SAMPLE PROCET AND PRACTICES OF FLUID MECHANICS

Project: What is *Bernoulli Equation? Please Give* Examples of Use of the Bernoulli Equation.

Solution:

The Bernoulli equation states that the sum of the flow, kinetic, and potential energies of a fluid particle along a streamline is constant. Therefore, the kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. This phenomenon can be made more visible by multiplying the Bernoulli equation by the density ρ,

$$P + \frac{1}{2}\rho V^2 + \rho g z = constant\ along\ streamline$$

Each term in this equation has pressure units, and thus each term represents some kind of pressure:

• P is the **static pressure** (it does not incorporate any dynamic effects); it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.

•$\frac{1}{2}\rho V^2$ is the **dynamic pressure;** it represents the pressure rise when the fluid in motion is brought to a stop isentropically.

• $\rho g z$ is the **hydrostatic pressure,** which is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., of fluid weight on pressure.

*The sum of the static, dynamic, and hydrostatic pressures is called the **total pressure.** Therefore, the Bernoulli equation states that the total pressure along a streamline is constant*

*The sum of the static and dynamic pressures is called the **stagnation pressure,*** and it is expressed as

$$P_2 = P_{stag} = P_1 + \frac{\rho V_1^2}{2}$$

The pressure at the stagnation point is greater than the static pressure, P_1, by an amount $\frac{\rho V_1^2}{2}$, the dynamic pressure. It can be shown that there is a stagnation point on any stationary body that is placed into a flowing fluid (Fig.1). Some of the fluid flows "over" and some "under" the object. The dividing line (or surface for two-dimensional flows) is termed the *stagnation streamline* and terminates at the stagnation point on the body. For symmetrical objects (such as a baseball) the stagnation point is clearly at the tip or front of the object.

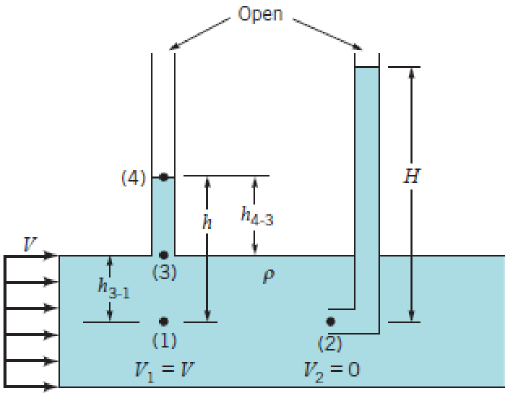


Figure 1. Measurement of static and stagnation pressures.

If elevation effects are neglected, the ***stagnation pressure,*** is the largest pressure obtainable along a given streamline. It represents the conversion of all of the kinetic energy into a pressure rise. The stagnation pressure represents the pressure at a point where the fluid is brought to a complete stop isentropically. The static, dynamic, and stagnation pressures are shown in the below Fig.2. When static and stagnation pressures are measured at a specified location, the fluid velocity at that location can be calculated from

$$V_1 = V = \sqrt{\frac{2(P_2 - P_1)}{\rho}}$$

This equation is useful in the measurement of flow velocity when a combination of a static pressure tap and a Pitot tube is used, as illustrated in Fig.2.

