Chapter 6: Circular Motion and Other Applications of Newton's Laws

PHY0101/PHY(PEN)101

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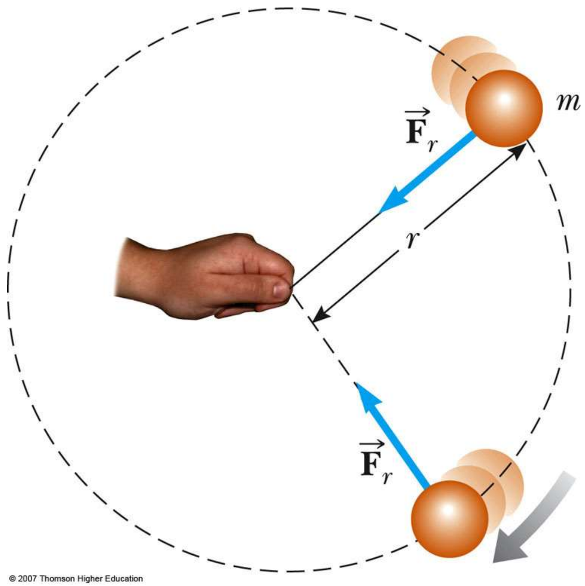
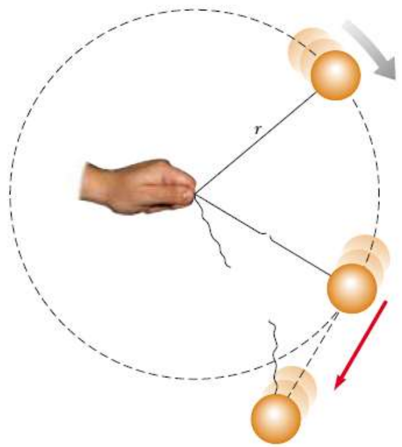
Outline

6.1 Newton's Second Law Applied to Uniform Circular Motion
6.2 Nonuniform Circular Motion
6.3 Motion in Accelerated Frames

Now we apply Newton's laws to objects traveling in circular paths. Also, we discuss motion observed from an accelerating frame of reference.

Uniform Circular Motion





Consider a particle moving in a circle of radius *r*, at a *constant speed* *v*.

$$a_c = (v^2/r)$$

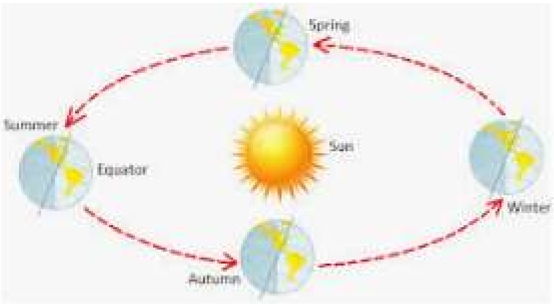
The string exerts a radial force F_r to move the ball along a circle.

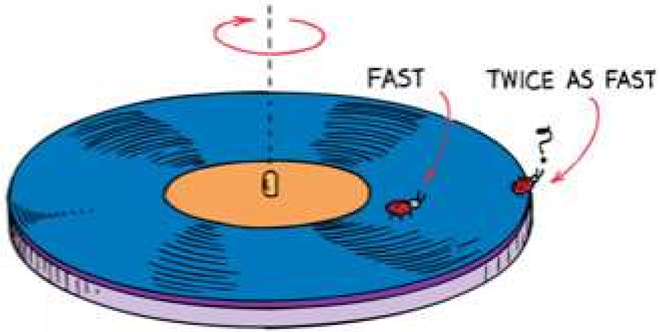
Apply Newton's second law along the radial direction to find F_r :

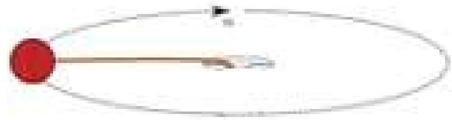
$$\sum F = ma_c = m\frac{v^2}{r}$$

Is centripetal force a new kind of force?

Solution: No. Centripetal force is not a separate force. Consider some examples.







F_r is due to gravity. F_r is due to friction. F_r is due to tension in the string

Furthermore, the centripetal force could be a combination of two or more forces.

