Chapter 4: Motion in Two Dimensions

PHY0101/PHY(PEN)101

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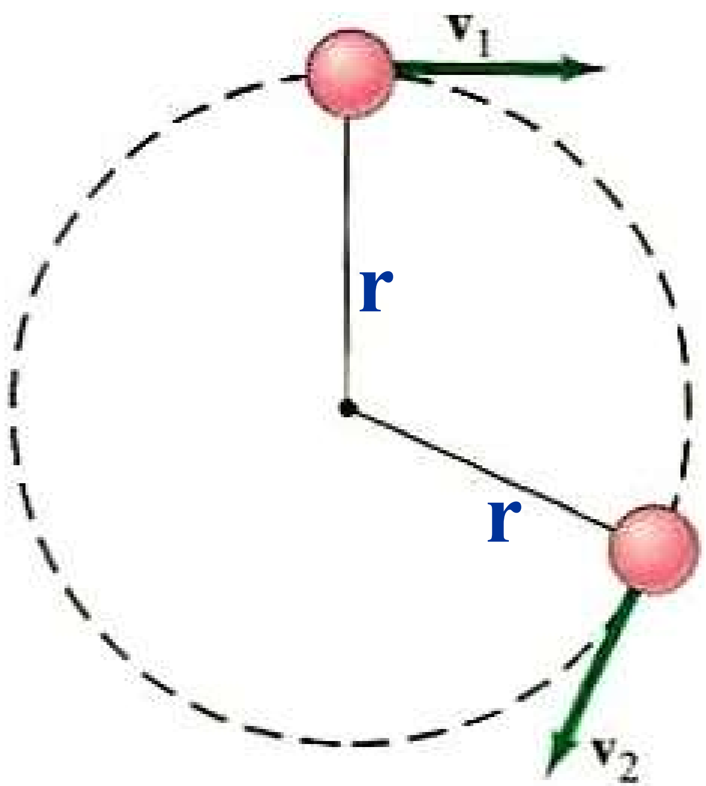
Outline

4.1 The Position, Velocity, and Acceleration Vectors
4.2 2D Motion with Constant Acceleration
4.3 Projectile Motion
4.4 Uniform Circular Motion
4.5 Tangential and Radial Acceleration
4.6 Relative Velocity and Relative Acceleration

Uniform Circular Motion

- Motion of a mass in a circle at ***constant speed***.
- **Constant *speed*** $v = |v| = \text{constant}$

⇒ **The Magnitude** (size) of the velocity vector **v** is constant. ***BUT*** the ***DIRECTION*** of ***v*** changes continually!



A small object moving in a circle, showing how the velocity changes. Note that at each point, the instantaneous velocity is in a direction tangent to the circular path.

$$v \perp r$$

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- For a mass moving in circle at ***constant speed***.

Acceleration ≡ **Rate of Change of Velocity**

$$a = (\Delta v/\Delta t)$$

Constant Speed ⇒ **The Magnitude** (size) of the velocity vector ***v*** is constant. $v = |v| =$ **constant**

BUT the ***DIRECTION*** of ***v*** changes continually!

⇒ ***An object moving in a circle undergoes acceleration!***

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Centripetal (Radial) Acceleration

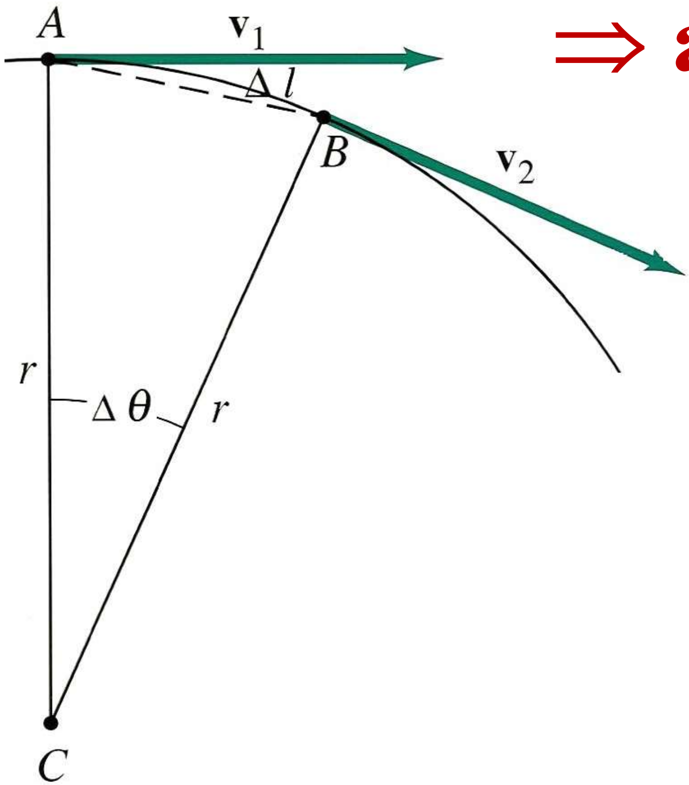
Consider motion in a circle from **A** to point **B**.