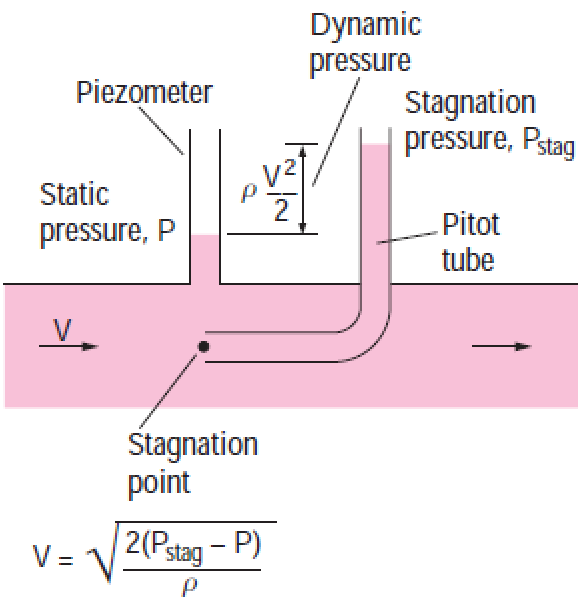


Figure 2. The static, dynamic, and stagnation pressures

A **static pressure tap** is simply a small hole drilled into a wall such that the plane of the hole is parallel to the flow direction. It measures the static pressure. A **Pitot tube** is a small tube with its open end aligned *into* the flow so as to sense the full impact pressure of the flowing fluid. It measures the stagnation pressure. In situations in which the static and stagnation pressure of a flowing *liquid* are greater than atmospheric pressure, a vertical transparent tube called a **piezometer tube** (or simply a **piezometer**) can be attached to the pressure tap and to the Pitot tube.The liquid rises in the piezometer tube to a column height (*head*) that is proportional to the pressure being measured. If the pressures to be measured are below atmospheric, or if measuring pressures in *gases*, piezometer tubes do not work. However, the static pressure tap and Pitot tube can still be used, but they must be connected to some other kind of pressure measurement device such as a U-tube manometer or a pressure transducer. Sometimes it is convenient to integrate static pressure holes on a Pitot probe. The result is a **Pitot-static probe,** as shown in the below Fig.3. A Pitot-static probe connected to a pressure transducer or a manometer measures the dynamic pressure (and thus fluid velocity) directly.

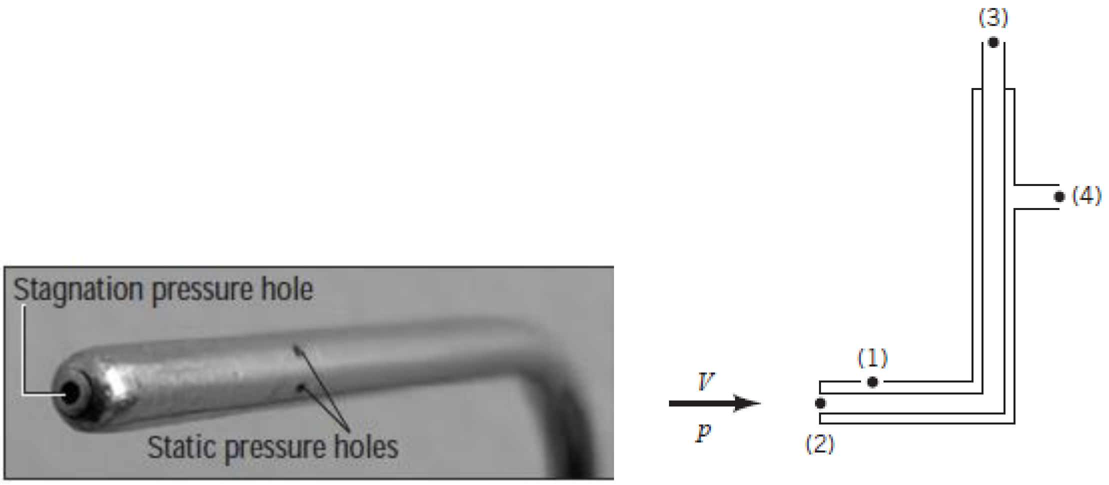


Figure 3. Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes

As shown in Fig.4.7. two concentric tubes are attached to two pressure gages (or a differential gage) so that the values of P_3 and P_4 (or the difference P_3-P_4) can be determined. The center tube measures the stagnation pressure at its open tip. If elevation changes are negligible,

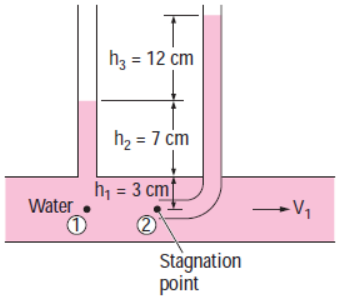
$$P_3 = P + \frac{1}{2}\rho V^2$$

Where *p* and *V* are the pressure and velocity of the fluid upstream of point (2). The outer tube is made with several small holes at an appropriate distance from the tip so that they measure the static pressure. If the effect of the elevation difference between (1) and (4) is negligible, then $P=P_4=P_1$

By combining these two equations we see that

$P_3 - P_4 = \frac{1}{2}\rho V^2$ which can be rearranged to give $V = \sqrt{\frac{2(P_3-P_4)}{\rho}}$

Example: A piezometer and a Pitot tube are tapped into a horizontal water pipe, as shown in Fig. to measure static and stagnation (static + dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe. The flow is steady and incompressible.



