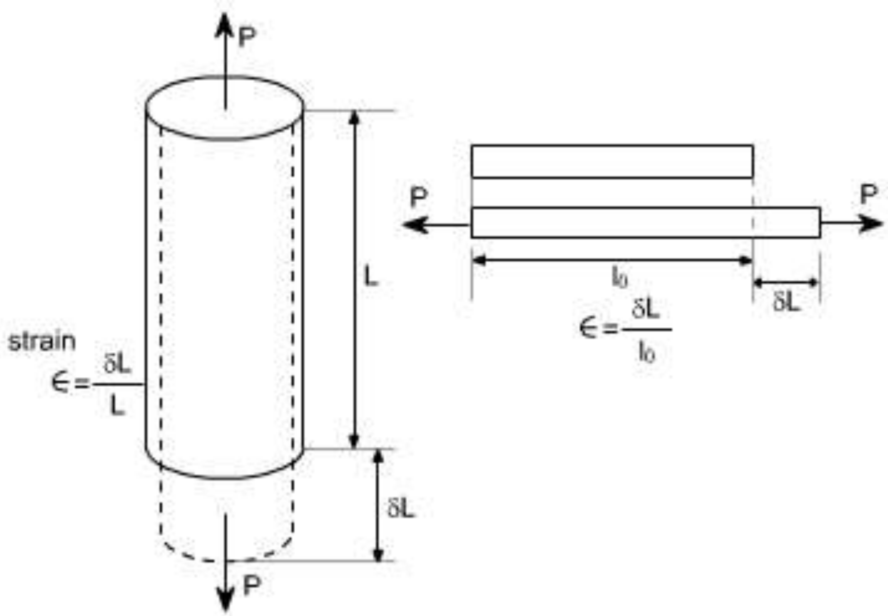
STRAIN CONCEPT

In case of any type of external force is applied on a rigid/deformable body, some change takes place in the dimension of the body. The ratio of this change of dimension in the body to its actual length is called strain.

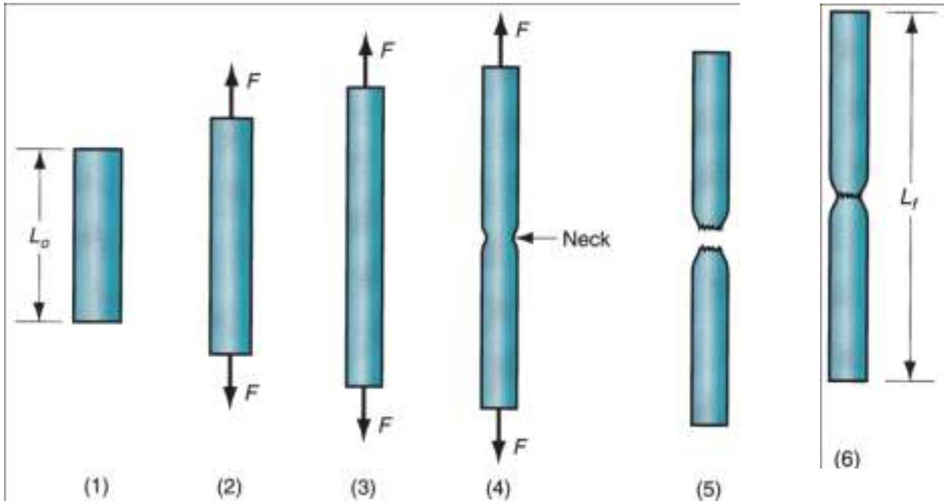
Tensile strain: The strain produced in a body due to "*Tensile Force*". Tensile force always results in increment of the length and decrease in the cross section area of the body. In this case the ratio of the increase in length to the original length is called tensile strain.

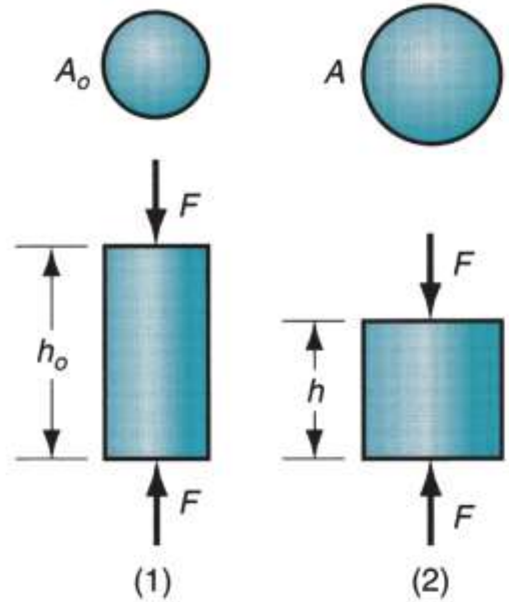
Compressive strain: In compressive force there is a decrease in the dimension of the body. The ratio of the decrease in the length of the body to the original length is called compressive strain.



Tensile Stress and elongation

Compressive Stress and shortening





Tensile Stress (Extension)

1. no load
2. uniform elongation and area reduction
3. maximum load
4. Necking
5. Fracture
6. putting pieces back together to measure final length

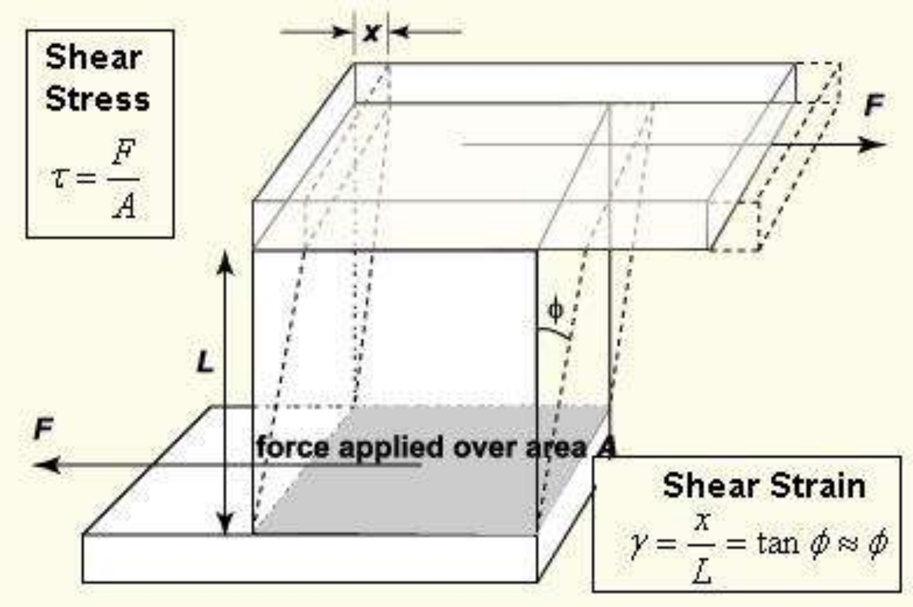
Compression

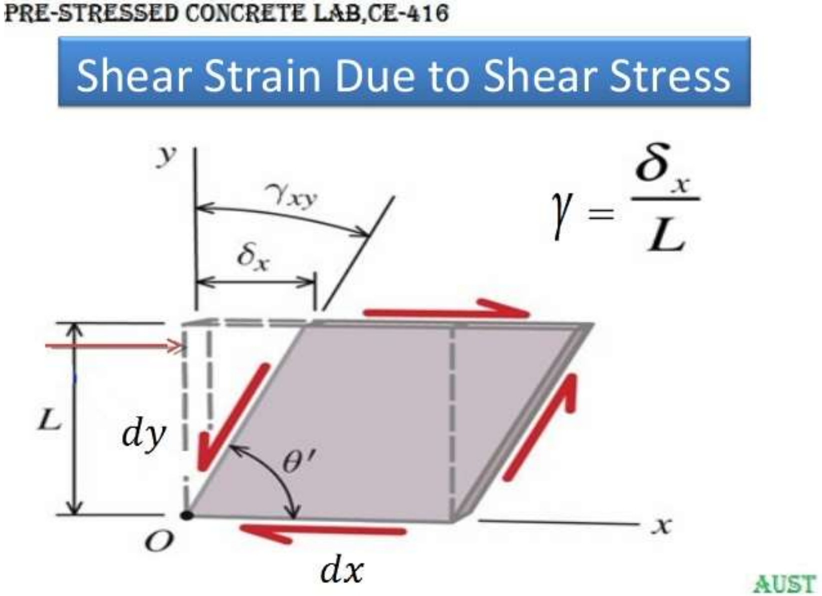
As the specimen is compressed, height is reduced and cross-sectional area is increased