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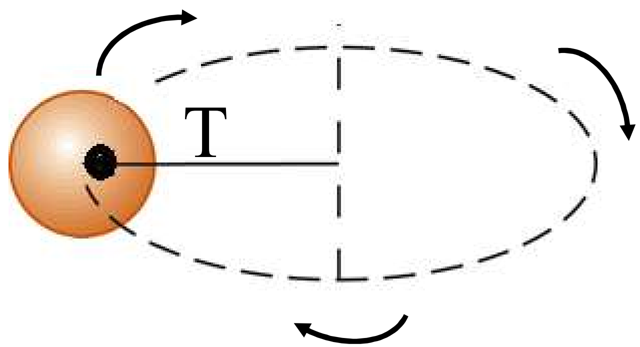
Example 6.1 **How Fast Can It Spin?**

A ball of mass 0.500 kg is attached to the end of a cord 1.50 m long. The ball is whirled in a horizontal circle. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed at which the ball can be whirled before the cord breaks?



Solution: The force causing the centripetal acceleration is the force T exerted by the cord.

$$T = m\frac{v^2}{r}$$

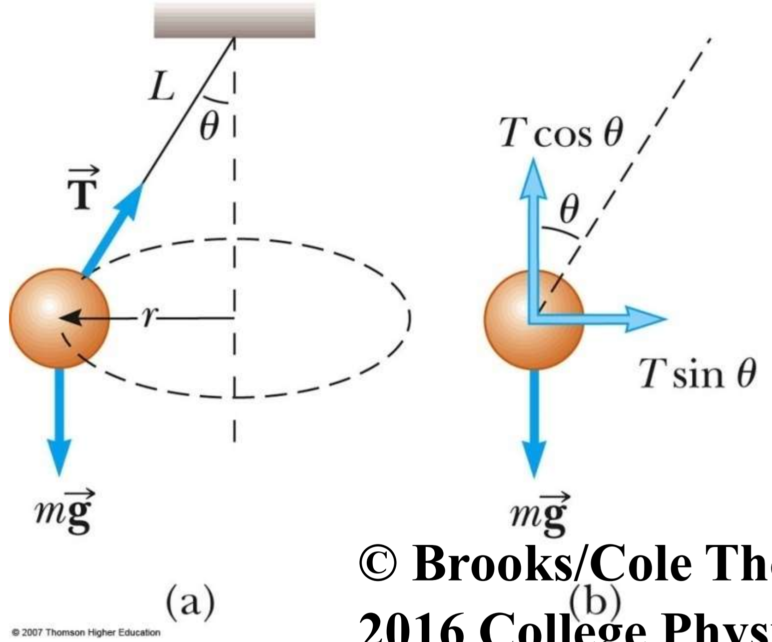
$$v = \sqrt{\frac{Tr}{m}}$$

$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50.0\ \text{N})\ (1.50\ \text{m})}{0.500\ \text{kg}}} = 12.2\ \text{m/s}$$

This shows that *v* increases with *T* and decreases with larger *m*.

Example 6.2 **The Conical Pendulum**

A small object of mass *m* is suspended from a string of length *L*. The object revolves with constant speed *v* in a horizontal circle of radius *r*. Find an expression for *v*.



$T_x = T\sin\theta$, $T_y = T\cos\theta$.

Newton's 2nd Law: $\sum F_x = T\sin\theta = ma_c = m(v^2/r)$ (1)

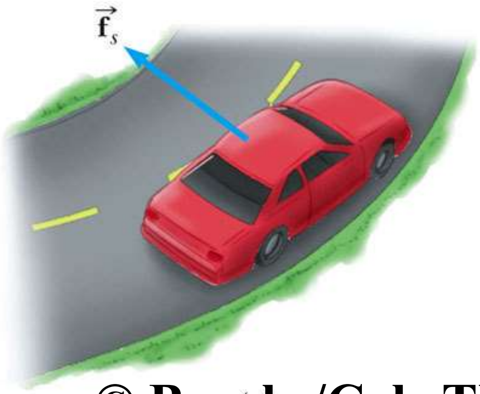
$\sum F_y = T\cos\theta - mg = 0$; $T\cos\theta = mg$ (2)

Dividing (1) by (2) gives: $\tan\theta = [v^2/(rg)]$, or $v = (rg\tan\theta)^{1/2}$

From trig, $r = L\sin\theta$ so, $v = (Lg\sin\theta\tan\theta)^{1/2}$

Example 6.3 **Maximum speed of the car**

A 1 500-kg car moving on a flat, horizontal road negotiates a curve, as shown in Figure. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.500, find the maximum speed the car can have and still make the turn successfully.



