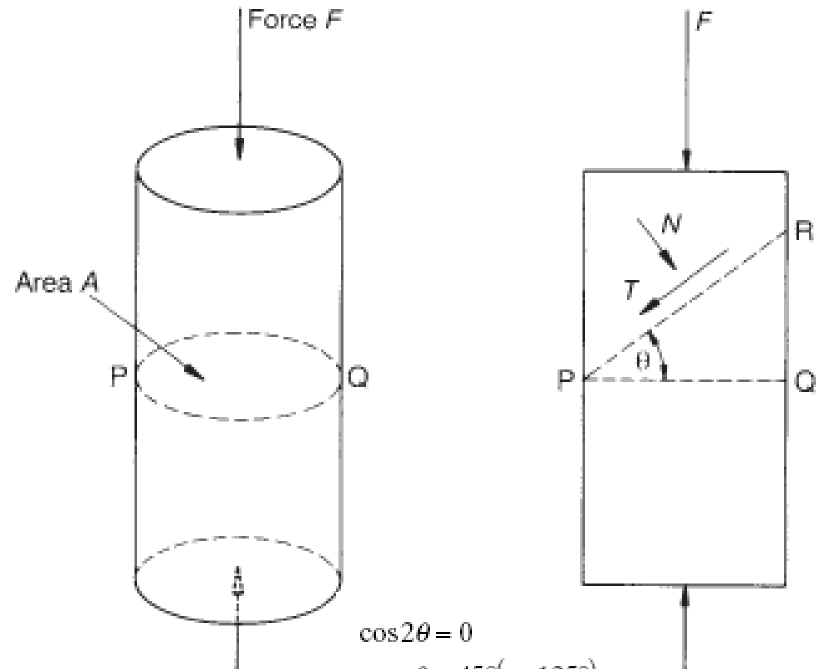
$$\sigma_{n\theta} = \frac{N\cos\theta}{A} = \frac{F}{A}\cos^2\theta \qquad (1.3a)$$

$$\tau_\theta = \frac{T\cos\theta}{A} = \frac{F}{2A}\sin 2\theta \qquad (1.3b)$$

It is obvious by inspection that the maximum normal stress, equal to F/A, acts on radial planes. The magnitude and direction of the maximum value of τ_θ can be found by differentiating equation 1.3b:

$$\frac{d\tau_\theta}{d\theta} = \frac{F}{A}\cos 2\theta$$

The maximum value of τ_θ is found by putting $d\tau_\theta/d\theta = 0$, thus:



$$\cos 2\theta = 0$$

$$\theta = 45^\circ \left(\text{or } 135^\circ\right)$$

$$\tau_{\theta\max} = \frac{F}{2A}$$

Ref : Mohr Circles, Stress Paths and Geotechnics. (Parry 2004)

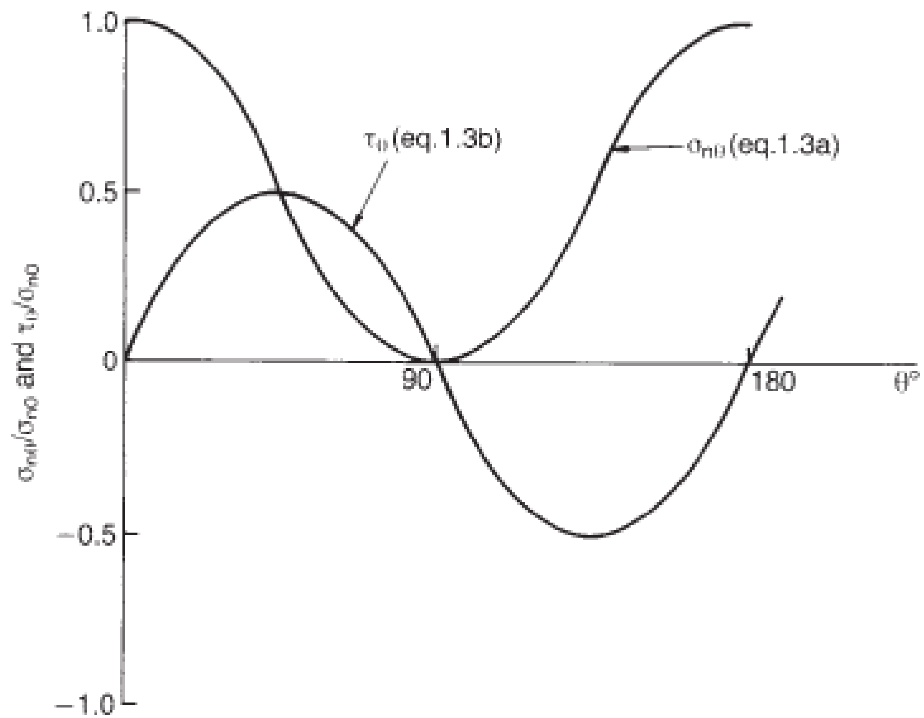


Figure 1.2 Variation of normal stress $\sigma_{n\theta}$ and shear stress τ_θ with angle of plane θ in cylindrical test specimen.

The variations of $\sigma_{n\theta}$ and τ_θ with θ, given by equations 1.3a and l.3b, are shown in Figure 1.2. It can be seen that $\tau_{\theta max}$ occurs on a plane with $\theta = 45°$ and $\sigma_{n\theta max}$ on a plane with $\theta = 0°$.