$$\sigma_e = -\frac{F}{A_o}$$

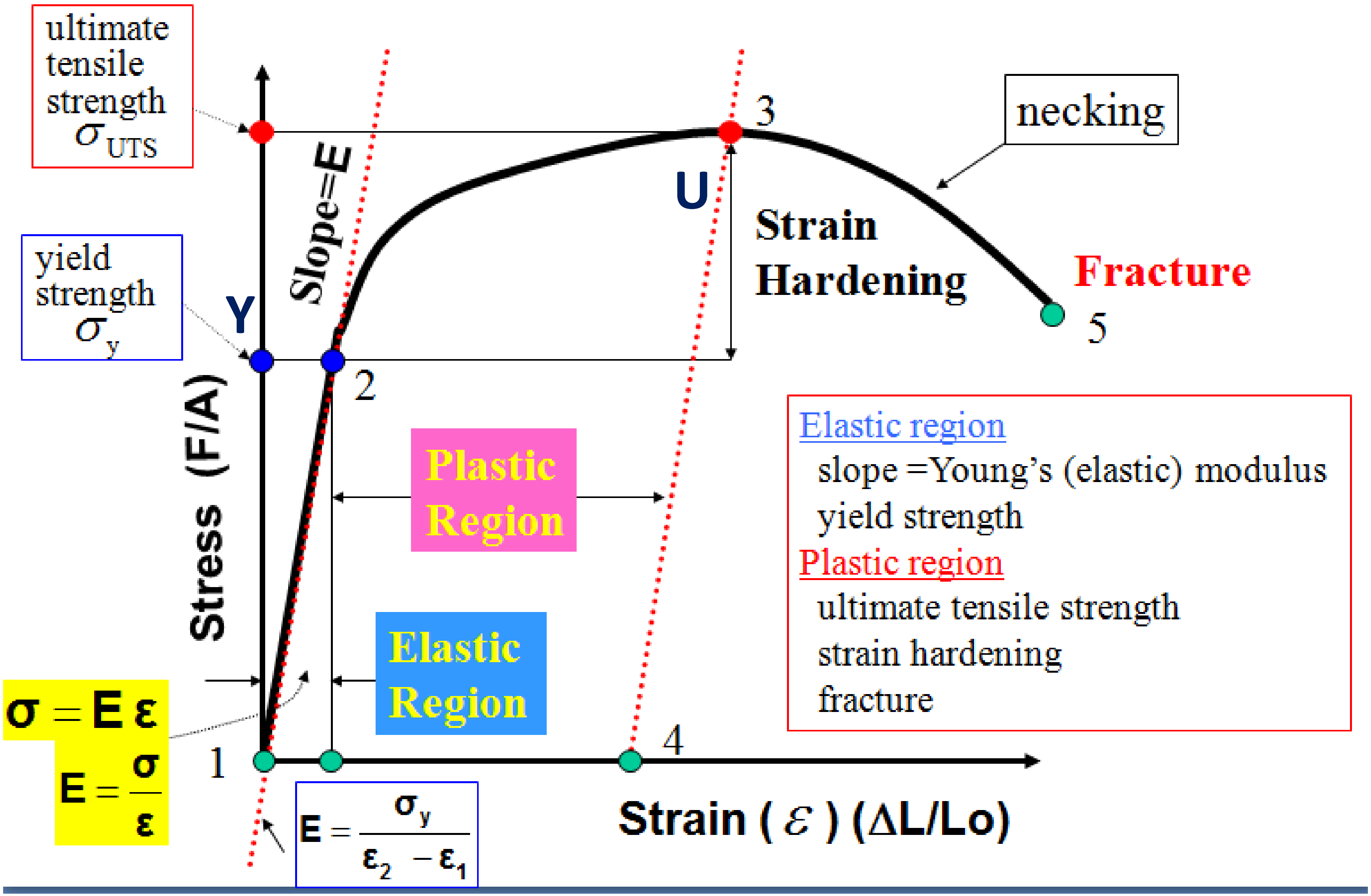
A_o = original area of the specimen

<u>Shear strain:</u> The strain which is produced in a body due to shear force is called shear strain. It is the radian measure through which the body is distorted due to shearing.





3. Volumetric strain: The ratio of the change in the volume of a body to the original volume is called the volumetric strain. In volumetric strain there is a change in the volume of the body due to application of the external forces.



Generalized "**STRESS – STRAIN**" relation of materials

Proportional Limit : *The ordinate of the point "**2**" is known as the proportional limit (i.e., the maximum stress that may be developed during a simple tension test) such that the stress is a linear function of strain.*

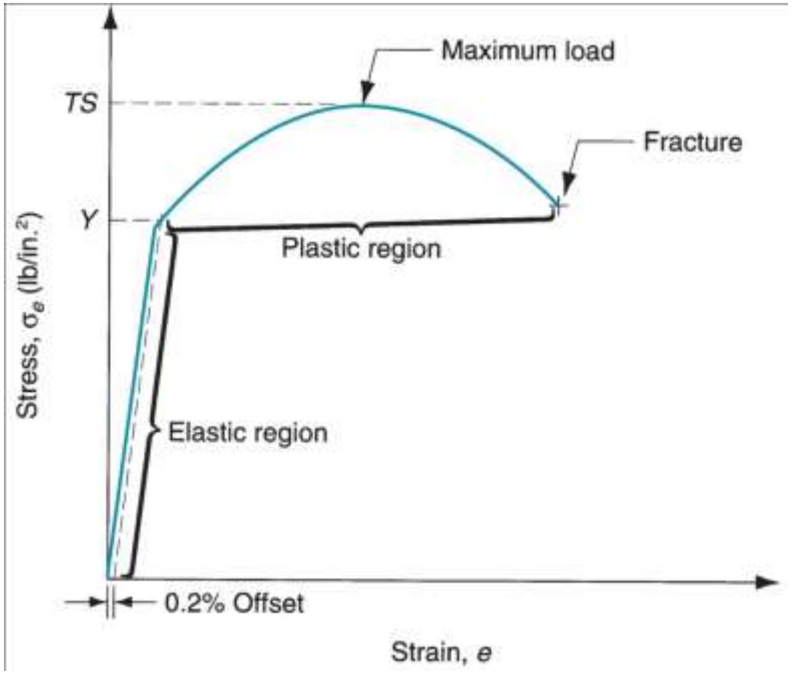
Elastic Limit : *The ordinate of a point almost coincident with "**2**" is known as the elastic limit (i.e., the maximum stress that may be developed during a simple tension test such that there is no permanent or residual deformation when the load is entirely removed). In such case, no permanent strain occurs*

Elastic and Plastic Regions : *The region of the stress-strain curve extending from the origin to the proportional limit is called the elastic range. The region of the stress-strain curve extending from the proportional limit to the point of rupture is called the plastic range. There will be some permament strain after the plastic region (limit) is reached.*

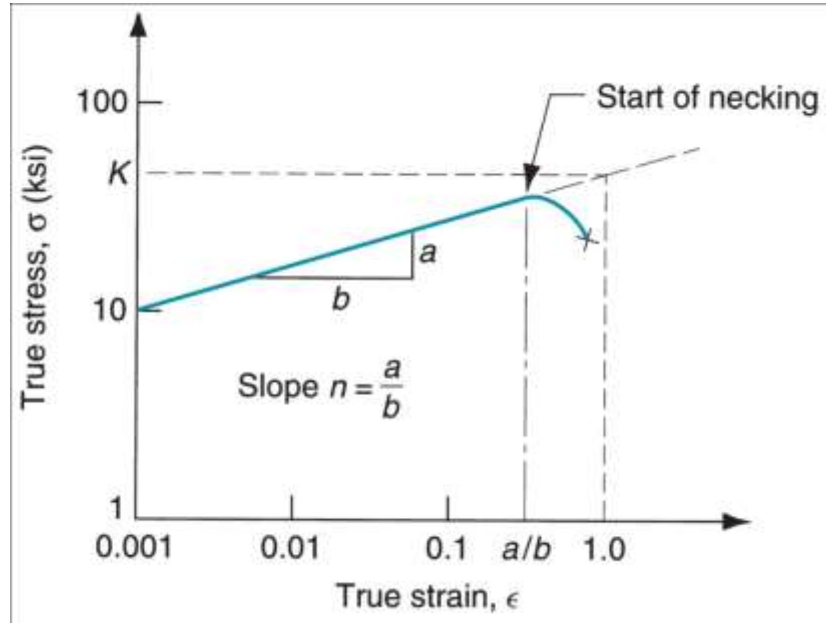
Yield Point : *The ordinate of the point "**Y**" at which there is an increase in strain with no increase in stress, is known as the yield point of the material. After loading has progressed to the point Y, yielding is said to take place.*

Ultimate Strength or Tensile Strength : *The ordinate of the point "**U**" is known either as the ultimate strength or the tensile/compressive strength of the material.*

Fracture (Breaking Strength) : *The ordinate of the point"**5**" is called the breaking strength of the material. After such point is reached, the material is subjected to physical breakdown*