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The force causing the turn is the friction force.

Solution: Newton's 2nd Law

(let + x be to left) is:

$\sum F_x = f_s = ma_c = m(v^2/r)$ **(1)**

$\sum F_y = 0 = n - mg; \; n = mg$ **(2)**

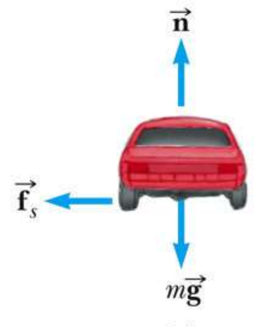
The maximum static friction force is (using (2))

$f_s^{(max)} = \mu_s n = \mu_s mg$ **(3)**

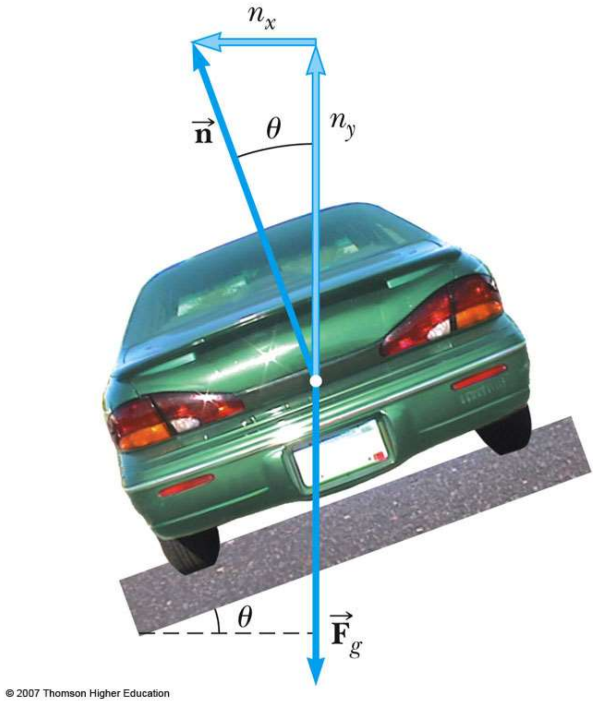
$\Rightarrow$ **If** $m(v^2/r) > f_s^{(max)}$, **so** v_{max} **is the solution to** $\mu_s mg = m[(v_{max})^2/r]$

Or, $v_{max} = (\mu_s gr)^{1/2}$

Putting in numbers gives: $v_{max} = 13.1$ **m/s**



Example 6.4 **Banked Curves**



Engineers design curves which are banked (tilted towards the inside of the curve) to keep cars on the road. If $r = 35$ m and we need $v = 13.4$ m/s, calculate the angle θ of banking needed (without friction).

Solution: From free body diagram, the radial and vertical components of the force **n** normal to the surface are:

$$n_x = n \sin\theta, \; n_y = n \cos\theta,$$

Newton's 2nd Law

$$\sum F_x = n \sin\theta = m(v^2/r) \qquad (1)$$

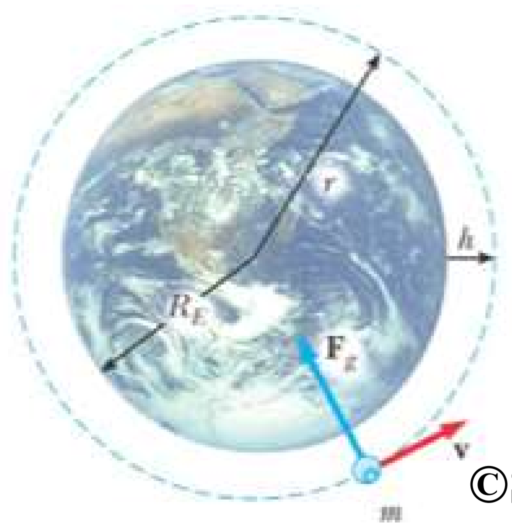
$$\sum F_y = 0 = n \cos\theta - mg; \; n \cos\theta = mg \qquad (2)$$

Dividing **(1)** by **(2)** gives: $\tan\theta = [(v^2)/(gr)]$

Putting in numbers gives: $\tan\theta = 0.523$ **or**

$$\theta = 27.6^\circ$$

Example 6.5 **Satellite Motion**



Consider a satellite of mass m moving in a circular orbit around the Earth at a constant speed V and at an altitude h above the Earth's surface. (a) Determine the speed of the satellite. (b) Determine the satellite's period of revolution.