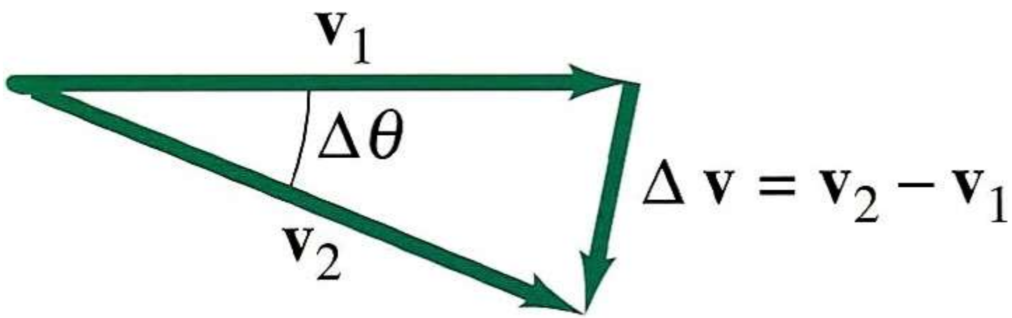
The velocity **v** is tangent to the circle.

$$\mathbf{a} = \lim_{\Delta t \to 0} (\Delta \mathbf{v}/\Delta t) = \lim_{\Delta t \to 0}[(\mathbf{v}_2 - \mathbf{v}_1)/(\Delta t)]$$

As $\Delta t \to 0$, $\Delta \mathbf{v} \to \perp \mathbf{v}$ & $\Delta \mathbf{v}$ is in the ***radial direction***

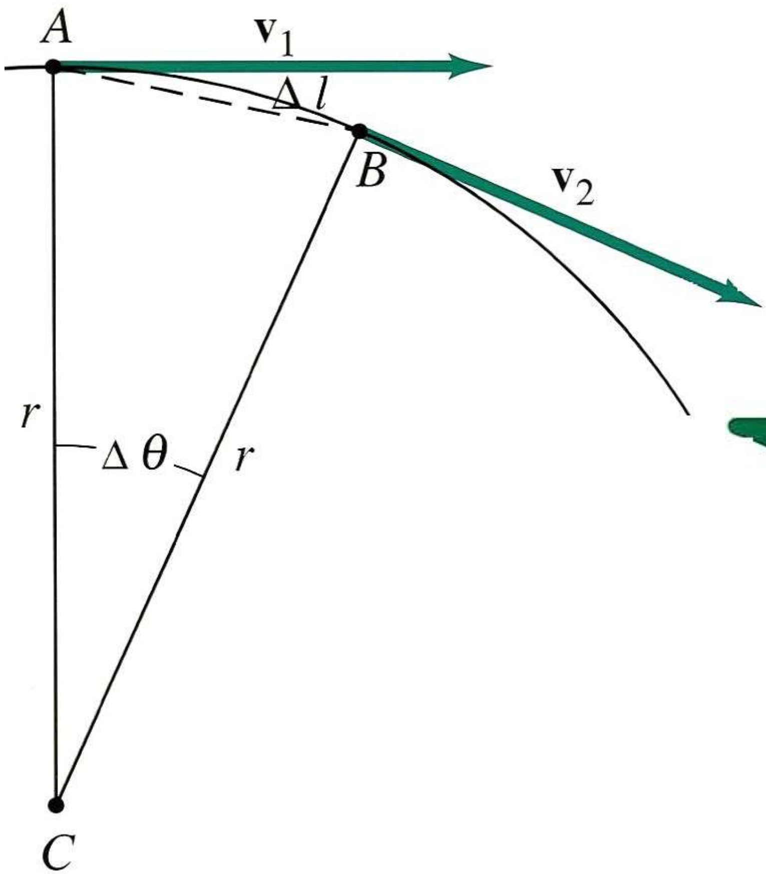
$\Rightarrow \mathbf{a} \equiv \mathbf{a}_c$ **is** ***radial!***