Solution: We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the tip of the Pitot tube. This is a steady flow with straight and parallel streamlines, and the gage pressures at points 1 and 2 can be expressed as

$P_1=\rho g(h_1+h_2)$ $P_2=\rho g(h_1+h_2+h_3)$

Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \rightarrow \frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions gives

$$\frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g}=\frac{\rho g(h_1+h_2+h_3)-\rho g(h_1+h_2)}{\rho g}=h_3$$

Solving for V_1 and substituting,

$$V_1=\sqrt{2\times 9.81\times 0.12}=1.53\ m/s$$

Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot tube.

Examples of Use of the Bernoulli Equation

In this section we illustrate various additional applications of the Bernoulli equation. Between any two points, (1) and (2), on a streamline in steady, inviscid, incompressible flow the Bernoulli equation can be applied in the form.

$$P_1+\frac{1}{2}\rho V_1^2+\gamma z_1=P_2+\frac{1}{2}\rho V_2^2+\gamma z_2$$

Obviously if five of the six variables are known, the remaining one can be determined. In many instances it is necessary to introduce other equations, such as the conservation of mass.

Free Jet

One of the oldest equations in fluid mechanics deals with the flow of a liquid from a large reservoir. A modern version of this type of flow involves the flow of coffee from a coffee urn as indicated by the below Fig.4. The exit pressure for an incompressible fluid jet is equal to the surrounding pressure.

Figure 4. The flow of coffee from a coffee urn

The basic principles of this type of flow are shown in the below Fig.5. where a jet of liquid of diameter *d* flows from the nozzle with velocity *V*. (A nozzle is a device shaped to accelerate a fluid.). Application of the above Equation between points (1) and (2) on the streamline shown gives

$$V = \sqrt{2gh}$$

Which is the modern version of a result obtained in 1643 by Torricelli 11608–16472, an Italian physicist.

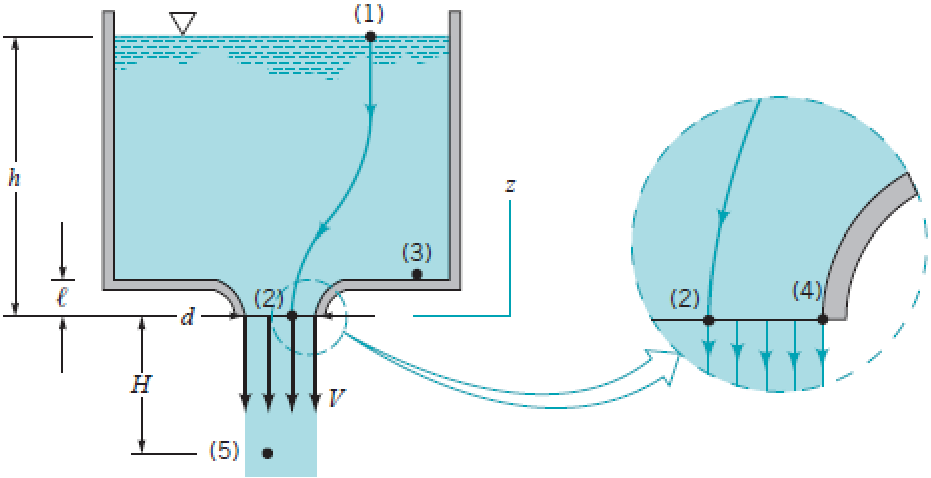


Figure 5. Vertical flow from a tank

For the horizontal nozzle of Fig.6a, the velocity of the fluid at the centerline, V_2 will be slightly greater than that at the top, V_1, and slightly less than that at the bottom, V_3, due to the differences in elevation. In general, $d \ll h$ as shown in Fig.4.10*b* and we can safely use the centerline velocity as a reasonable "***average velocity***." From another assumption ***a velocity factor*** can be used for real velocity.

Velocity factor $C_v = \frac{real\ velocity}{theoretical\ velocity} = \frac{V_r}{V_t} = \frac{V_r}{\sqrt{2gh}}$

If the exit is not a smooth, well-contoured nozzle, but rather a flat plate as shown in Fig.6c, the diameter of the jet, d_j, will be less than the diameter of the hole, d_h. This phenomenon, called **a *vena contracta*** effect, is a result of the inability of the fluid to turn the sharp 90° corner indicated by the dotted lines in the figure.