Typical srtess-strain curve



Flow Curve

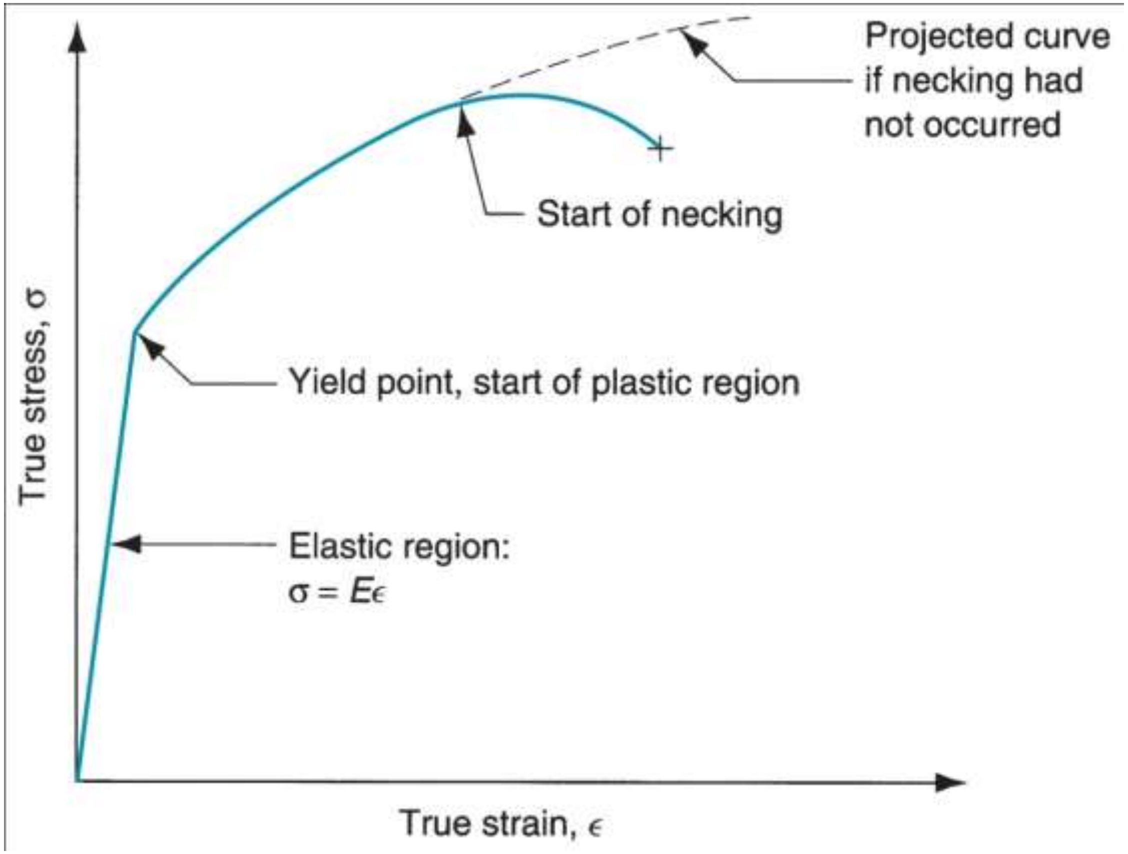
Because it is a straight line in a log-log plot, the relationship between true stress and true strain in the plastic region is

$$\sigma = K\varepsilon^n$$

K = strength coefficient;

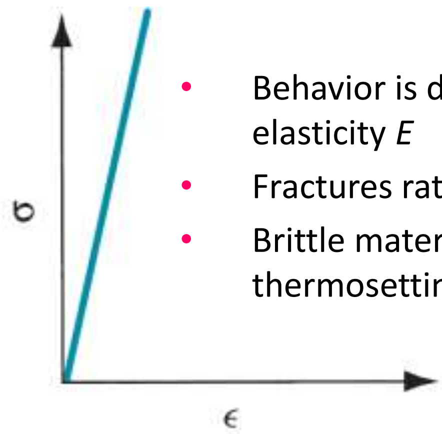
n = strain hardening exponent

True Stress-Strain Curve



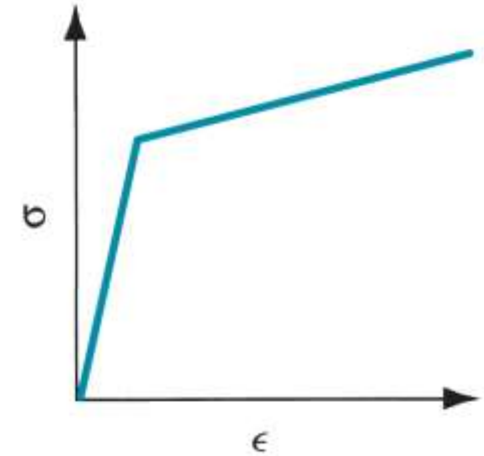
- Note that true stress increases continuously in the plastic region until necking
  - In the engineering stress-strain curve, the significance of this was lost because stress was based on the original area value
- It means that material gets stronger as strain increases
  - This is the property called ***STRAIN HARDENING***

PERFECTLY ELASTIC



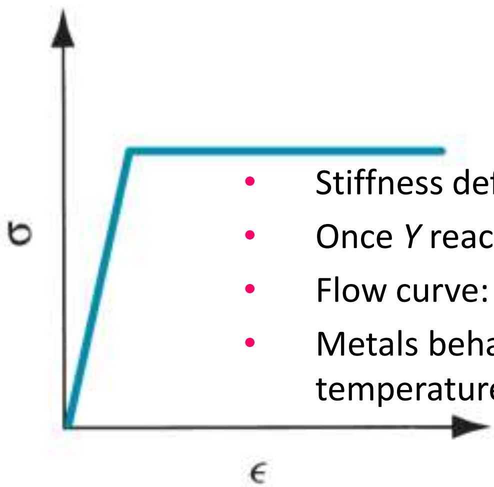
- Behavior is defined completely by modulus of elasticity *E*
- Fractures rather than yielding to plastic flow
- Brittle materials: ceramics, many cast irons, and thermosetting polymers

ELASTIC AND STRAIN HARDENING

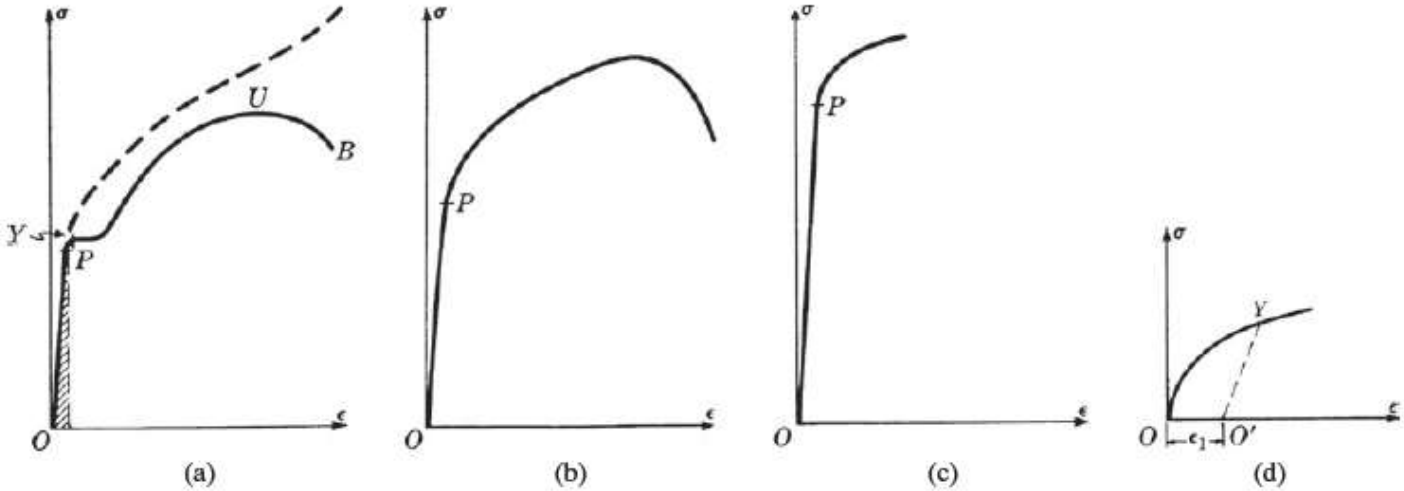


- Hooke's Law in elastic region, yields at *Y*
- Flow curve: $K > Y$, $n > 0$
- Most ductile metals behave this way when cold worked

PERFECTLY PLASTIC



- Stiffness defined by *E*
- Once *Y* reached, deforms plastically at same stress level
- Flow curve: $K = Y$, $n = 0$
- Metals behave like this when heated to sufficiently high temperatures (above recrystallization)



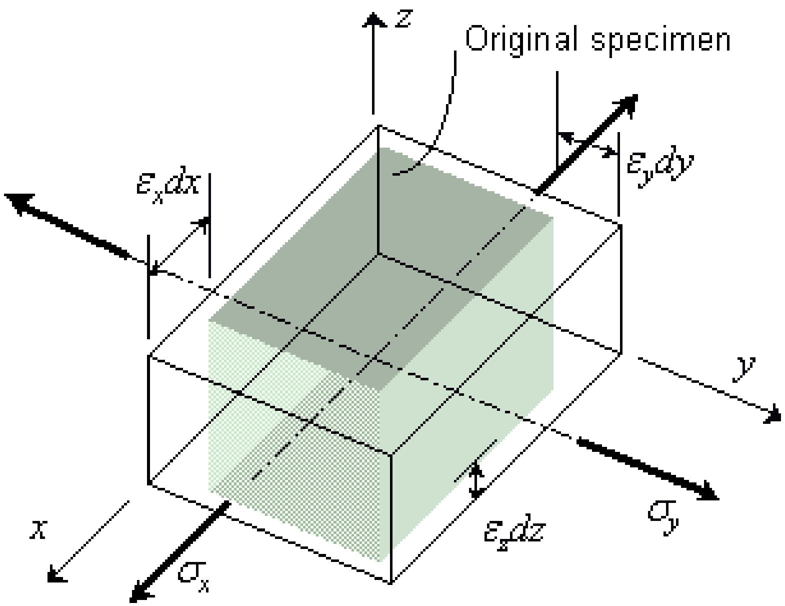
(a) Carbon-structured steel
(b) Alloy steel
(c) Hard steel
(d) Cast iron

Ductile and Brittle Materials

Metallic engineering materials are commonly classified as either *ductile* or *brittle* materials. A *ductile material* is one having a relatively large tensile strain up to the point of rupture (for example, structural steel or aluminum) whereas a *brittle material* has a relatively small strain up to this same point. An arbitrary strain of 0.05 mm/mm is frequently taken as the dividing line between these two classes of materials. Cast iron and concrete are examples of brittle materials.