Solution:

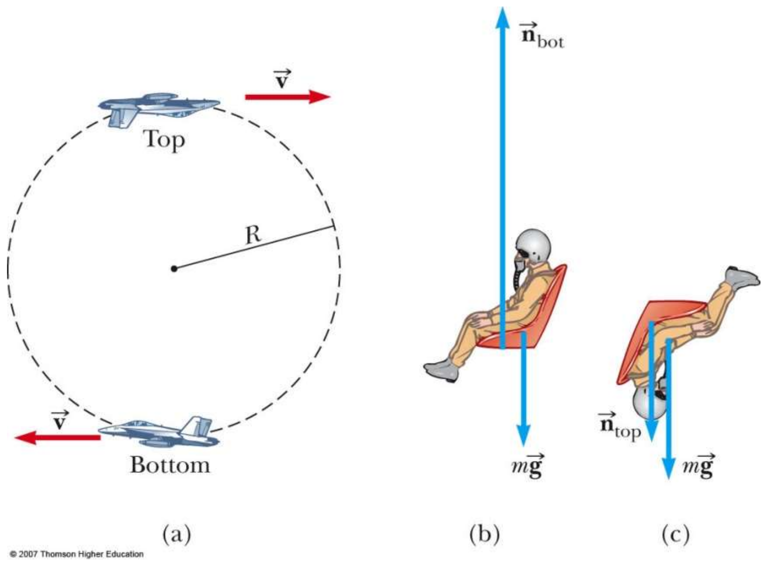
(a)

$$F = G\frac{M_{earth}m_{satellite}}{r^2} = \frac{m_{satellite}\vartheta^2}{r}$$

$$\vartheta = \sqrt{\frac{GM_e}{r}} \Rightarrow \vartheta = \sqrt{\frac{GM_e}{R_{earth}+h}}$$

(b) $$T_p = \frac{2\pi r}{\vartheta} = \frac{2\pi r}{\sqrt{\frac{GM_e}{r}}} \Rightarrow T_p = \frac{2\pi r^{3/2}}{\sqrt{GM_e}} \qquad T_p{}^2 \propto r^3$$

Example 6.6 "**Loop-the-Loop**"!



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A pilot, mass *m*, in a jet does a "loop-the-loop. The plane, Fig. (a), moves in a vertical circle, radius r = 2.7 km = 2,700 m at a constant speed v = 225 m/s.

a) Calculate the force, n_{bot} (normal force), exerted by the seat on the pilot at the bottom of the circle, Fig. (b).

b) Calculate this force, n_{top}, at the top of the circle, Fig. (c).

TOP: Fig. (b). Newton's 2nd Law in the radial (y) direction (up is "+").

$\sum F_y = n_{bot} - mg = m(v^2/r)$ **so** $n_{bot} = m(v^2/r) + mg$ **or**

$n_{bot} = mg[1 + (v^2/rg)] = 2.91\ mg$ **(putting in numbers) he feels "heavier".**

BOTTOM: Fig. (c). Newton's 2nd Law in the radial (y) direction (down is "+").

$\sum F_y = n_{top} + mg = m(v^2/r)$ **so** $n_{top} = m(v^2/r) - mg$ **or**

$n_{top} = mg[(v^2/rg) - 1] = 0.913\ mg$ **(putting in numbers) he feels "lighter".**