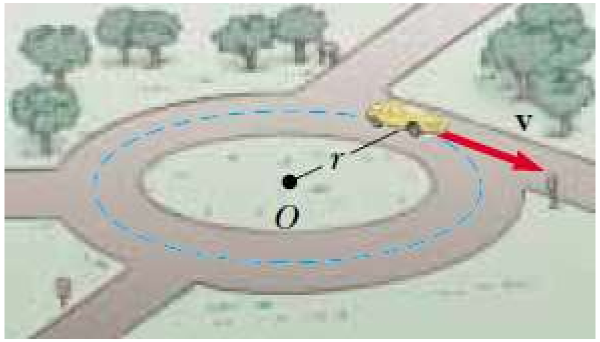


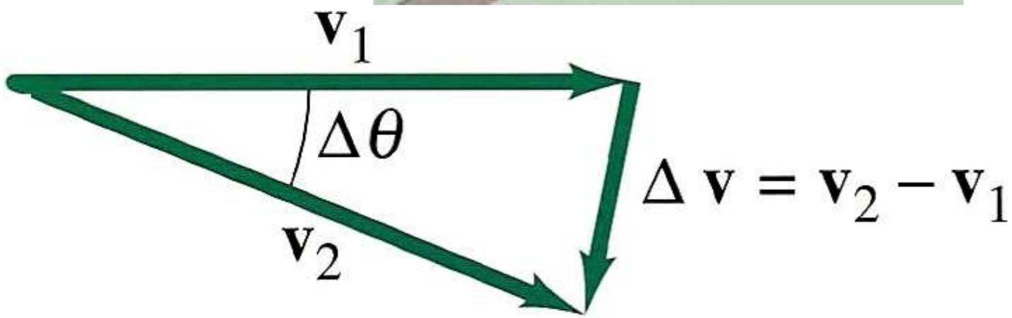


(b)

$\Rightarrow$ **a $\equiv$ a$_c$ is *Radial!!***







(b)

Similar Triangles $\Rightarrow$ $(\Delta v/v) \approx (\Delta \ell / r)$

As $\Delta t \rightarrow 0$, $\Delta\theta \rightarrow 0$, **A$\rightarrow$ B**

$$(\Delta v/v) = (\Delta \ell/r) \Rightarrow \Delta v = (v/r)\Delta \ell$$

- Note that the acceleration (*radial*) is

$$a_c = (\Delta v/\Delta t) = (v/r)(\Delta \ell/\Delta t)$$

As $\Delta t \to 0$, $(\Delta \ell/\Delta t) \to v$ and

Magnitude: $\boxed{a_c = (v^2/r)}$

Direction: **Radially *inward!***

"Centripetal" ≡ "Towards the Center"

Centripetal Acceleration is acceleration ***towards the center.***

- A typical figure for a particle moving in uniform circular motion, radius **r** (speed **v = constant**) is shown here:

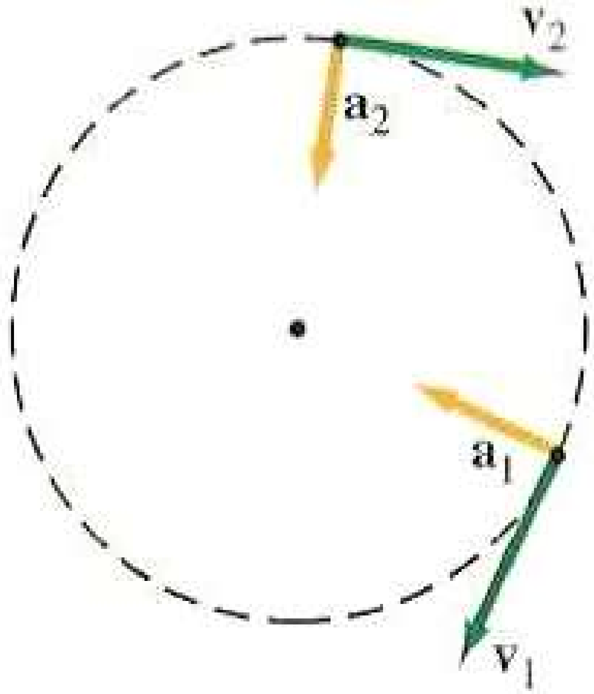
Velocity vector v is always
Tangent to the circle!!