HOOKE'S LAW

Hooke's law states that, within the limit of elasticity of a material, strain is proportional to applied stress on materials. The linear relationship between stress and strain applies for $0 \le \sigma \le \sigma_{Yield}$



$$\epsilon = \frac{\sigma}{E}$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \quad \text{or} \quad \epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \quad \text{or} \quad \epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E} \quad \text{or} \quad \epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

E : Young's Modulus and ν: Poisson Ratio

The generalized Hooke's Law also reveals that strain can exist without stress. For example, if the member is experiencing a load in the y-direction (which in turn causes a stress in the y-direction), the Hooke's Law shows that strain in the x-direction does not equal to zero. This is because as material is being pulled outward by the y-plane, the material in the x-plane moves inward to fill in the space once occupied, just like an elastic band becomes thinner as you try to pull it apart. In this situation, the x-plane does not have any external force acting on them but they experience a change in length. Therefore, it is valid to say that strain exist without stress in the x-plane.

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \varepsilon_{yz}\\ \varepsilon_{zx}\\ \varepsilon_{xy}\end{bmatrix}=\frac{1}{E}\begin{bmatrix}1 & -\nu & -\nu & 0 & 0 & 0\\ -\nu & 1 & -\nu & 0 & 0 & 0\\ -\nu & -\nu & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1+\nu & 0 & 0\\ 0 & 0 & 0 & 0 & 1+\nu & 0\\ 0 & 0 & 0 & 0 & 0 & 1+\nu\end{bmatrix}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{zx}\\ \sigma_{xy}\end{bmatrix}$$

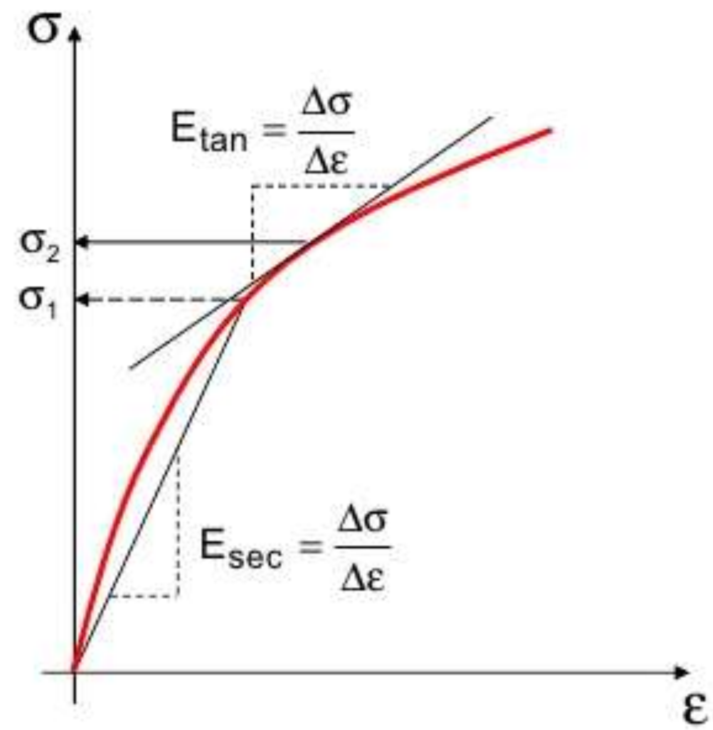
Compliance matrix of Hooke's law

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{zx}\\ \sigma_{xy}\end{bmatrix}=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu & 0 & 0 & 0\\ \nu & 1-\nu & \nu & 0 & 0 & 0\\ \nu & \nu & 1-\nu & 0 & 0 & 0\\ 0 & 0 & 0 & 1-2\nu & 0 & 0\\ 0 & 0 & 0 & 0 & 1-2\nu & 0\\ 0 & 0 & 0 & 0 & 0 & 1-2\nu\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \varepsilon_{yz}\\ \varepsilon_{zx}\\ \varepsilon_{xy}\end{bmatrix}$$

Stiffness matrix of Hooke's law (inverse of compliance matrix)

YOUNG MODULUS (MODULUS of ELASTICITY)

- Some materials (cast iron, concrete) do not have a linear elastic portion of the stress-strain curve
- For this *nonlinear* behaviour, *Secant* and *Tangent* Modulii are used
- Secant modulus:
  - Slope of straight line between origin and a point on stress-strain curve
- Tangent modulus:
  - Slope of the stress-strain curve at a specified level of stress



Ref : Structures and Materials- Section 4 Behaviour of Materials (Loughborough Univ.)

SHEAR MODULUS : The ratio of shear stress to engineering shear strain on the loading plane. The shear modulus ”***G***” is also known as the “**Rigidity modulus**”, and is equivalent to the 2nd Lamé constant mmentioned in books on continuum theory. Common sense and the 2nd Law of Thermodynamics require that a positive shear stress leads to a positive shear strain. Therefore, the shear modulus *G* is required to be positive for all materials.

$$G = \frac{\sigma_{xy}}{\varepsilon_{xy} + \varepsilon_{yx}} = \frac{\sigma_{xy}}{2\varepsilon_{xy}} = \frac{\sigma_{xy}}{\gamma_{xy}} \qquad \gamma_{xy} = \varepsilon_{xy} + \varepsilon_{yx} = 2\varepsilon_{xy}$$

$$= \frac{E}{2(1+\nu)}$$

BULK MODULUS : When an isotropic material specimen is subjected to hydrostatic pressures, all shear stress will be zero and the normal stress will be uniform ($\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma$). In response to the hydrostatic load, the specimen will change its volume. Its resistance to do so is quantified as the bulk modulus ”***K***”, also known as the modulus of compression. Technically, *K* is defined as the ratio of hydrostatic pressure to the relative volume change (which is related to the direct strains). Common sense and the 2nd Law of Thermodynamics require that a positive hydrostatic load leads to a positive volume change. Therefore, the bulk modulus *K* is required to be nonnegative for all materials

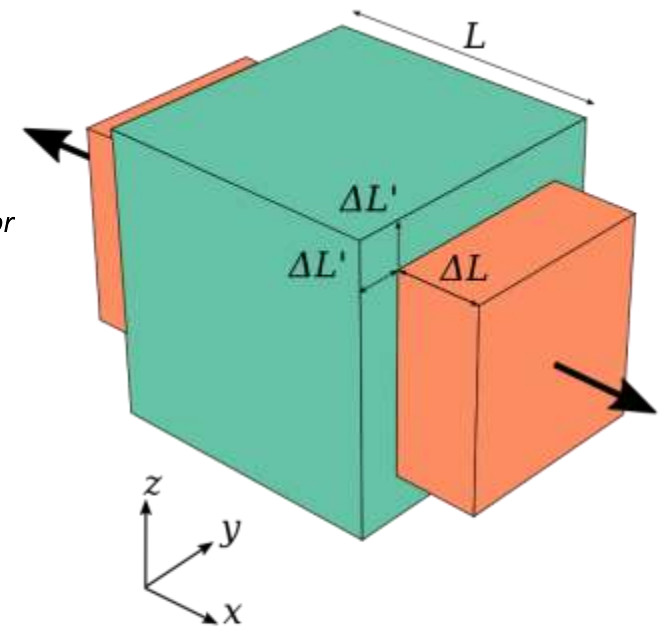
$$K = \frac{\sigma}{\Delta V / V} = \frac{\sigma}{\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}}$$

$$= \frac{E}{3(1-2\nu)}$$

POISSON'S RATIO

Poisson's ratio is a measure of the **Poisson effect**, the phenomenon in which a material tends to expand in directions perpendicular to the direction of compression. Conversely, if the material is stretched rather than compressed, it usually tends to contract in the directions transverse to the direction of stretching. It is a common observation when a rubber band is stretched, it becomes noticeably thinner. Again, the Poisson ratio will be the ratio of relative contraction to relative expansion and will have the same value as above. In certain rare cases, a material will actually shrink in the transverse direction when compressed (or expand when stretched) which will yield a negative value of the Poisson ratio.The Poisson's ratio of a stable, isotropic, linear elastic material will be greater than −1.0 or less than 0.5 because of the requirement for Young's modulus, the shear modulus and bulk modulus to have positive values. Most materials have Poisson's ratio values ranging between 0.0 and 0.5.