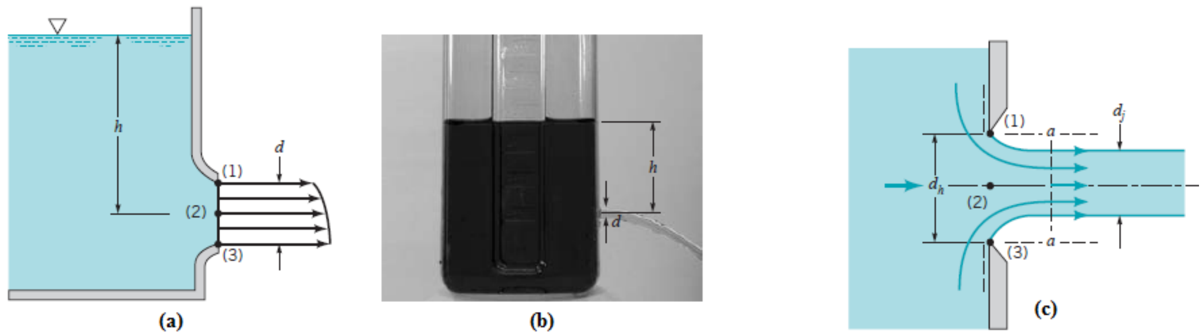


Figure 6. Horizontal flow from a tank (a and b) and Vena contracta effect for a sharp-edged orifice (c).

The vena contracta effect is a function of the geometry of the outlet. Some typical configurations are shown in the below Fig.7 along with typical values of the experimentally obtained ***contraction coefficient***, $C_c = \frac{A_j}{A_h} = (\frac{d_j}{d_h})^2$. *Where* A_j *and*

A_h are the areas of the jet at the vena contracta and the area of the hole, respectively. $A_j = \frac{\pi d_j^2}{4}$ and $A_h = \frac{\pi d_h^2}{4}$. Then the flow rate for free jet $Q = C_v C_c A_h \sqrt{2gh} = C_d A_h \sqrt{2gh}$. $C_d = C_c C_v$ can be taken to be 0.62 for free jet.

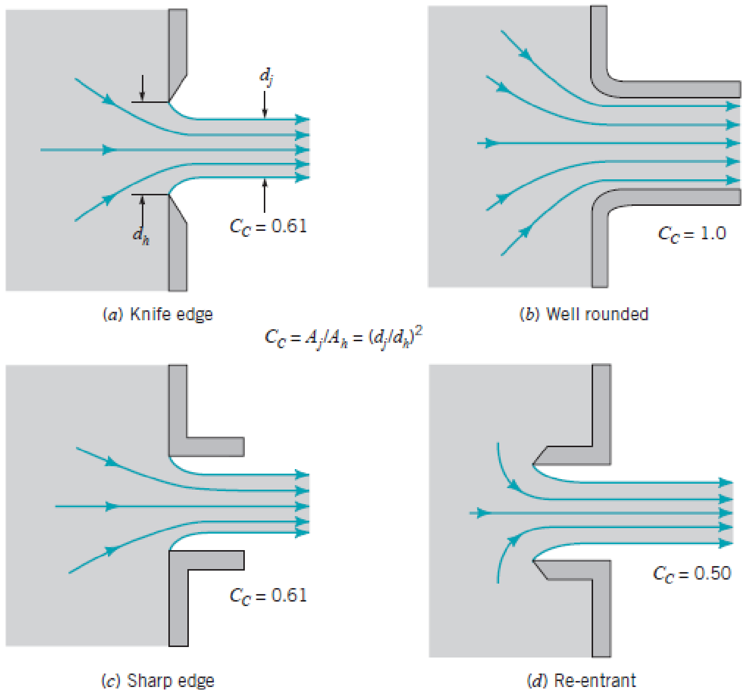


Figure 7. Typical flow patterns and contraction coefficients for various round exit configurations. (*a*) Knife edge, (*b*) Well rounded, (*c*) Sharp edge, (*d*) Re-entrant.

