Chapter 2: Vectors

PHY0101 and PHY/PEN 101

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Outline

3.1 Coordinate Systems
3.2 Vector and Scalar Quantities
3.3 Some Properties of Vectors
3.4 Components of a Vector and Unit Vectors
3.5 Scalar Product of Two Vectors
(a later chapter in Serway!)
3.6 Vector Product of Two Vectors
(a later chapter in Serway!)

Coordinate Axes

- Usually, we define a reference frame using a standard coordinate axes. (The choice of reference frame is arbitrary & up to us!).
- **<u>Rectangular or Cartesian Coordinates</u>:**
- A point in the plane is denoted as **(x,y)** or **(x,y,z)**

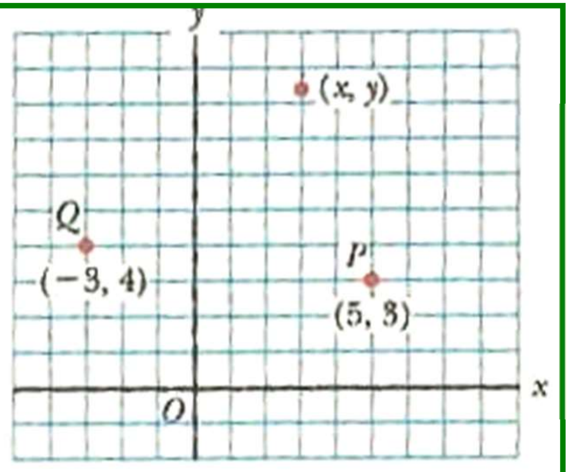
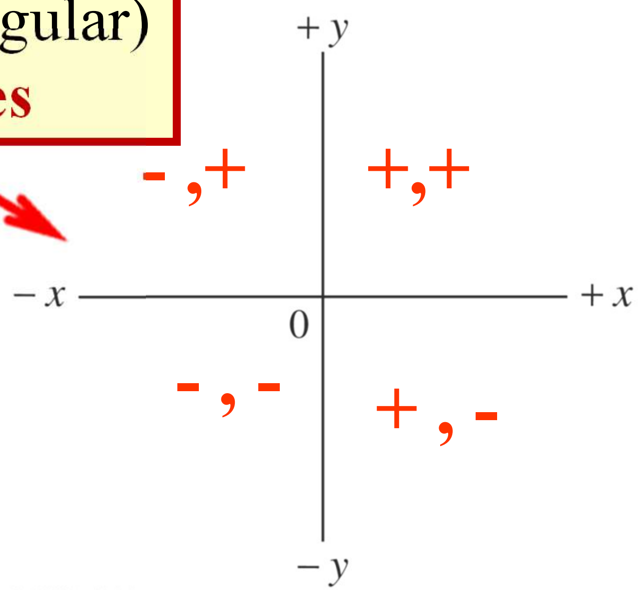


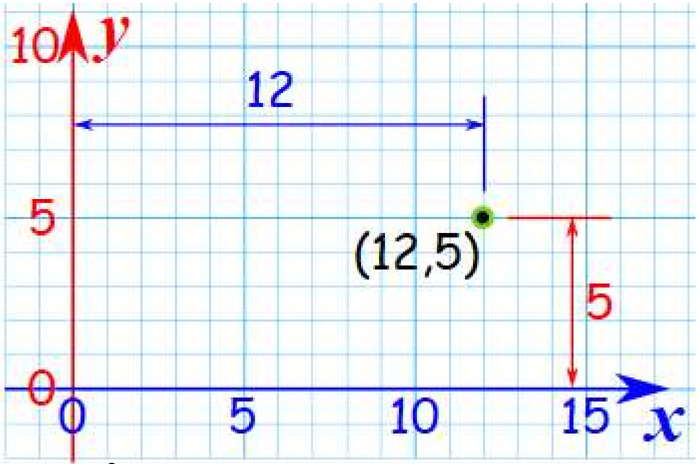
Figure 3.1 Designation of points in a Cartesian coordinate system. Every point is labeled with coordinates (x, y).

Standard sets of xy
(Cartesian or rectangular)
coordinate axes



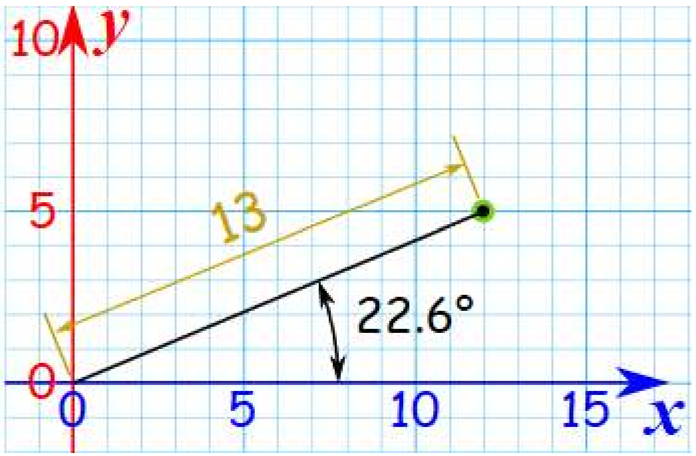
3.1.1 Cartesian Coordinates

Using Cartesian Coordinates we mark a point by how far along and how far up it is:

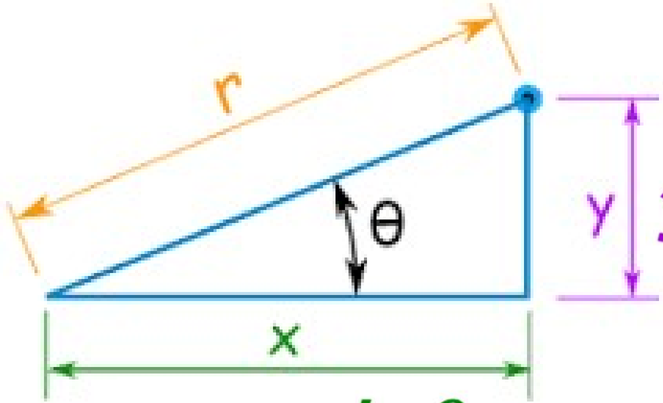


3.1.2 Polar Coordinates

Using Polar Coordinates we mark a point by how far away, and what angle it is:



When we know a point in Cartesian Coordinates (x,y) and we want it in Polar Coordinates (r,θ)



$$y = r\cos\theta$$

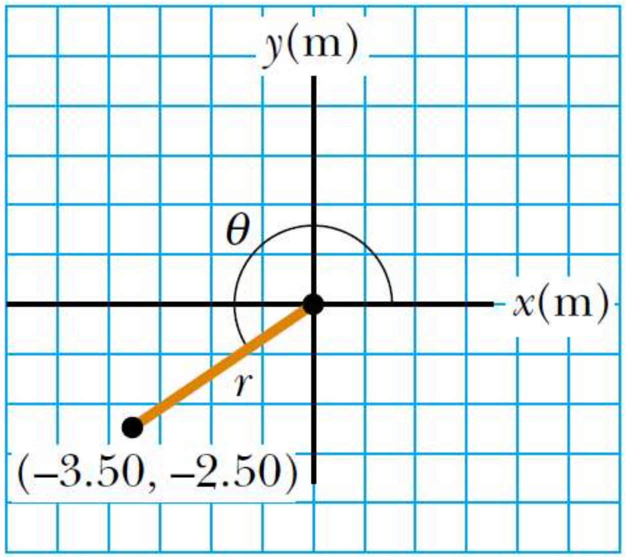
$$x = r\sin\theta$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

Positive θ is an angle measured counterclockwise from the positive x axis.