https://commons.wikimedia.org/wiki/File:PoissonRatio.svg



$$\nu = -\frac{\text{Strain in direction of load}}{\text{Strain at right angle to load}}$$

$$\nu = -\frac{\varepsilon_{lateral}}{\varepsilon_{axial}}$$

Material	Young's Modulus		Shear Modulus		Bulk Modulus	
	10^{10} N/m²	*10^{6} lb/in²*	*10^{10} N/m²*	*10^{6} lb/in²*	*10^{10} N/m²*	*10^{6} lb/in²*
Aluminum	7.0	10	2.4	3.4	7.0	10
Brass	9.1	13	3.6	5.1	6.1	8.5
Copper	11	16	4.2	6.0	14	20
Glass	5.5	7.8	2.3	3.3	3.7	5.2
Iron	9.1	13	7.0	10	10	14
Lead	1.6	2.3	0.56	0.8	0.77	1.1
Steel	20	29	8.4	12	16	23

Ref: Engineering toolbox web site

STRESS and STRAIN in GEOLOGY

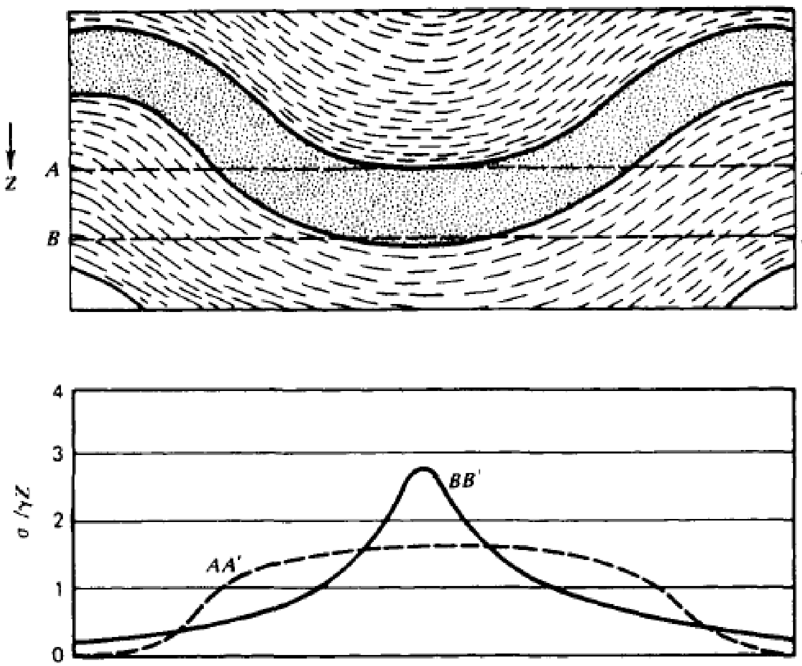


Figure 4.3 The influence of folds in heterogeneous, layered rock on vertical stresses.

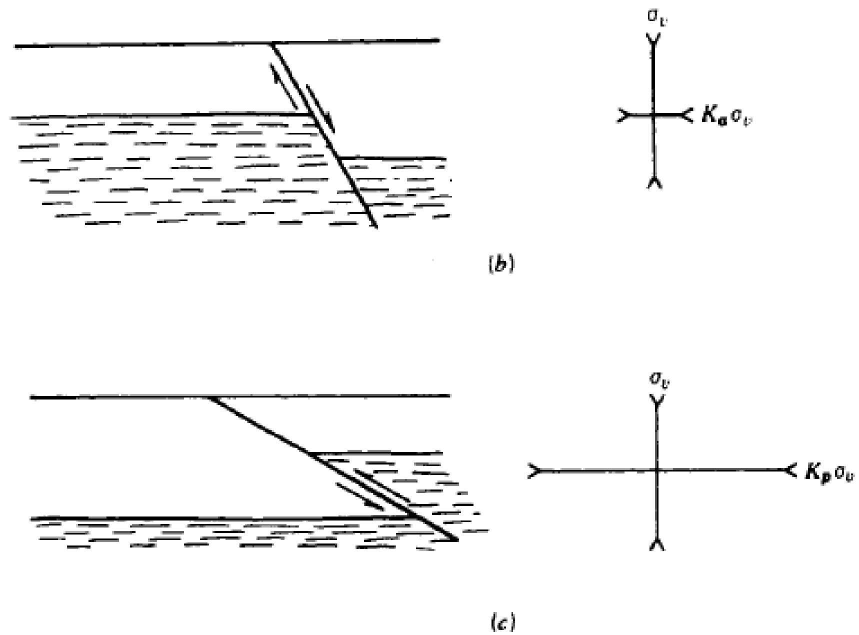
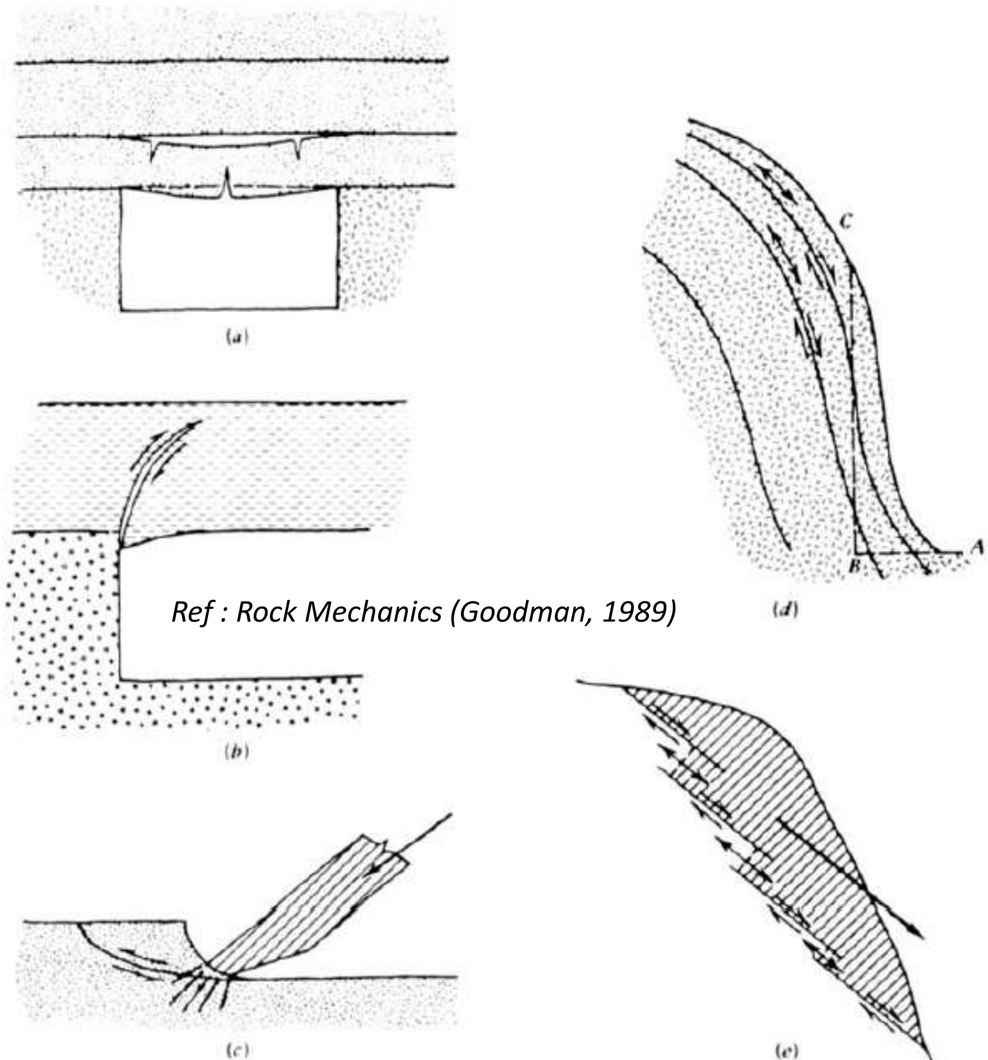


Figure 4.6 Stresses required to initiate normal and reverse faults.