$\boldsymbol{a} = \boldsymbol{a}_c =$
Centripetal Acceleration

a_c is Radially Inward
always!

$\Rightarrow a_c \perp$ v ***always!!***

Magnitude: $a_c = (v^2/r)$



For uniform circular motion, **a** is always perpendicular to **v**.

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Period & Frequency

- Consider again a particle moving in uniform circular motion of radius **r** (speed **v** = constant)
- Description in terms of ***period*** **T** & ***frequency*** **f**:
- Period **T** ≡ The time for one revolution (time to go around once), usually in seconds.
- Frequency **f** ≡ the number of revolutions per second.

$$\Rightarrow \mathbf{T = (1/f)}$$

revolution: devir

- Particle moving in uniform circular motion, radius **r** (speed **v** = constant)
- Circumference

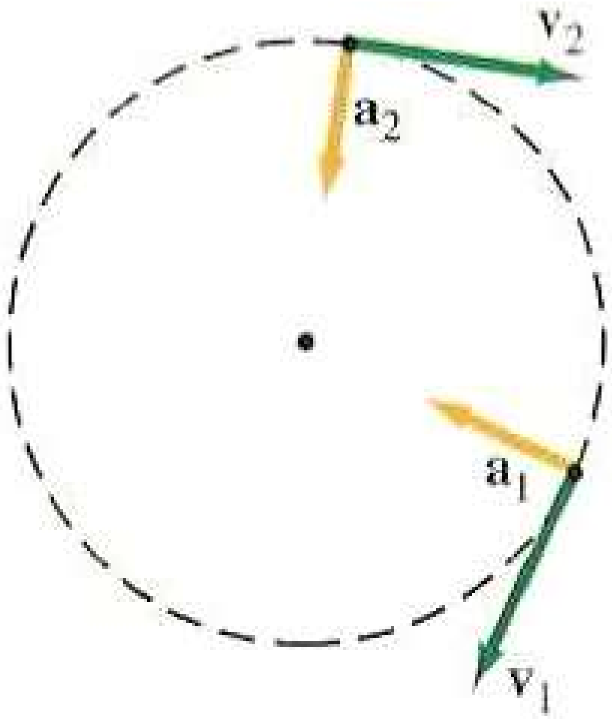
  = distance around= $\mathbf{2\pi r}$

⇒ Speed:

$$\mathbf{v = (2\pi r/T) = 2\pi r f}$$

⇒ **Centripetal acceleration:**

$$\mathbf{a_c = (v^2/r) = (4\pi^2 r/T^2)}$$



For uniform circular motion, **a** is always perpendicular to **v**.

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Example: Centripetal Acceleration of the Earth

Problem 1: Calculate the centripetal acceleration of the Earth as it moves in its orbit around the Sun.

NOTE: The radius of the Earth's orbit (from a table) is

r = 1.496 × 10^{11} m

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(1.496 \times 10^{11}\,\mathrm{m})}{(1\,\mathrm{yr})^2}\left(\frac{1\,\mathrm{yr}}{3.156 \times 10^7\,\mathrm{s}}\right)^2$$

$$= 5.93 \times 10^{-3}\,\mathrm{m/s^2}$$

Tangential & Radial Acceleration

- Consider an object moving in a curved path. If the speed **|v|** of object is changing, **there is an acceleration in the direction of motion.**

$\equiv$ ***Tangential Acceleration***

$$a_t \equiv |dv/dt|$$

- But, **there is also always a radial (or centripetal) acceleration perpendicular to the direction of motion.**