Non-Uniform Circular Motion

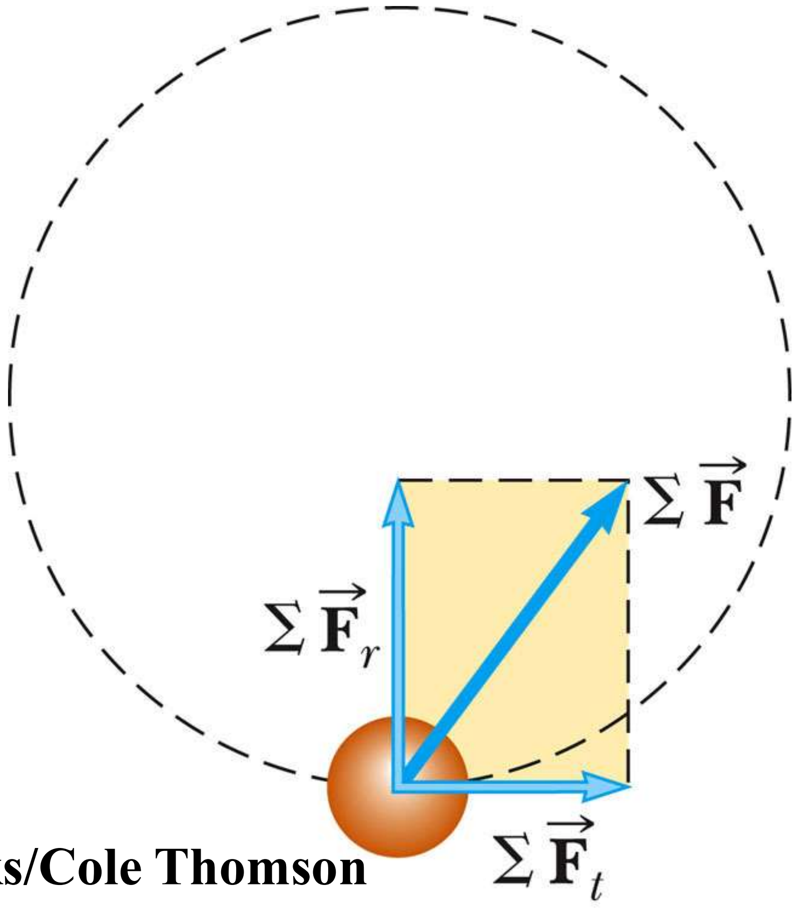
If particle moves with a varying speed

The acceleration and force have centripetal and tangential components:

- $\vec{F}_r$ produces the centripetal acceleration
- $\vec{F}_t$ produces the tangential acceleration

$$\sum \vec{a} = \sum \vec{a}_r + \sum \vec{a}_t$$

$$\sum \vec{F} = \sum \vec{F}_r + \sum \vec{F}_t$$



Vertical Circle with Non-Uniform Speed

Determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

Solution:

$$\sum F_t = mg \sin \theta = ma_t$$

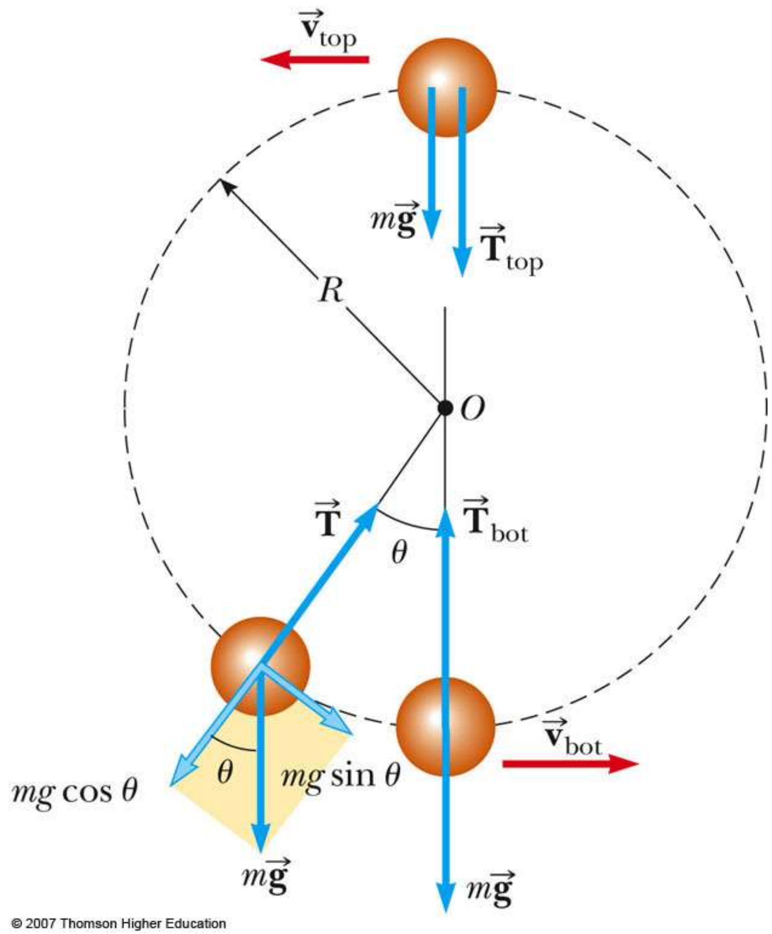
$$a_t = g \sin \theta$$

$$a_t = dv/dt$$

$$\sum F_r = T - mg \cos \theta = \frac{mv^2}{R}$$

The tension at any point can be found:

$$T = mg\left(\frac{v^2}{Rg} + \cos\theta\right)$$



What If? What if we set the ball in motion with a slower speed? What speed would the ball have as it passes over the top of the circle if the tension in the cord goes to zero instantaneously at this point?