Example 3.1 The Cartesian coordinates of a point in the *xy* plane are $(x, y) = (-3.50, -2.50)$ m, as shown in Figure 3.3. Find the polar coordinates of this point.



$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2} = 4.30\text{ m}$$

$$\tan\theta = \frac{y}{x} = \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$

$$\theta = 216°$$

Scalar quantities

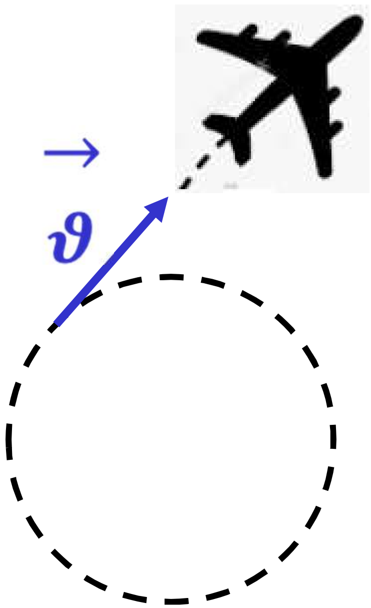
Scalar ≡ A quantity with **magnitude only (no direction).**

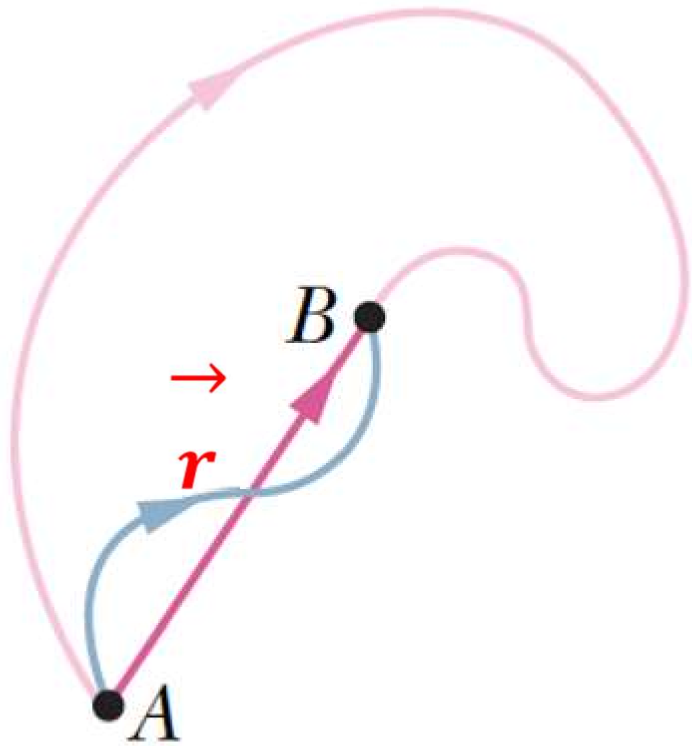
- Examples of scalar quantities are volume, mass, speed, time intervals, mass, temperature, energy.
- The rules of ordinary arithmetic are used to manipulate scalar quantities.

Vector quantities

Vector ≡ A quantity with **magnitude and direction.**

- Examples of vector quantities are displacement, velocity, acceleration, force, momentum.
- Vectors need to account for direction for algebraic operations.





Displacement is a vector quantity shown by the arrow drawn from A to B.

A vector quantity can be shown as: $\vec{r}$ or $\mathbf{r}$

The magnitude of a vector: $|\vec{r}|$ or r

The magnitude of a vector is *always* a positive number.

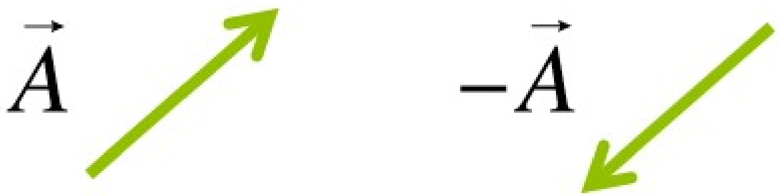
Equality of two vectors

2 vectors, **A** & **B**.