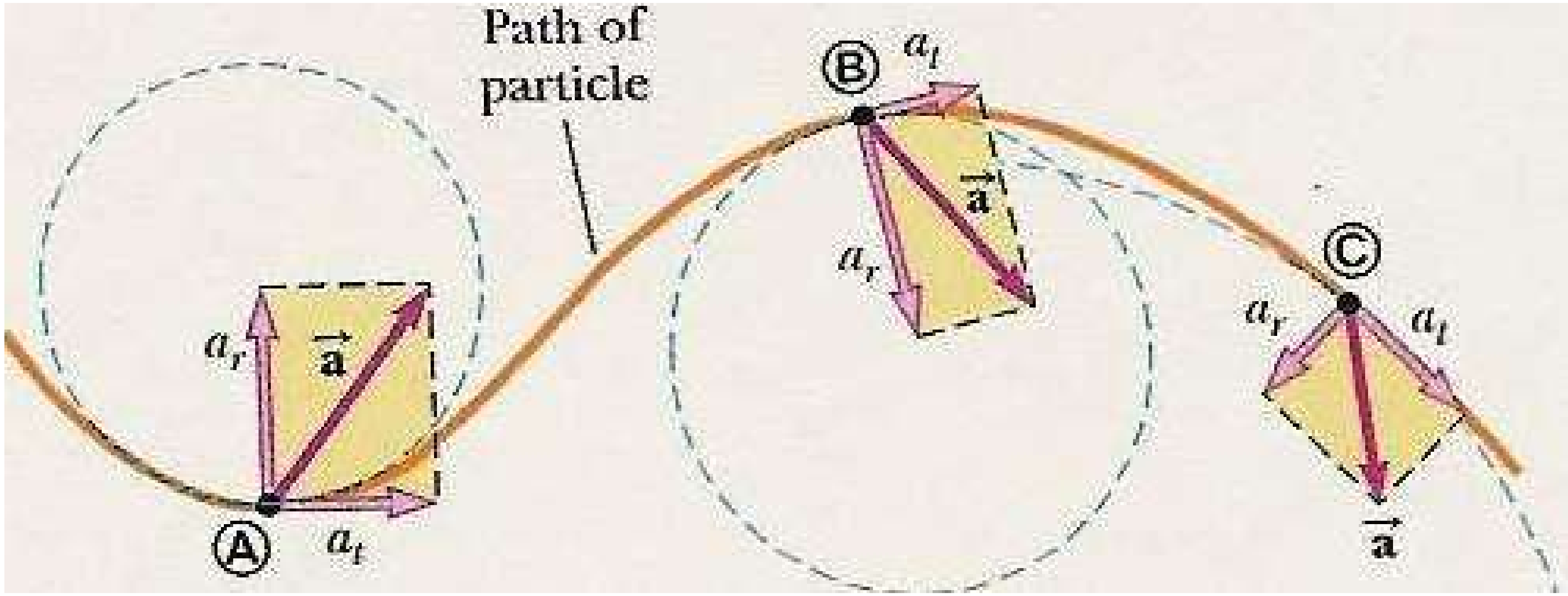
$\equiv$ ***Radial (Centripetal) Acceleration***

$$a_r = |a_c| \equiv (v^2/r)$$

Total Acceleration

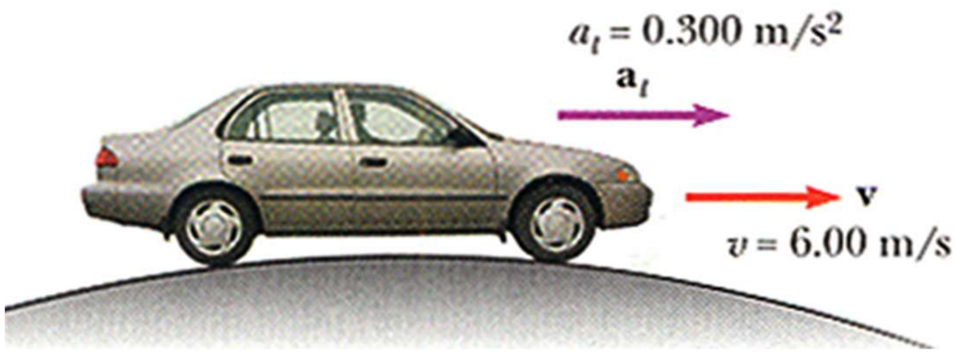
⇒ In this case there are always two ⊥ **vector** components of the acceleration:

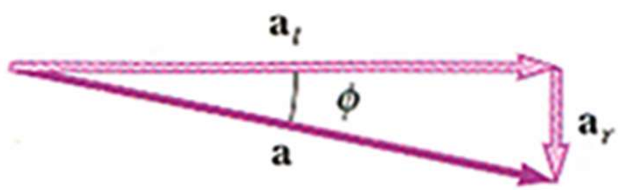
- **Tangential:** $a_t = |dv/dt|$ & **Radial:** $a_r = (v^2/r)$
- **Total Acceleration** is the ***vector sum***: $a = a_R + a_t$



4.5 Tangential & Radial acceleration
Example: Over the Rise

Problem: A car undergoes a constant acceleration of **a = 0.3 m/s²** ***parallel to the roadway***. It passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius **r = 500 m.** At the moment it is at the top of the rise, its velocity vector **v** is horizontal & has a magnitude of **v = 6.0 m/s**. What is the direction of the total acceleration vector **a** for the car at this instant?





