Work-Energy Theorem

- When work is done by a net force on an object and the only change in the object is its speed, the work done is equal to the change in the object's kinetic energy

$$W_{net} = K_f - K_i = \Delta K$$

  - Speed will increase if work is positive
  - Speed will decrease if work is negative

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\ 2}$$

Work with Varying Forces

- On a graph of force as a function of position, the total work done by the force is represented by the area under the curve between the initial and the final position
- Straight-line motion

$$W = F_{ax}\Delta x_a + F_{bx}\Delta x_b + ...$$

$$W = \int_{x_1}^{x_2} F_x dx$$

- Motion along a curve

$$W = \int_{P_1}^{P_2} F\cos\phi dl = \int_{P_1}^{P_2} F_{||} dl = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l}$$

Figure 7.7
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Work-Energy with Varying Forces

- Work-energy theorem $W_{tol} = \Delta K$ holds for varying forces as well as for constant ones

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = v_x \frac{dv_x}{dx}$$

$$W_{tot} = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} ma_x dx = \int_{x_1}^{x_2} mv_x \frac{dv_x}{dx} dx$$

$$W_{tot} = \int_{v_1}^{v_2} mv_x dv_x$$

$$W_{tot} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \Delta K$$

Spring Force: a Varying Force

- Involves the *spring constant*, k
- Hooke's Law gives the force

$$\vec{F} = -k\vec{x}$$

  - F is in the opposite direction of x, always back towards the equilibrium point.
  - k depends on how the spring was formed, the material it is made from, thickness of the wire, etc

Figure 7.10
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Measuring Spring Constant

- spring at its natural equilibrium length.
- Hang a mass on spring and let it hang to distance d (stationary)
- From $F_x = kx - mg = 0$

$$k = \frac{mg}{d}$$

- get spring constant.

Figure 7.12
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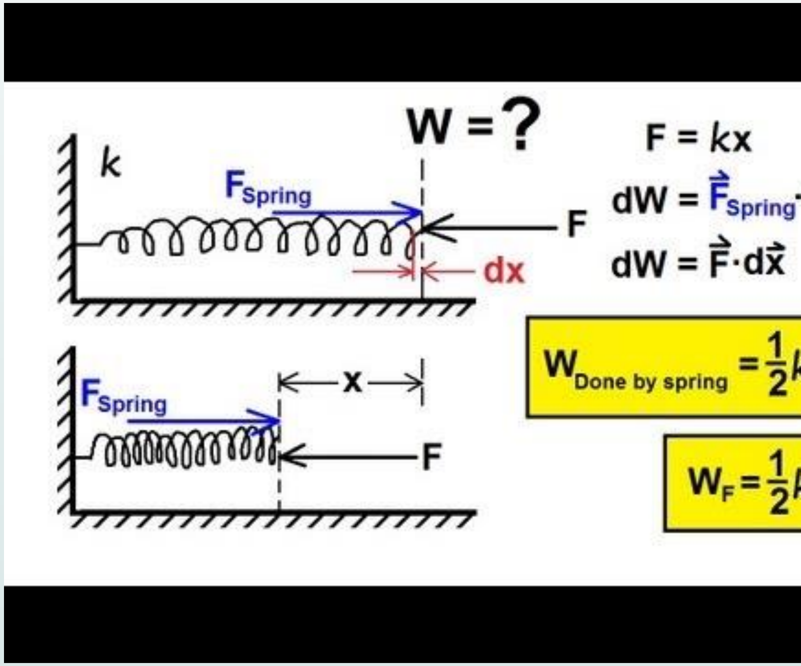
Work done on a Spring

- We have to do work to extend the spring
- We apply equal and opposite forces to the ends of spring and gradually increase the forces
- The work we must do to stretch the spring from x1 to x2

$$W = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} kx dx = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

- Work done on a spring is not equal to work done by a spring



Power

- Work is not tied to work time interval
- The transfer rate of energy is important in practical device design and use.
- The time ratio of energy transfer is called power
- The average power is given by

$$\overline{P} = \frac{W}{\Delta t}$$

Instantaneous Power

- Power is the time rate of energy transfer. Power applies to any energy transfer vehicle
- Other expression

$$\overline{P} = \frac{W}{\Delta t} = \frac{F\Delta x}{\Delta t} = F\overline{v}$$

- A more general definition of instantaneous power

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

Units of Power

- The SI unit of power is called the watt
  - 1 watt = 1 joule / second = 1 kg · m^2 / s^3
- Units of power can also be used to express units of work or energy
  - 1 kWh = (1000 W)(3600 s) = 3.6 x10^6 J

Power Delivered by an Elevator Motor

- 1000 kg lift, carries a maximum load of 800 kg. The continuous friction force of 4000 N retards the upward movement. What minimum power should the engine use to lift the fully loaded elevator at a constant speed of 6 m / s?

$$F_{net,y} = ma_y$$

$$T - f - Mg = 0$$

$$T = f + Mg = 2.16\times10^4 N$$

$$P = Fv = (2.16\times10^4 N)(3m/s)$$

$$= 13\times10^4 W$$

Figure 7.19
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