A = B means that **A** & **B** have the same **magnitude & direction.**

Negativity of a vector

$\vec{A}$ $-\vec{A}$



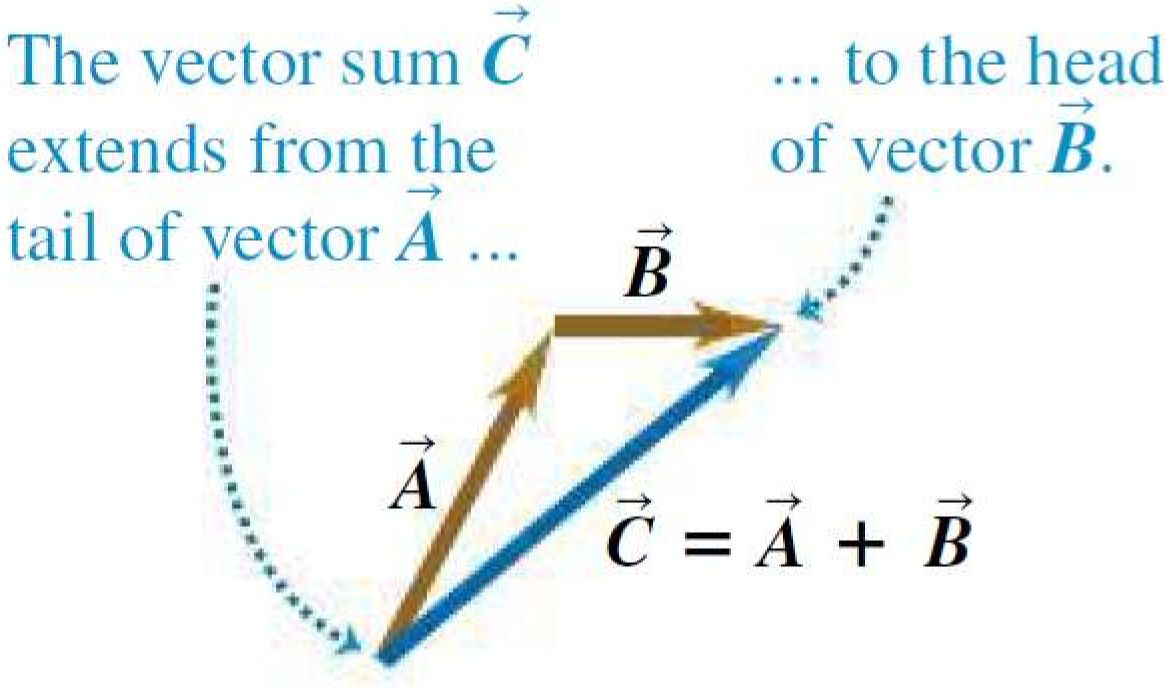
Vector Addition and Subtraction

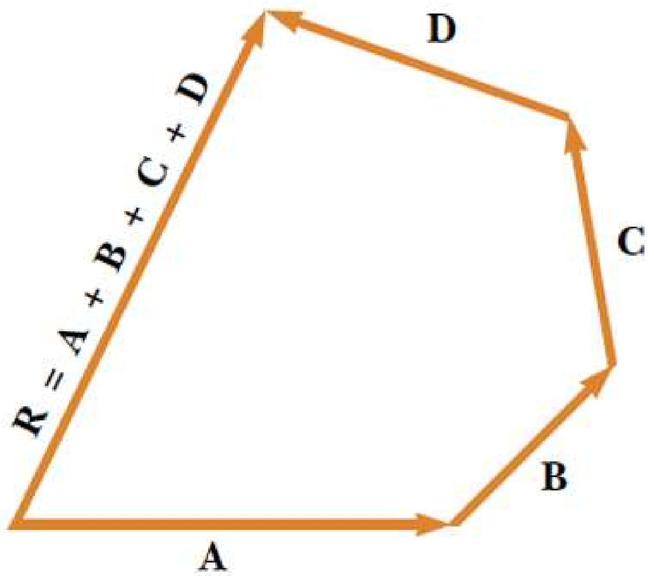
- Adding Vectors Geometrically

  (i)Adding vectors from head to tail

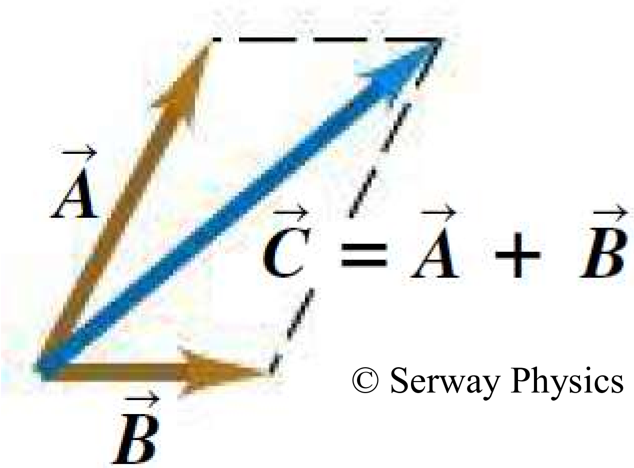
  (ii) Adding vectors by using paralellogram
- Adding Vectors by Using Trigonometry

(i)



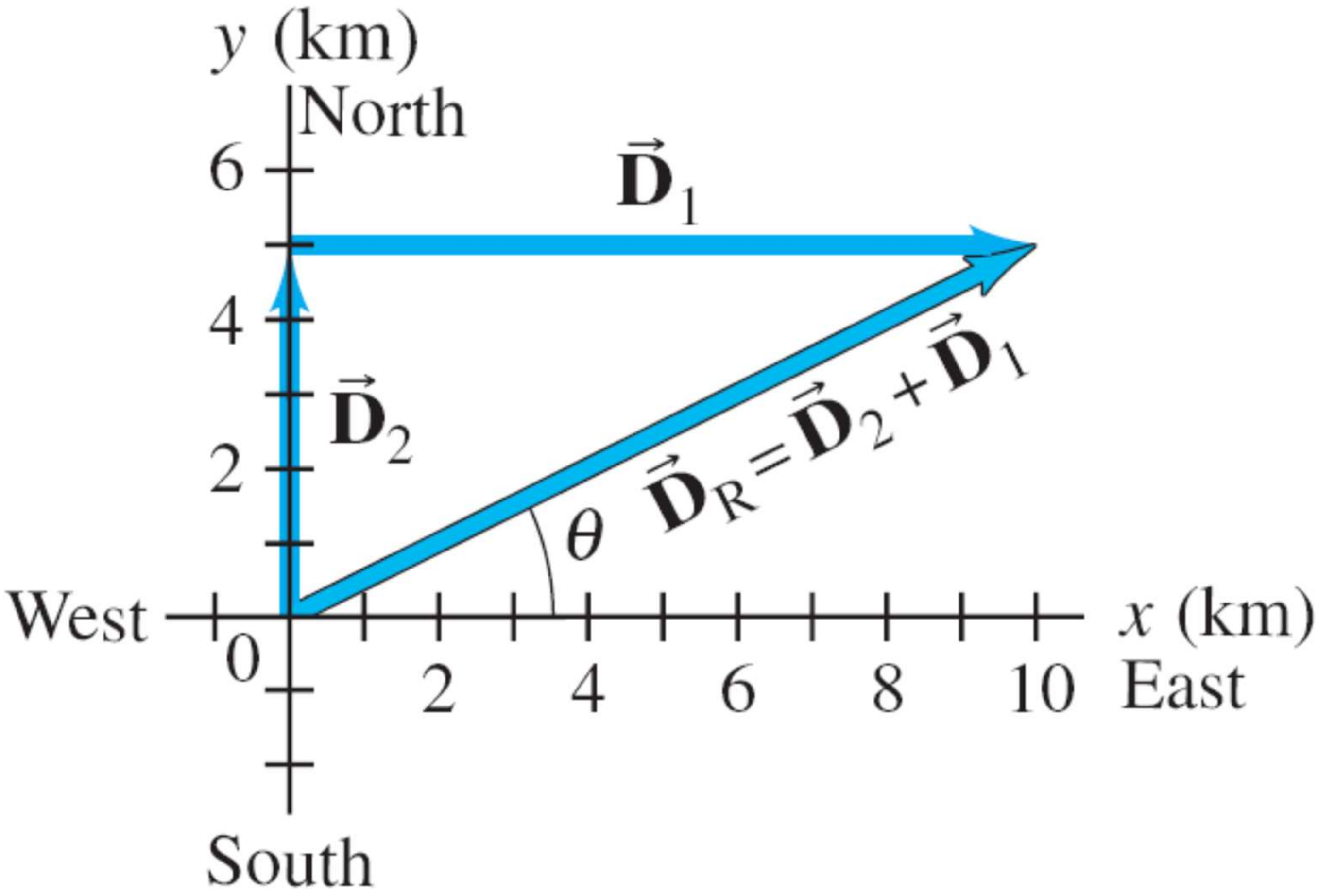


(ii) We can also add two vectors by placing the vectors tail to tail and constructing a paralelogram:



Commutative law of addition

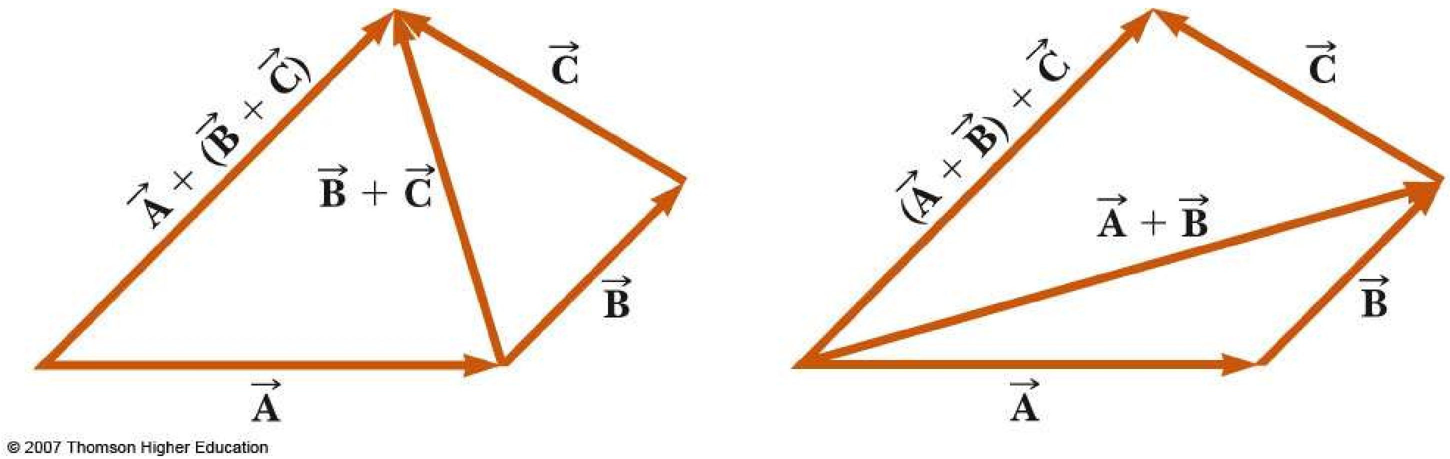
Adding vectors in the opposite order gives the same result. In the example, $\mathbf{D_R = D_1 + D_2 = D_2 + D_1}$



Associative law of addition:

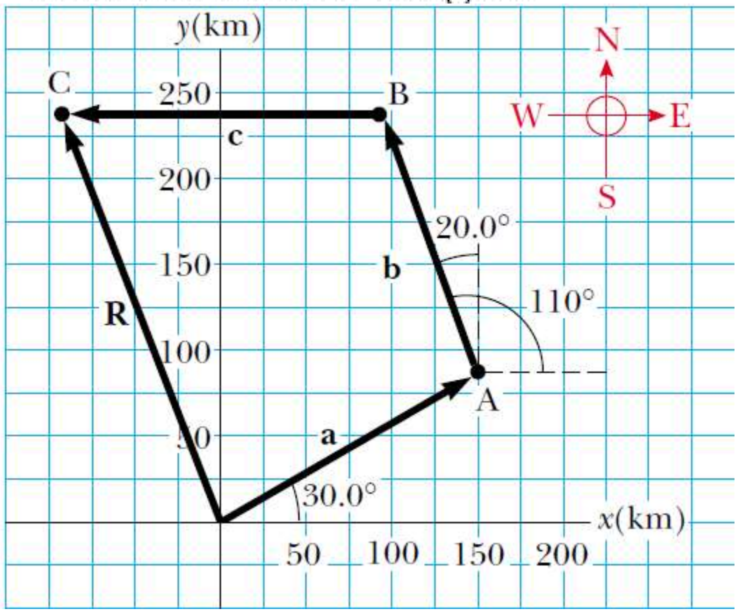
When adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped:

$$\vec{\mathbf{A}} + \left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right) = \left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right) + \vec{\mathbf{C}}$$



Example 3.6 **Let's Fly Away!**

A commuter airplane takes the route shown in Figure 3.20. First, it flies from the origin of the coordinate system shown to city A, located 175 km in a direction 30.0° north of east. Next, it flies 153 km 20.0° west of north to city B. Finally, it flies 195 km due west to city C. Find the location of city C relative to the origin.



$$a_x = a\cos(30.0°) = (175 \text{ km})(0.866) = 152 \text{ km}$$

$$a_y = a\sin(30.0°) = (175 \text{ km})(0.500) = 87.5 \text{ km}$$

$$b_x = b\cos(110°) = (153 \text{ km})(-0.342) = -52.3 \text{ km}$$

$$b_y = b\sin(110°) = (153 \text{ km})(0.940) = 144 \text{ km}$$

$$c_x = c\cos(180°) = (195 \text{ km})(-1) = -195 \text{ km}$$

$$c_y = c\sin(180°) = 0$$

$$R_x = a_x + b_x + c_x = 152 \text{ km} - 52.3 \text{ km} - 195 \text{ km}$$

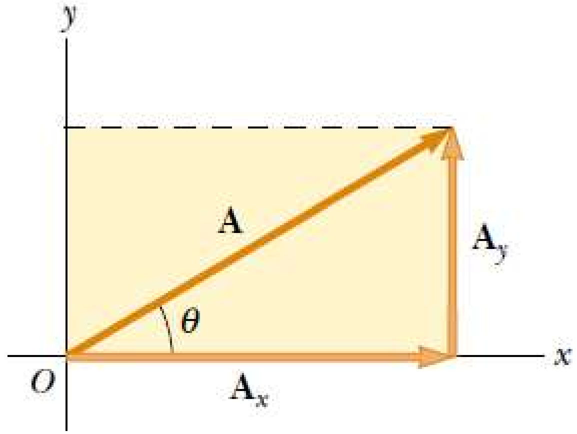
$$= -95.3 \text{ km}$$

$$R_y = a_y + b_y + c_y = 87.5 \text{ km} + 144 \text{ km} + 0 = 232 \text{ km}$$