Ref : Rock Mechanics (Goodman, 1989)



Ref : Rock Mechanics (Goodman, 1989)

Figure 3.1 Examples of failure modes involving breakage of rock. (*a*) Flexure. (*b*) Shear. (*c*) Crushing and tensile cracking, followed by shear. (*d* and *e*) Direct tension.

$$G = \frac{E}{2(1+\nu)} \qquad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

***Table 6.1** Modulus Ratio $\mathbf{E}/\mathbf{q}_u$ and Poisson's Ratio ν for the Rock Specimens of Table 3.1*[a]

Description	E/q_u	ν
Berea sandstone	261	0.38
Navajo sandstone	183	0.46
Tensleep sandstone	264	0.11
Hackensack siltstone	214	0.22
Monticello Dam greywacke	253	0.08
Solenhofen limestone	260	0.29
Bedford limestone	559	0.29
Tavernalle limestone	570	0.30
Oneota dolomite	505	0.34
Lockport dolomite	565	0.34
Flaming Gorge shale	157	0.25
Micaceous shale	148	0.29
Dworshak Dam gneiss	331	0.34
Quartz mica schist	375	0.31
Baraboo quartzite	276	0.11
Taconic marble	773	0.40
Cherokee marble	834	0.25
Nevada Test Site granite	523	0.22
Pikes Peak granite	312	0.18
Cedar City tonalite	189	0.17
Palisades diabase	339	0.28
Nevada Test Site basalt	236	0.32
John Day Basalt	236	0.29
Nevada Test Site tuff	323	0.29

[a] *E* reported here includes both recoverable and nonrecoverable deformation, mixed in unknown proportions.

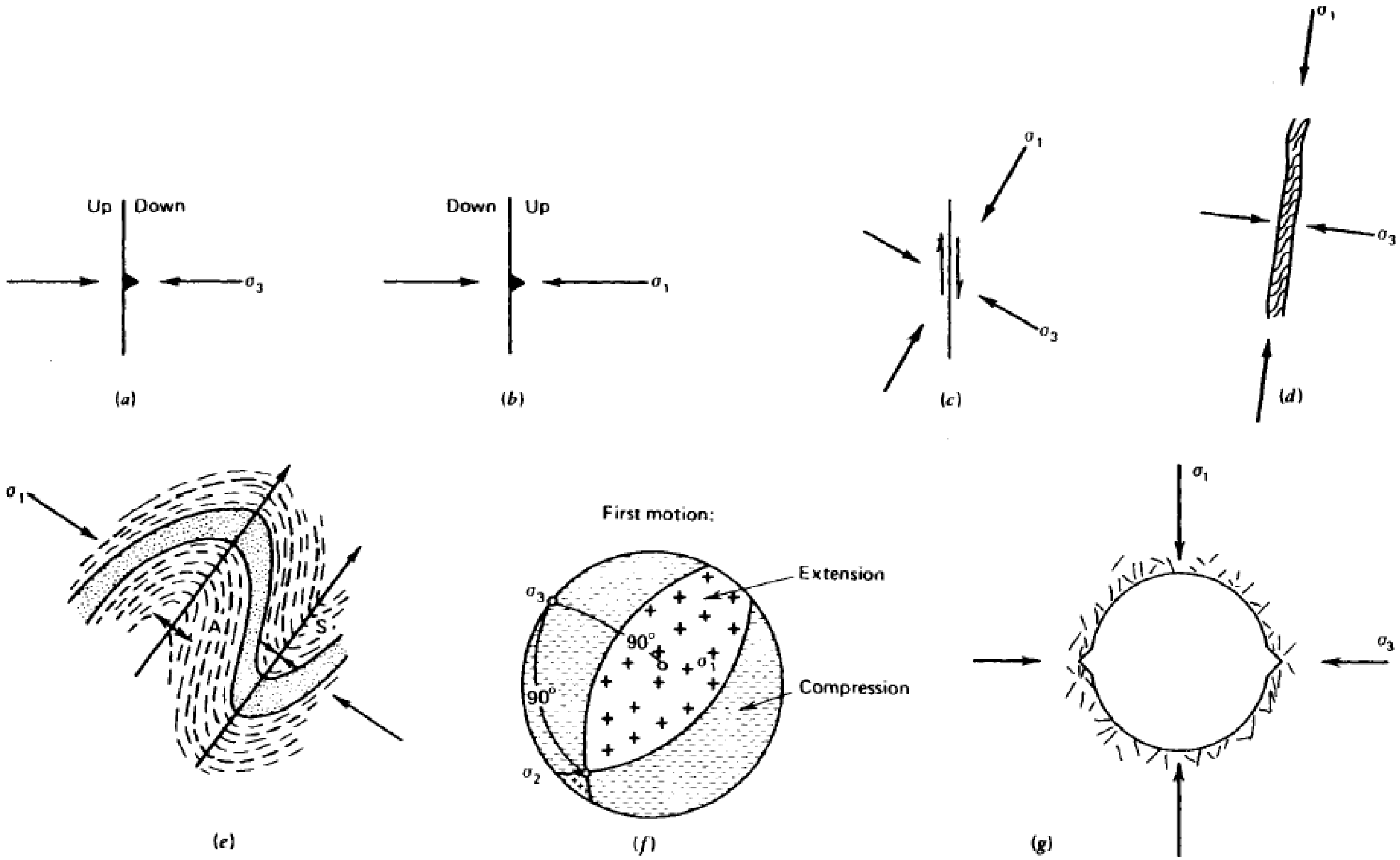
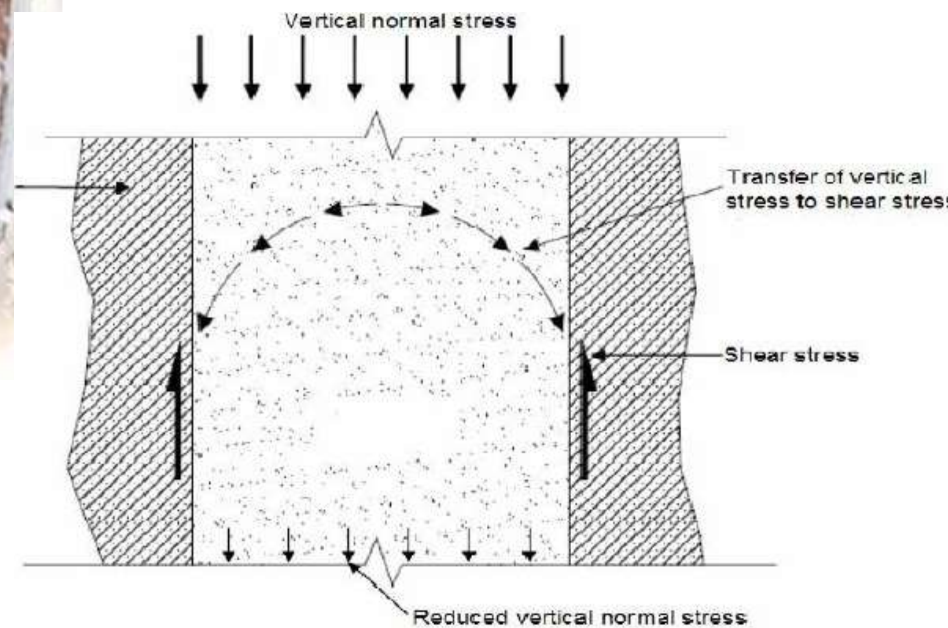
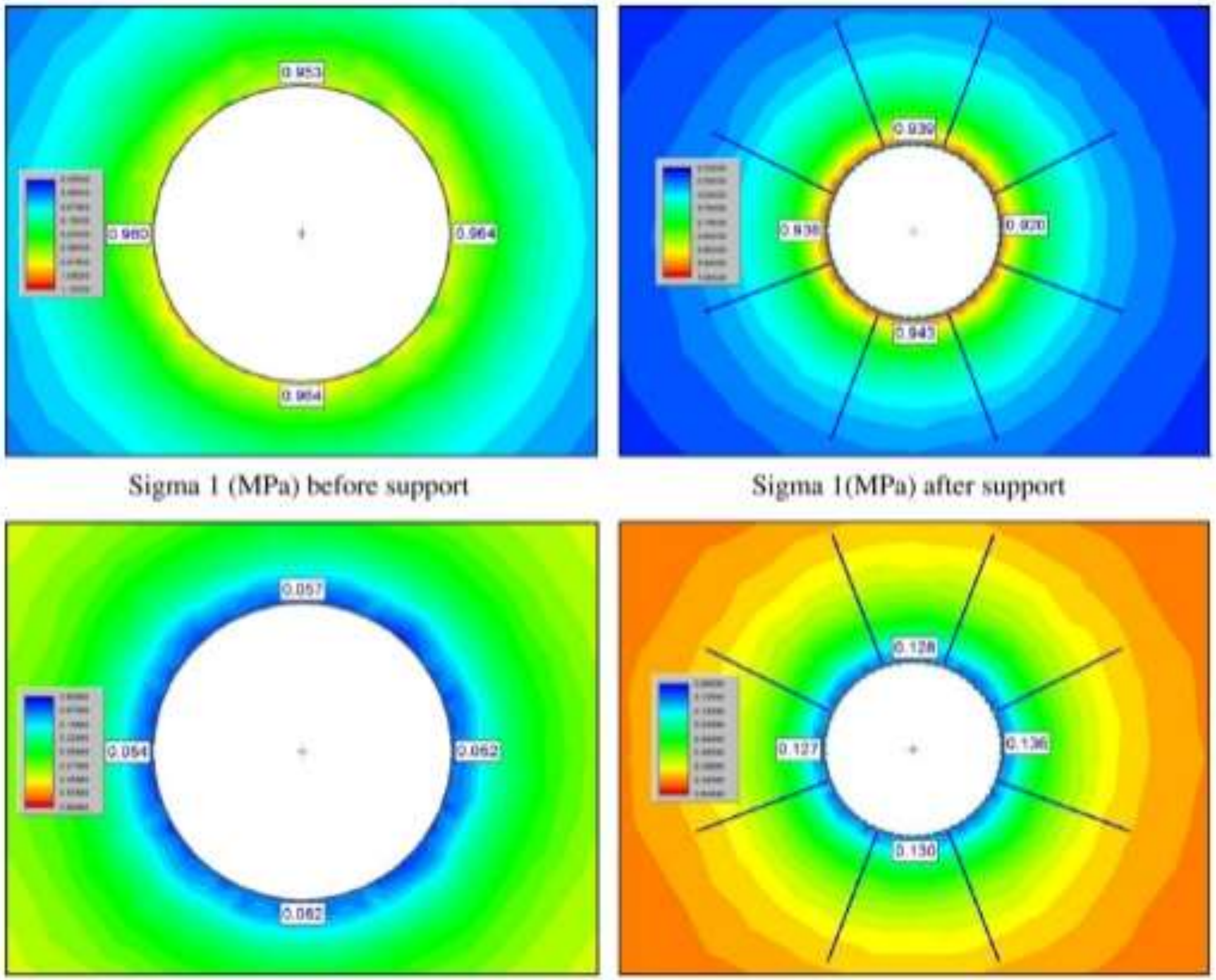