Answer: At the top of the path $\cos 180° = -1$

$$T_{\text{top}} = m\left(\frac{v_{\text{top}}^2}{R} - g\right)$$

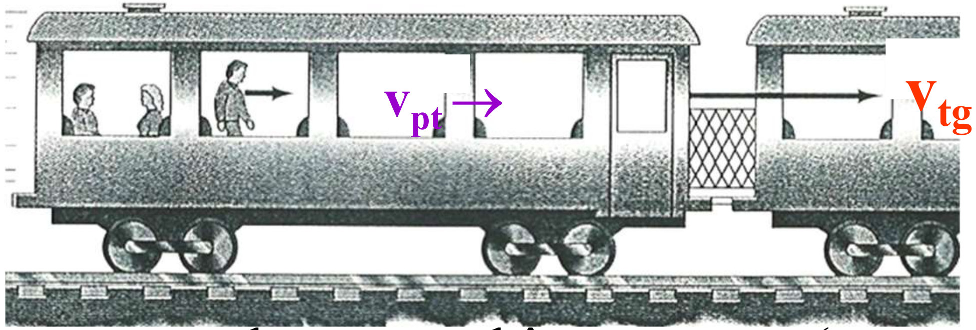
Let us set $T_{\text{top}} = 0$. Then,

$$0 = m\left(\frac{v_{\text{top}}^2}{R} - g\right)$$

$$v_{\text{top}} = \sqrt{gR}$$

6.3 Motion in Accelerated Frames

Recall, **Ch. 5:** ***Newton's Laws technically apply only in inertial (non-accelerated) reference frames***. Consider a **train** moving with respect to the **ground** at constant velocity $\mathbf{v}_{tg}$ to the right. A **person** is walking in the train at a constant velocity $\mathbf{v}_{pt}$ with respect to the train.



Newton's Laws hold for the person walking in the train as long as the train velocity $\mathbf{v}_{tg}$ with respect to the ground is constant (no acceleration). But, if the train accelerates with respect to the ground ($\mathbf{v}_{tg} \neq$ constant) and the person does an experiment, the result will ***appear to violate Newton's Laws*** because objects will undergo an acceleration in the opposite direction as the train's acceleration. This acceleration will (seem to be) unexplained because it occurs in the absence of forces!

More Discussion

- **Inertial Reference Frame:**
  - Any frame in which Newton's Laws are valid!
  - Any reference frame moving with uniform (***non-accelerated***) motion with respect to an "absolute" frame "fixed" with respect to the stars.
- By definition, ***Newton's Laws*** are only valid in inertial frames!!

$$\sum \mathbf{F} = m\mathbf{a}$$

Is not valid in a non-inertial frame!

- ***By definition***, a "***Force***", as defined in Chapter 5 is that it comes about from an interaction between objects.

Fictitious "Forces"

- Suppose a person on a train places a flat object on a flat, horizontal, frictionless surface. There are no horizontal forces on the object. The train accelerates forward. The person will see the flat object accelerate horizontally backward at the same time. This

  Appears to violate Newton's Laws

  because there is no horizontal force on the object (accelerations are ***caused by*** forces).

  This only appears to violate Newton's Laws, but actually doesn't,

  because the accelerating train is

  NOT AN Inertial Reference Frame.
- Based on this experiment, the person will say that there is a ***Fictitious (or Apparent) Force*** on the object.

Simple Example of a Fictitious Force

You ride in a car going around a curve at constant velocity **v**.

⇒ There is a centripetal acceleration ***on the car:***

$a_c = (v^2/r)$, towards the center of the curve

(**r** = curve radius).