In unit–vector notation,

$$\vec{R} = -95.3\hat{\imath} + 232\hat{\jmath} \qquad R = 250.8\ km, \theta = -67.7°$$

3.4 Components of a Vector and Unit Vectors

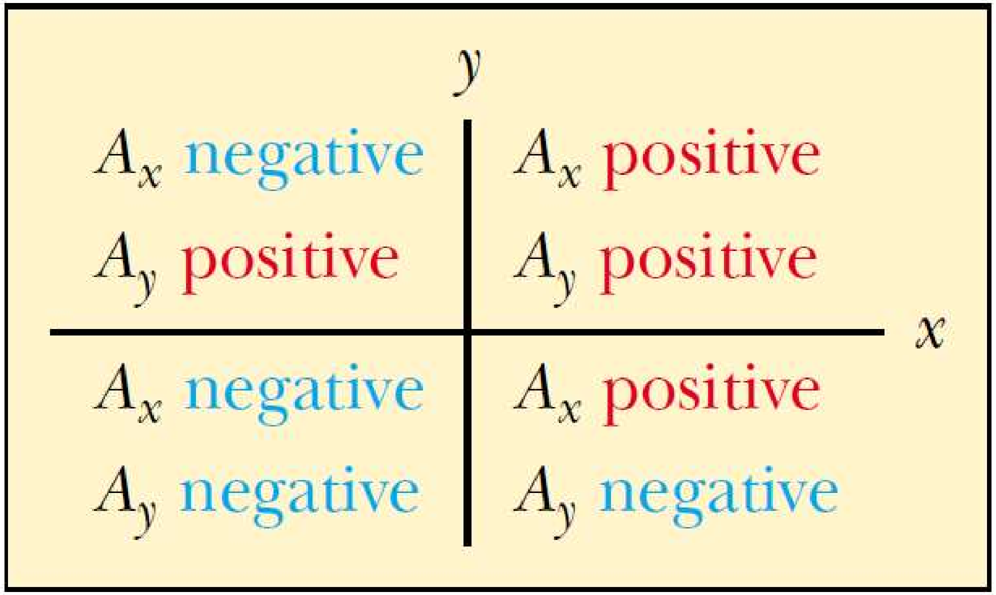


$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

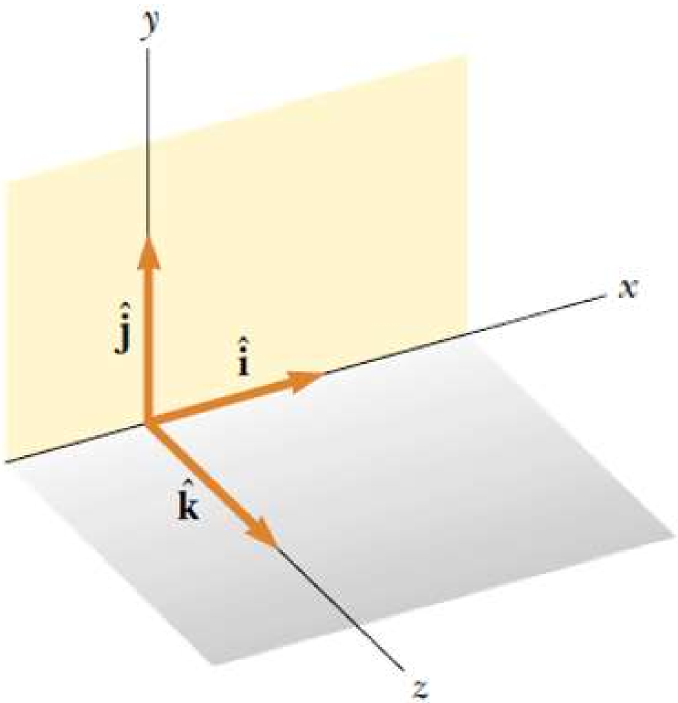
$$A = \sqrt{A_x^{\,2} + A_y^{\,2}}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



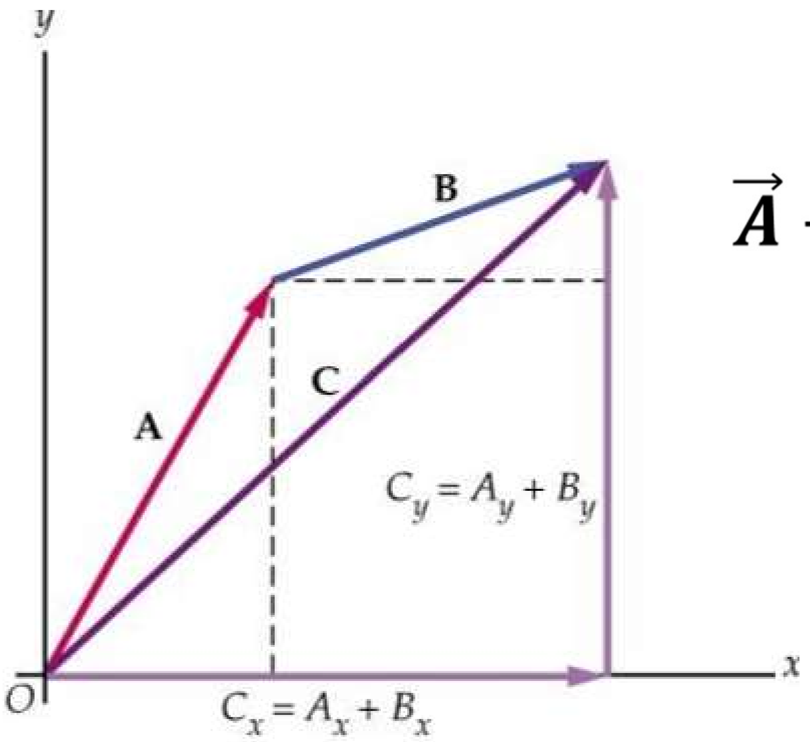
Unit Vectors

- Vector quantities often are expressed in terms of unit vectors.
- A unit vector is a dimensionless vector having a magnitude of exactly 1.
- Unit vectors are used to specify a given direction and have no other physical significance.