Figure 4.8 Directions of stresses inferred from geologic features. (*a*) to (*e*) are plan views. (*a*) Normal fault. (*b*) Reverse fault. (*c*) Strike slip fault. (*d*) Dike. (*e*) Folds. (*f*) Stereographic projection of first motion vectors from an earthquake. (*g*) Relation of stress directions to bore-hole breakouts.

Ref : Rock Mechanics (Goodman, 1989)



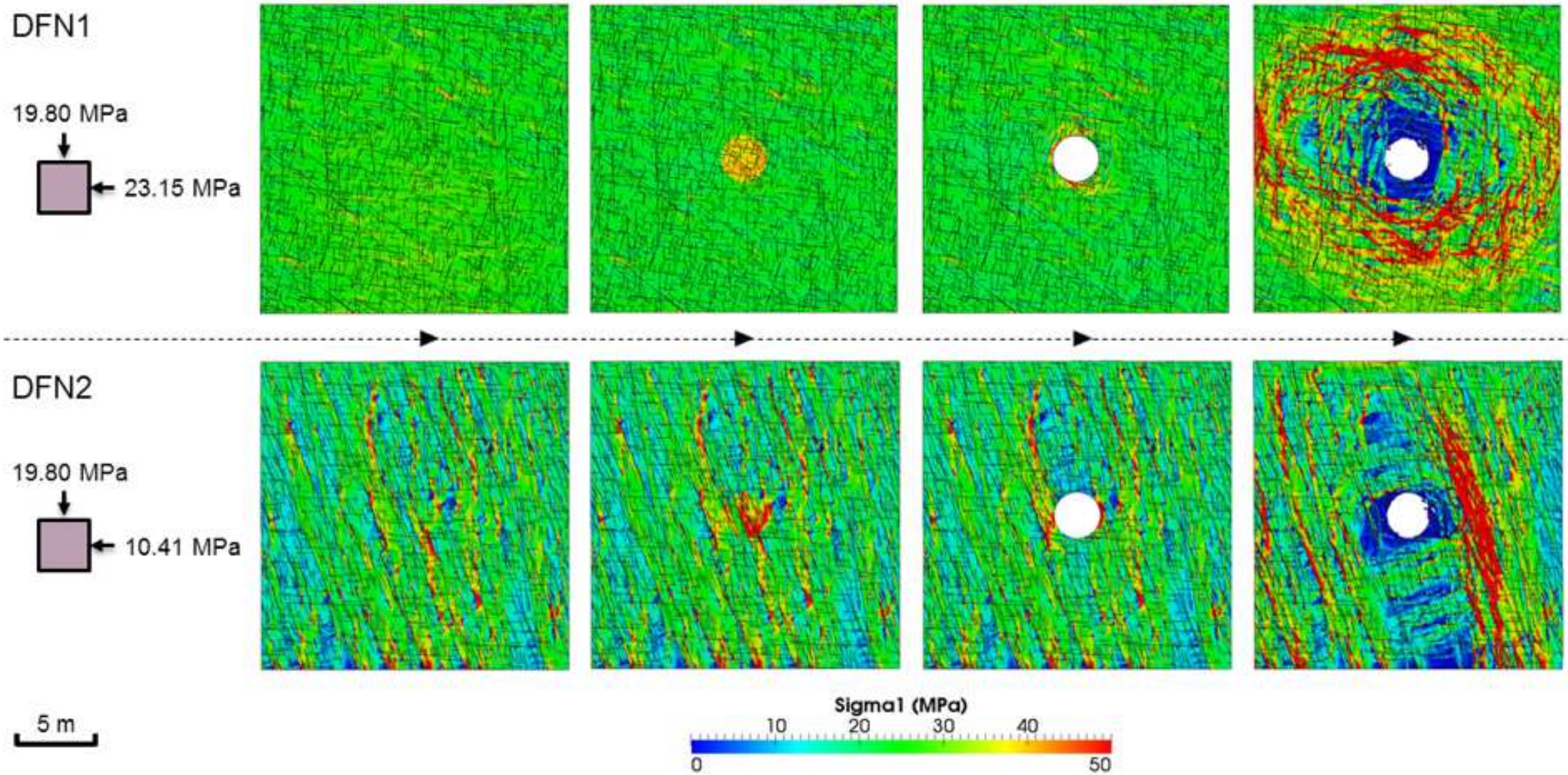
Distribution of stresses in a rock mass



Sigma 1 (MPa) before support — Sigma 1(MPa) after support

Sigma 3 (MPa) before support — Sigma 3(MPa) after support

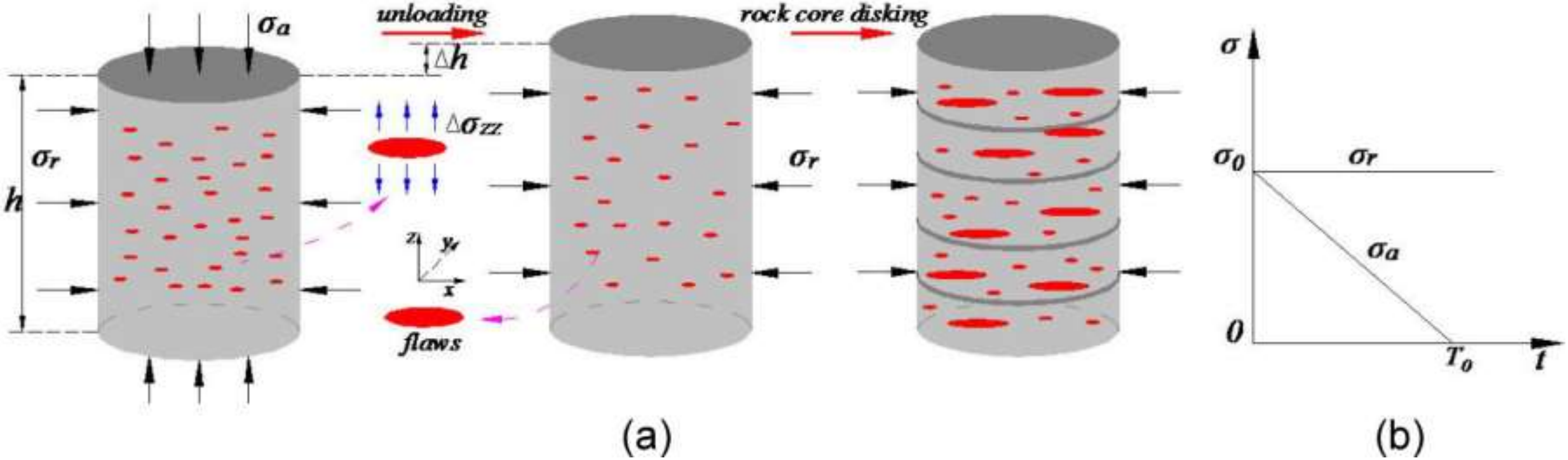
Ref: Stresses around tunnel before and after support for basalts (Gurocak, et al. 2007)



Ref: Excavation Damaged Zone Modelling. Solidity Project

Ref: Rock Engineering (Hoek, 2000)