Confined Flows

In many cases the fluid is physically constrained within a device so that its pressure cannot be prescribed a priori as was done for the free jet examples above. Such cases include nozzles and pipes of variable diameter for which the fluid velocity changes because the flow area is different from one section to another. For these situations it is necessary to use the concept of conservation of mass (***the continuity equation***) along with the Bernoulli equation. For the needs of this chapter we can use a simplified form of the continuity equation obtained from the following intuitive arguments. Consider a fluid flowing through a fixed volume (such as a syringe) that has one inlet and one outlet as shown in Fig.8*a*. If the flow is steady so that there is no additional accumulation of fluid within the volume, the rate at which the fluid flows into the volume must equal the rate at which it flows out of the volume (otherwise, mass would not be conserved).

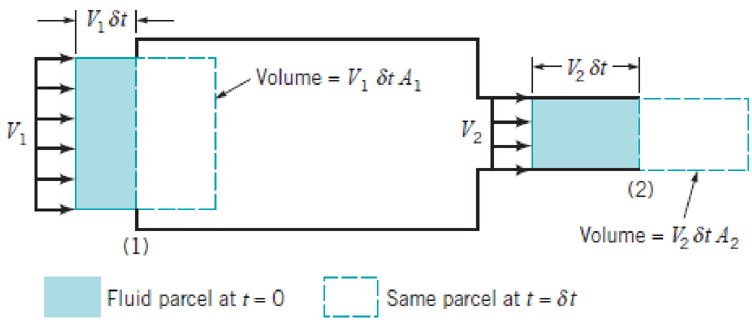
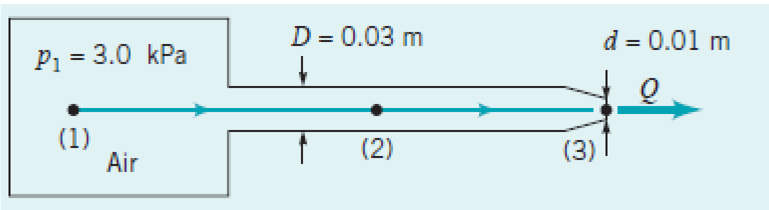


Figure 8. (*a*) Flow through a syringe. (*b*) Steady flow into and out of a volume.

The ***continuity equation*** for incompressible flow can be given as $Q_1=Q_2$ or $A_1V_1=A_2V_2$. Where; Q_1 is the inlet flow rate, Q_2 is the outlet flow rate, A_1 is the inlet cross section area, A_2 is the outler cross section area, V_1 is the inlet velocity of fluid, V_2 is the outlet velocity of fluid.

Example: Air flows steadily from a tank, through a hose of diameter and exits to the atmosphere from a nozzle of diameter as shown in Fig.. The pressure in the tank remains constant at 3.0 kPa (gage) and the atmospheric conditions are standard temperature and pressure. Determine the flowrate and the pressure in the hose.



Solution:

$$P_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 = P_3 + \frac{1}{2}\rho V_3^2 + \gamma z_3$$

With the assumption that $z_1=z_2=z_3$ (horizontal hose), $V_1=0$ (large tank), and $P_3=0$ (free jet), this becomes

$$V_3 = \sqrt{\frac{2P_1}{\rho}} \quad \text{and} \quad P_2 = P_1 - \frac{1}{2}\rho V_2^2 \quad (2)$$

The density of the air in the tank is obtained from the perfect gas law, using standard absolute pressure and temperature, as

$$\rho = \frac{P_1}{RT_1} == \frac{[3000 + 101000]}{286.9 \times (15 + 273)} = 1.26\ kg/m^3$$

Thus, we find that

$$V_3 = \sqrt{\frac{2 \times 3000}{1.26}} = 69\ m/s$$

$$Q = A_3V_3 = \frac{\pi d^2}{4}V_3 = \frac{\pi}{4}(0.01)^2 \times 69 = 0.00542\ m^3/s$$

The pressure within the hose can be obtained from Eq. 1 and the continuity equation $A_2V_2=A_3V_3$ Hence

$$V_2 = \frac{A_3V_3}{A_2} = (\frac{d}{D})^2 V_3 = (\frac{0.01}{0.03})^2(69) = 7.67\ m/s$$

And $P_2 = 3000 - \frac{1}{2} \times 1.26 \times 7.67 = 2963\ Pa$

In general, an increase in velocity is accompanied by a decrease in pressure. For example, the velocity of the air flowing over the top surface of an airplane wing is, on the average, faster than that flowing under the bottom surface. Thus, the net pressure force is greater on the bottom than on the top—the wing generates a