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Solution:

$$a_r = -\frac{v^2}{r} = -\frac{(6.00\ \text{m/s})^2}{500\ \text{m}} = -0.0720\ \text{m/s}^2$$

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(-0.0720)^2 + (0.300)^2}\ \text{m/s}^2$$

$$= 0.309\ \text{m/s}^2$$

$$\phi = \tan^{-1}\frac{a_r}{a_t} = \tan^{-1}\left(\frac{-0.0720\ \text{m/s}^2}{0.300\ \text{m/s}^2}\right) = -13.5^\circ$$

4.6 Relative Velocity and Relative Acceleration

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Velocity of A relative to B:

$\mathbf{V}_{AB}=\mathbf{V}_A-\mathbf{V}_B$

$\mathbf{v}_{AB}$: v of A with respect to B

$\mathbf{v}_B$: v of B with respect to a reference frame (ex.: the ground)

$\mathbf{v}_A$: v of A with respect to a reference frame (ex.: the ground)

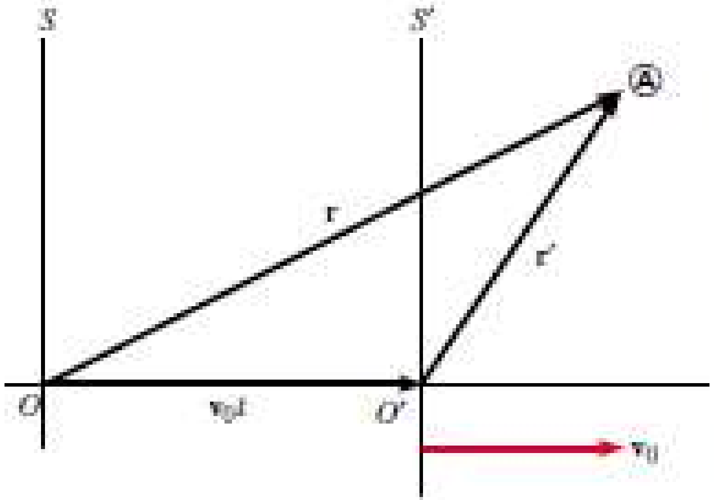

Figure 4.23 A particle located at Ⓐ is described by two observers, one in the fixed frame of reference S, and the other in the frame S′, which moves to the right with a constant velocity $\mathbf{v}_0$. The vector **r** is the particle's position vector relative to S, and **r**′ is its position vector relative to S′.

Galilean Transformation Equations

The vectors r and r' are related to each other through the expression:

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}_0 t$$

If we differentiate with respect to time and note that V_0 is constant, we obtain:

$$\mathbf{v}' = \mathbf{v} - \mathbf{v}_0$$

where V' and V is the velocity of the particle observed in the S' and S' frame, respectively.

Observers in two frames measure **the same acceleration** when V_0 is constant.

Example: A boad crossing a river

A boat heading due north crosses a wide river with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth.

Determine the velocity of the boat relative to an observer standing on either bank. (What is the resultant velocity of the boat relative to the shore?)

Solution:

$$\mathbf{v}_{bE} = \mathbf{v}_{br} + \mathbf{v}_{rE}$$

$$v_{bE} = \sqrt{v_{br}^2 + v_{rE}^2} = \sqrt{(10.0)^2 + (5.00)^2}\ \text{km/h}$$

$$= 11.2\ \text{km/h}$$

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{br}}\right) = \tan^{-1}\left(\frac{5.00}{10.0}\right) = 26.6^\circ$$

Take me back to my boat on the river…

- Position, Velocity, Acceleration in 2Ds
- Projectile motion
- Uniform circular motion
- Tangential and radial (centripetal) acceleration
- Relative velocity and relative acceleration