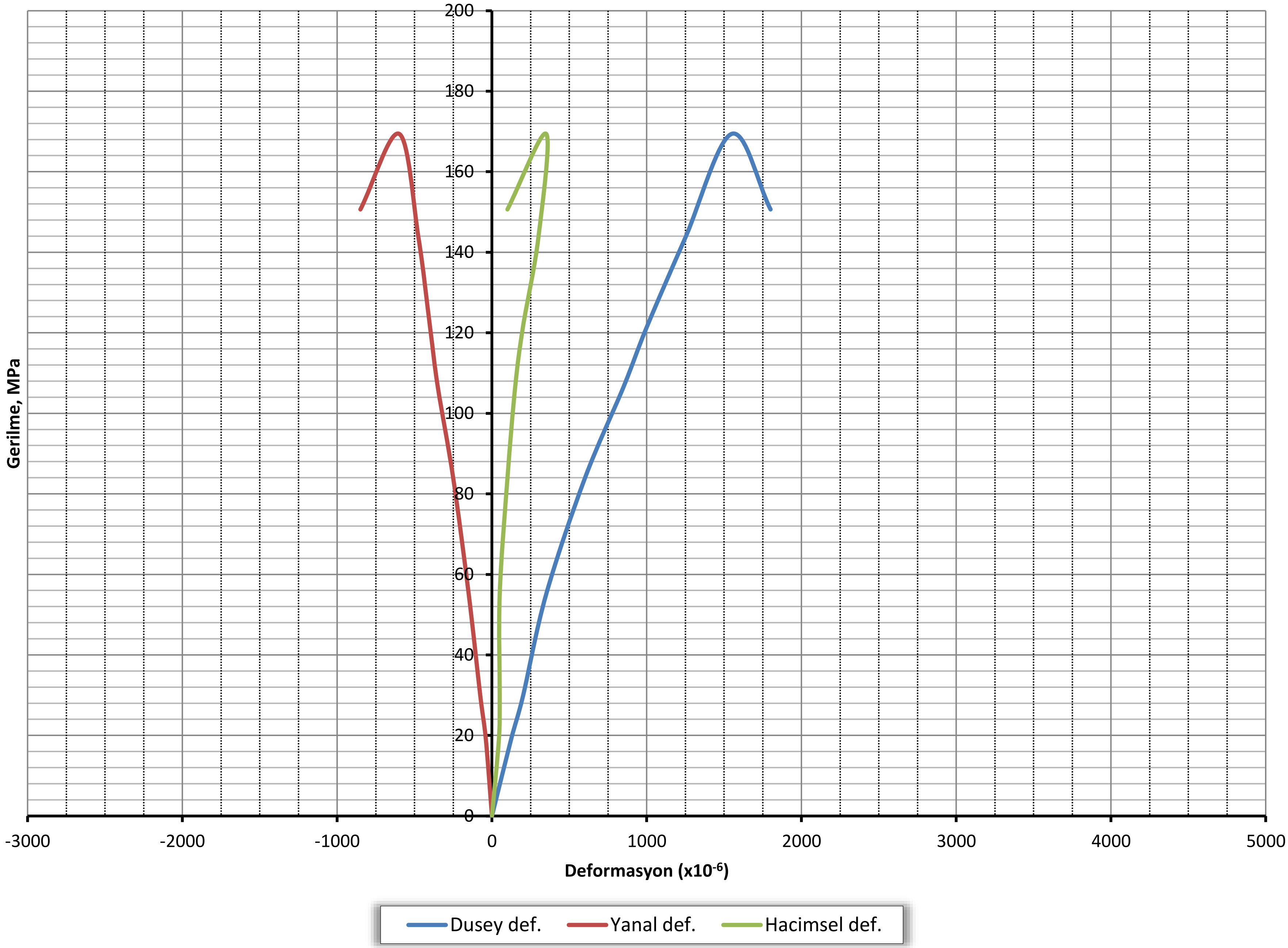
Figure 1.7: Disking of a 150 mm core of granite as a result of high in situ stresses.



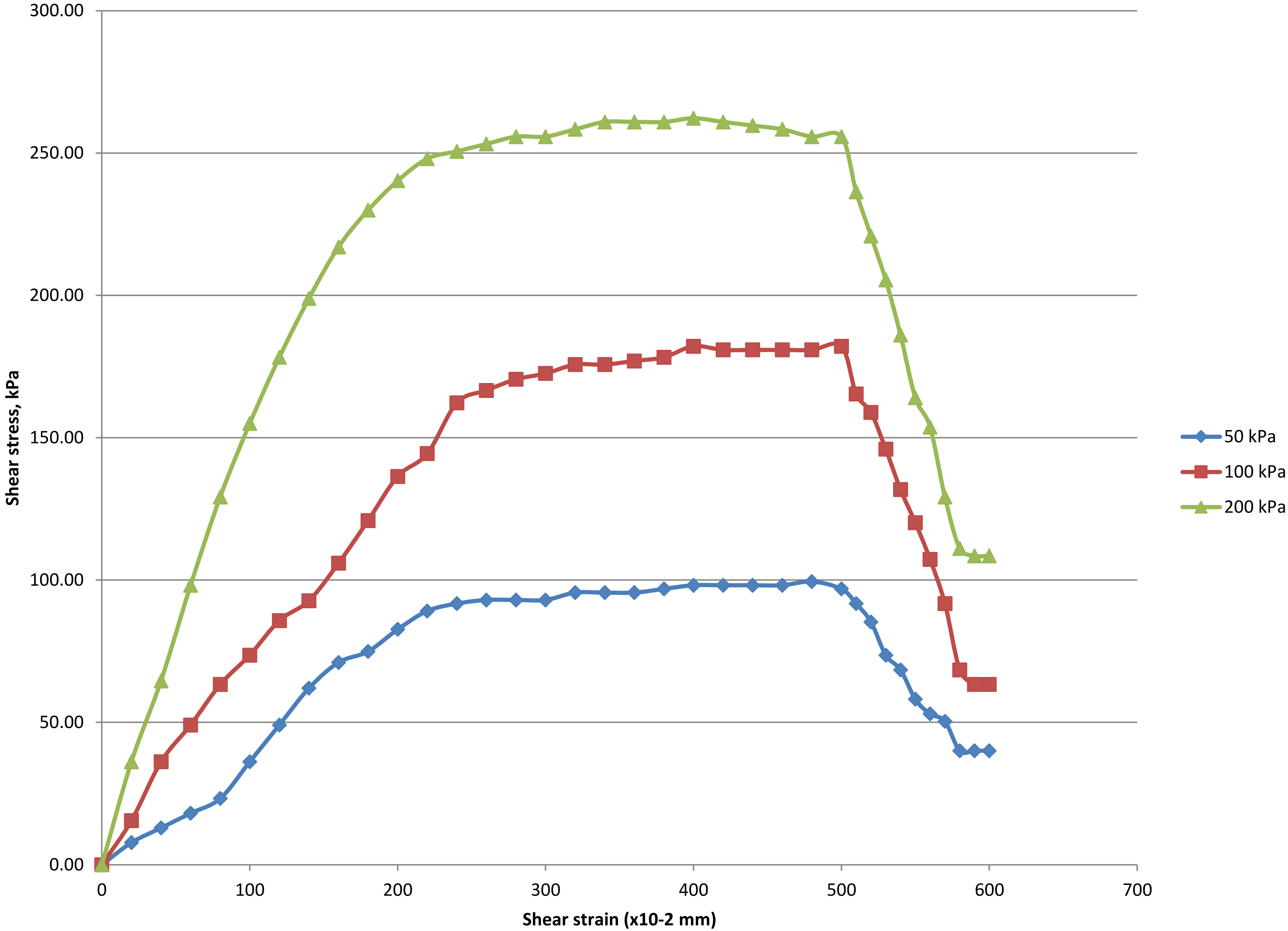
Ref: A Theoretical Explanation for Rock Core Disking in Triaxial Unloading Test by Considering Local Tensile Stress (Huang, et al. 2016)

Axial stress(Mpa)	Lateral strain ε_l (10^{-6})	Axial strainε_a(10^{-6})	Volumetric strainε_v (10^{-6})
0	0	0	0
19.21211933	-40	125	45
29.69145715	-75	200	50
55.01652354	-150	350	50
83.83470253	-250	600	100
106.5399345	-350	850	150
121.385663	-400	1000	200
143.6542559	-475	1250	300
169.4159614	-600	1550	350
150.6404811	-850	1800	100

Rock sample after normal vertical stress is applied


Gerilme, MPa
200
180
160
140
120
100
80
60
40
20
0
-3000
-2000
-1000
0
1000
2000
3000
4000
5000
Deformasyon (x10-6)
Dusey def.
Yanal def.
Hacimsel def.

Soil sample after shearing stress is applied



300.00
250.00
200.00
150.00
100.00
50.00
0.00
Shear stress, kPa
0
100
200
300
400
500
600
700
Shear strain (x10-2 mm)
50 kPa
100 kPa
200 kPa

SUMMARY

- Strain arises from varying types of loadings on materials
- Normal and shear strain are the main strain types
- When a material is loaded, there are typical limits where stress and strain are proportional and/or permanent strain occur
- Materials have mechanical properties i.e. modulus of elasticity, rigidity, poisson’s ratio, etc. which have their special characteristics
- Strain of materials might reach up to levels expressed by cm or metres
- Strain is one of the main elements of geological engineering interest