$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$

Adding Vectors by Using Trigonometry



$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + \left(A_y + B_y\right)\hat{\jmath}$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$C_x = A_x + B_x$$

$$C_y = A_y + B_y$$

Example 3.3 The Sum of Two Vectors

Find the sum of two vectors **A** and **B** lying in the *xy* plane and given by

$$\mathbf{A} = (2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\ \text{m} \qquad \text{and} \qquad \mathbf{B} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}})\ \text{m}$$

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (2.0 + 2.0)\hat{\mathbf{i}}\ \text{m} + (2.0 - 4.0)\hat{\mathbf{j}}\ \text{m}$$

$$= (4.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{j}})\ \text{m}$$

$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{(4.0\ \text{m})^2 + (-2.0\ \text{m})^2}$$

$$= 4.5\ \text{m}$$

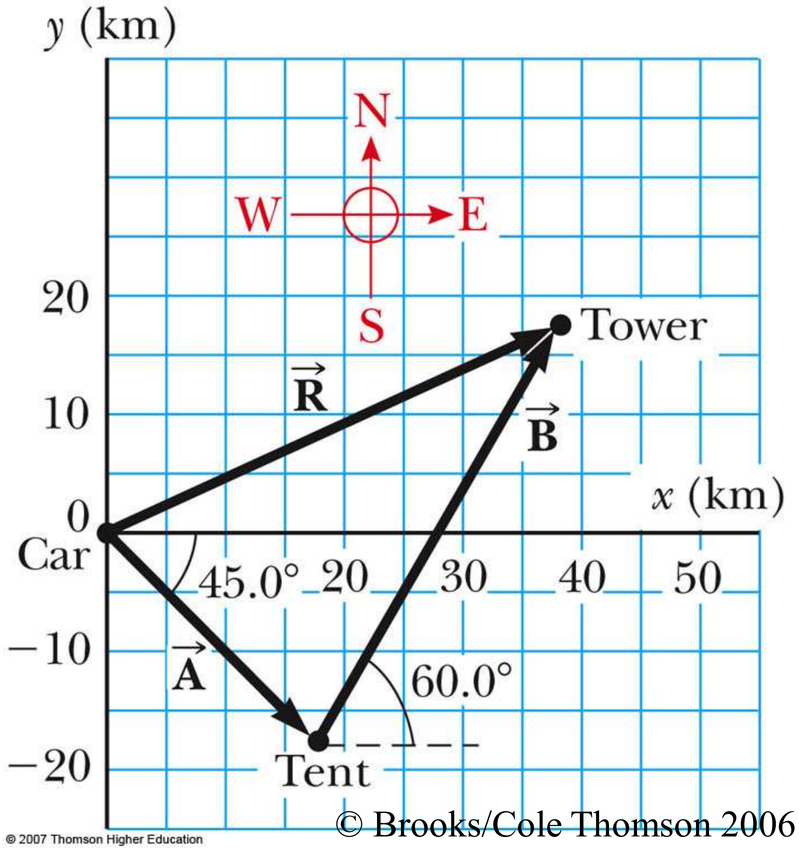
$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0\ \text{m}}{4.0\ \text{m}} = -0.50$$

$$\theta = 333^\circ$$

Example 3.5 – Taking a Hike

- A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.



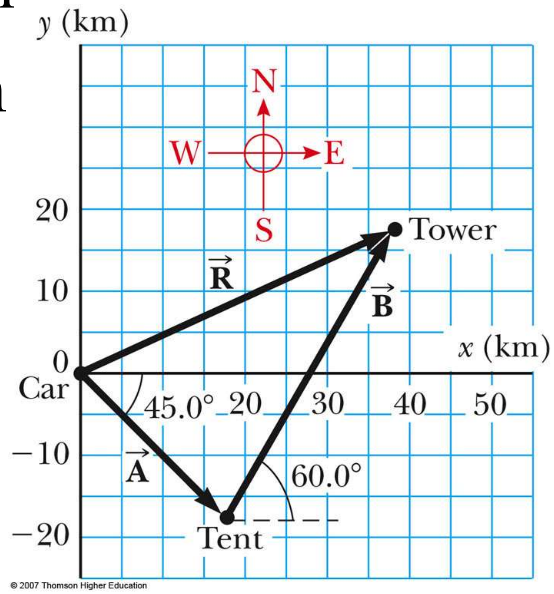
$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = 17.7\text{ km}$$

$$A_y = A\sin(-45.0°) = (25.0\text{ km})(-0.707) = -17.7\text{ km}$$

$$B_x = B\cos 60.0° = (40.0\text{ km})(0.500) = 20.0\text{ km}$$

$$B_y = B\sin 60.0° = (40.0\text{ km})(0.866) = 34.6\text{ km}$$

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$



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The resultant displacement for the trip has components given by

$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = 37.7\text{ km}$$

$$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = 16.9\text{ km}$$

In unit-vector form, we can write the total displacement as

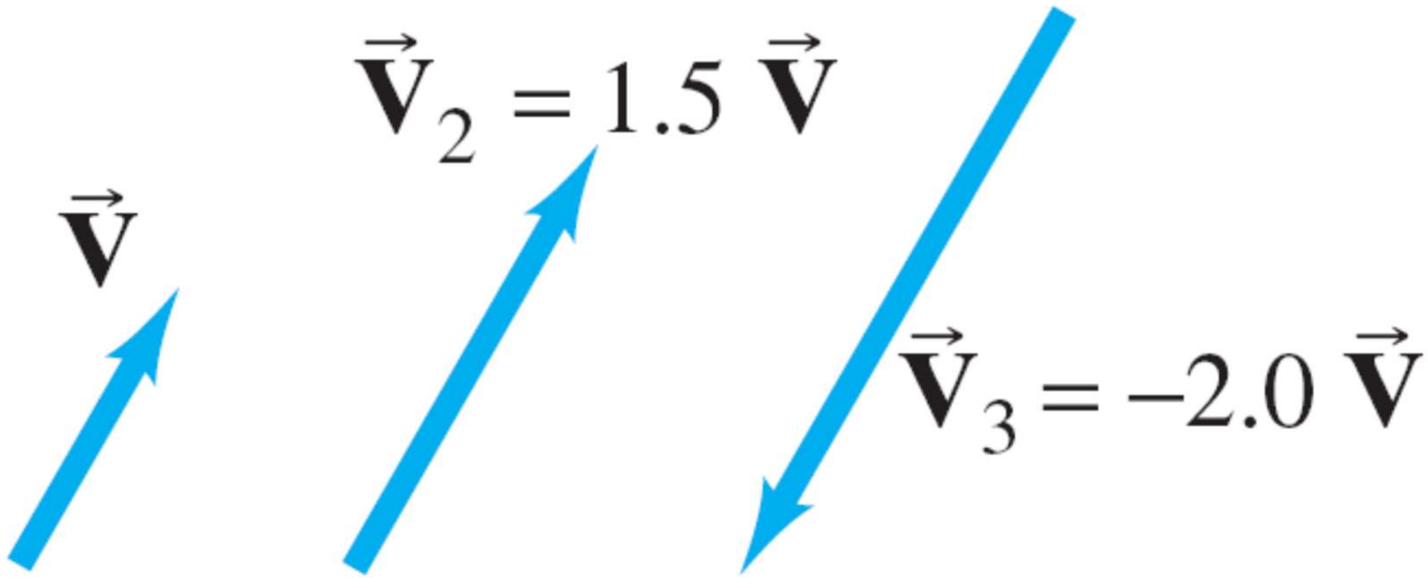
$$\mathbf{R} = (37.7\hat{\mathbf{i}} + 16.9\hat{\mathbf{j}})\text{ km}$$

Multiplication of Vectors
(i) Multiplication by a scalar

- A vector **V** can be multiplied by a scalar **c**

$$\mathbf{V'} = c\mathbf{V}$$

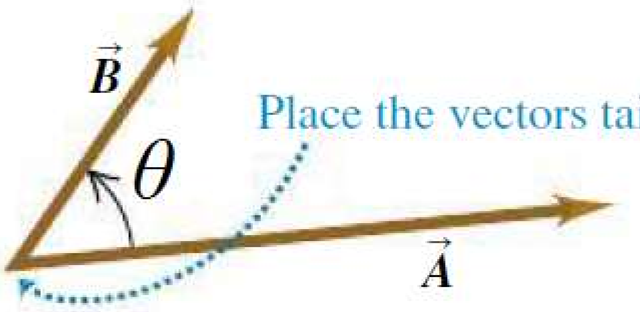
V' ≡ vector with magnitude **cV** the same direction as **V.**
If **c** is negative, the result is in the opposite direction.



(ii) Multiplication of Two Vectors
(ii.a) Scalar Product of Two Vectors

- The scalar product of two vectors is written as
  $\vec{R} = \vec{A}.\vec{B} \quad R = A.B.cos\theta$

- θ is the angle between A and B when placed tail to tail:



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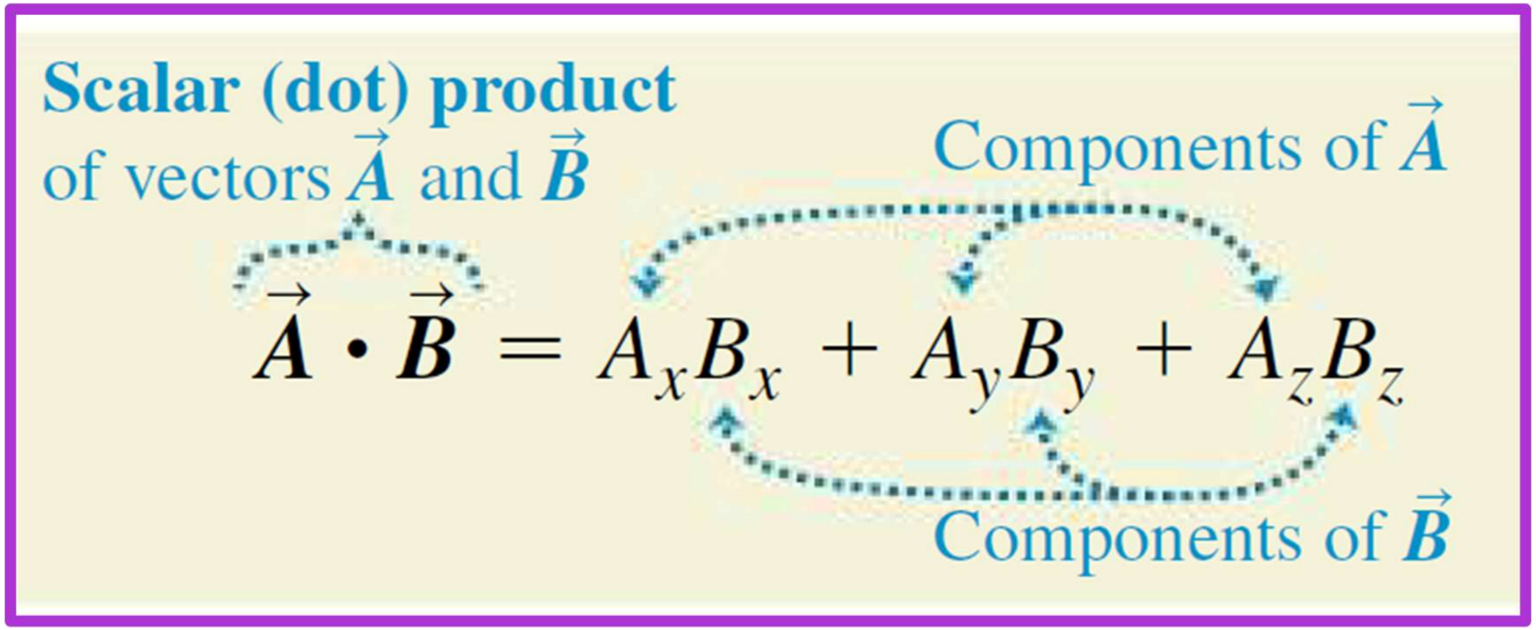
Scalar Product of Unit Vectors

$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = (1)(1)\cos 0^\circ = 1$$

$$\hat{\imath}\cdot\hat{\jmath} = \hat{\imath}\cdot\hat{k} = \hat{\jmath}\cdot\hat{k} = (1)(1)\cos 90^\circ = 0$$



Example 1.10 in the book University Physics ©

Find the angle between the vectors

$\vec{A} = 2\hat{\imath}+3\hat{\jmath}+1\hat{k}$ and $\vec{B} = -4\hat{\imath}+2\hat{\jmath}$ -1$\hat{k}$

$$\cos\phi = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$= (2.00)(-4.00) + (3.00)(2.00) + (1.00)(-1.00)$$

$$= -3.00$$

$$
\begin{aligned}
A &= \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(2.00)^2 + (3.00)^2 + (1.00)^2} \\
&= \sqrt{14.00} \\
B &= \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(-4.00)^2 + (2.00)^2 + (-1.00)^2} \\
&= \sqrt{21.00} \\
\cos\phi &= \frac{A_xB_x + A_yB_y + A_zB_z}{AB} = \frac{-3.00}{\sqrt{14.00}\,\sqrt{21.00}} = -0.175 \\
\phi &= 100^\circ
\end{aligned}
$$

(ii.b) Vector Product of Two Vectors

- We denote the vector product of two vectors as

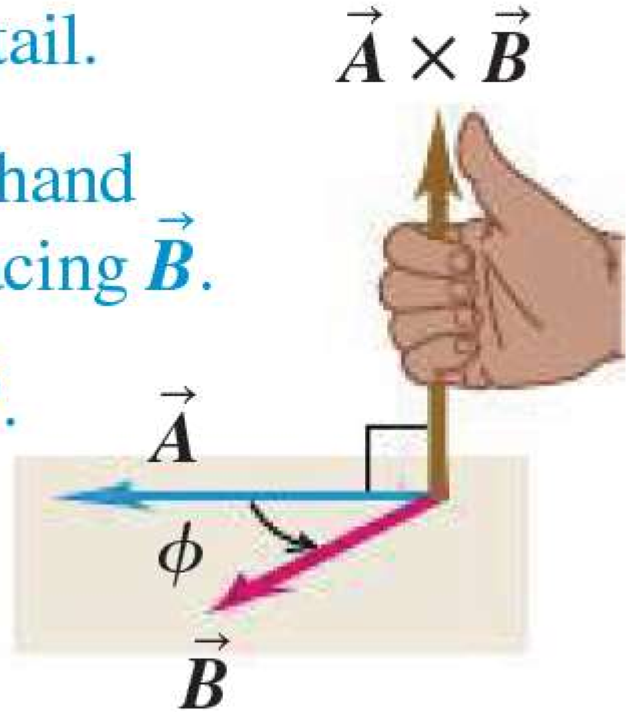
  $\vec{C} = \vec{A} \text{x} \vec{B} \quad C = ABsin\theta$

- As the name suggests, the vector product is itself a vector.
- The vector product is not commutative but instead is anticommutative:

$$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = -\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}}$$

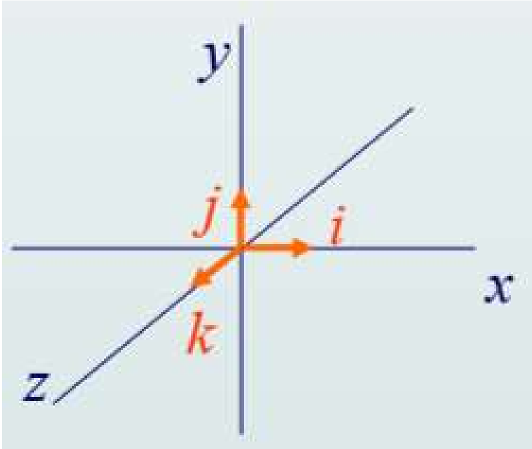
Using the right-hand rule to find the direction of $\vec{A} \times \vec{B}$

1. Place $\vec{A}$ and $\vec{B}$ tail to tail.
2. Point fingers of right hand along $\vec{A}$, with palm facing $\vec{B}$.
3. Curl fingers toward $\vec{B}$.
4. Thumb points in direction of $\vec{A} \times \vec{B}$.



Vector Multiplication of Unit Vectors

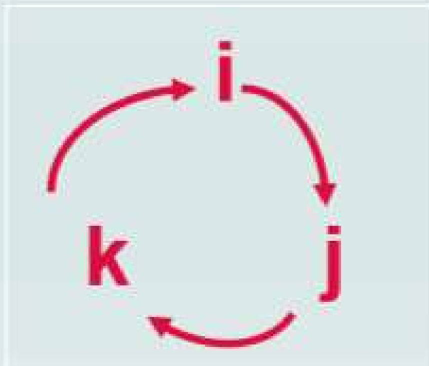
$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$



$$\hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath} = \hat{k}$$

$$\hat{\jmath} \times \hat{k} = -\hat{k} \times \hat{\jmath} = \hat{\imath}$$

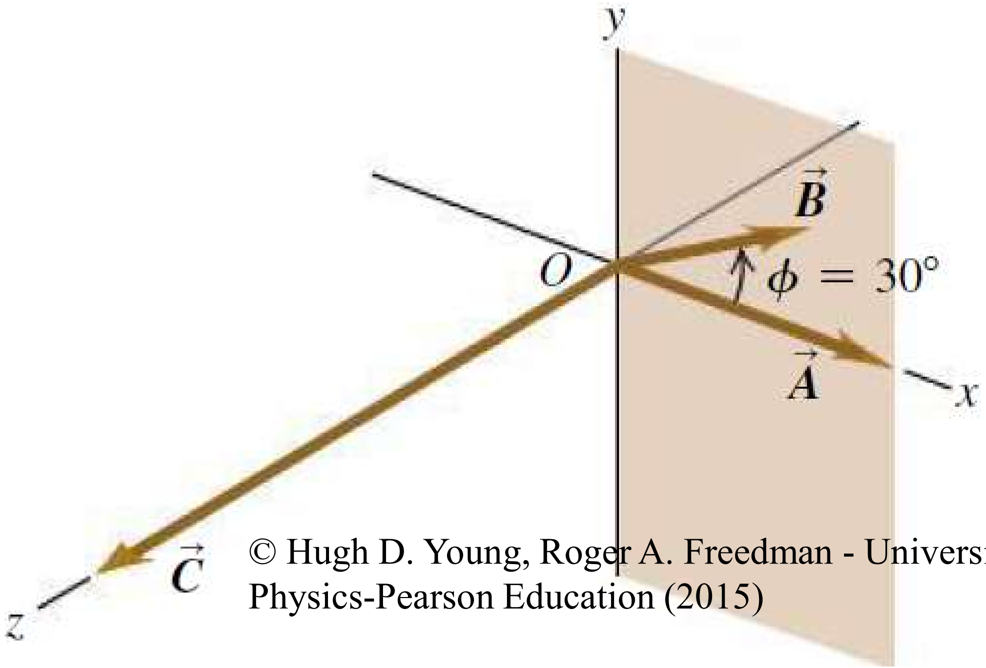
$$\hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k} = \hat{\jmath}$$



$$\vec{A} \times \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

Example 1.11 (page 48): Vector ***A*** has magnitude 6 units and is in the direction of the $+x$ axis. Vector ***B*** has magnitude 4 units and lies in the xy-plane, making an angle of 30° with the $+x$ axis (**Fig. 1.33**). Find the vector product $\vec{C} = \vec{A} \times \vec{B}$.



$$AB \sin \phi = (6)(4)(\sin 30°) = 12$$

$$A_x = 6$$

$$B_x = 4 \cos 30° = 2\sqrt{3} \qquad B_y = 4 \sin 30° = 2$$

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

$$C_z = (6)(2) - (0)(2\sqrt{3}) = 12$$

$$\vec{C} = 12\hat{k}$$

Summary

- A vector in cartesian coordinates and polar coordinates.
- Unit vectors, $\hat{i}, \hat{j}, \hat{k}$
- Vector quantities have direction as well as magnitude and combine according to the rules of vector addition.
- The scalar product of two vectors
- The vector product of two vectors