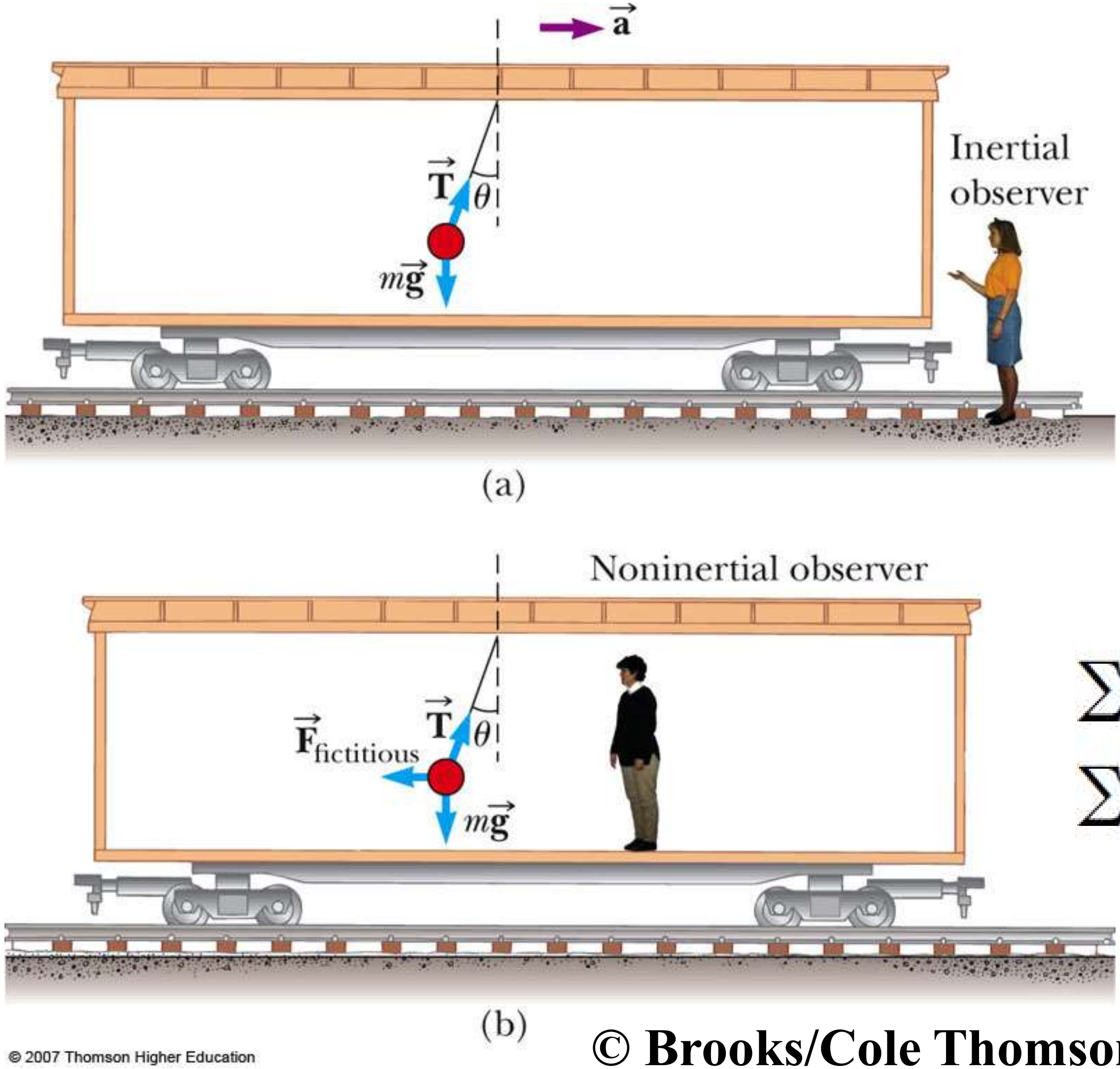
The car takes the curve fast enough that, in the passenger seat, you slide to the right and bump against the door.

Your reference frame, the car, is a <u>NON-INERTIAL</u> frame.

You feel an *apparent* force moving you towards the door (*away* from the center of the curve). A popular and <u>*WRONG*</u> explanation is that you feel a "Centrifugal Force" away from the center of the the curve. "Centrifugal Force" " is an example of a fictitious force.

Prof. Charles W. Myles

Example 6.8 Fictitious Forces in Linear Motion



Inertial observer

$$\sum F_x = T\sin\theta = ma$$

$$\sum F_y = T\cos\theta - mg = 0$$

Noninertial observer

$$\sum F'_x = T\sin\theta - F_{\text{fictitious}} = 0$$

$$\sum F'_y = T\cos\theta - mg = 0$$

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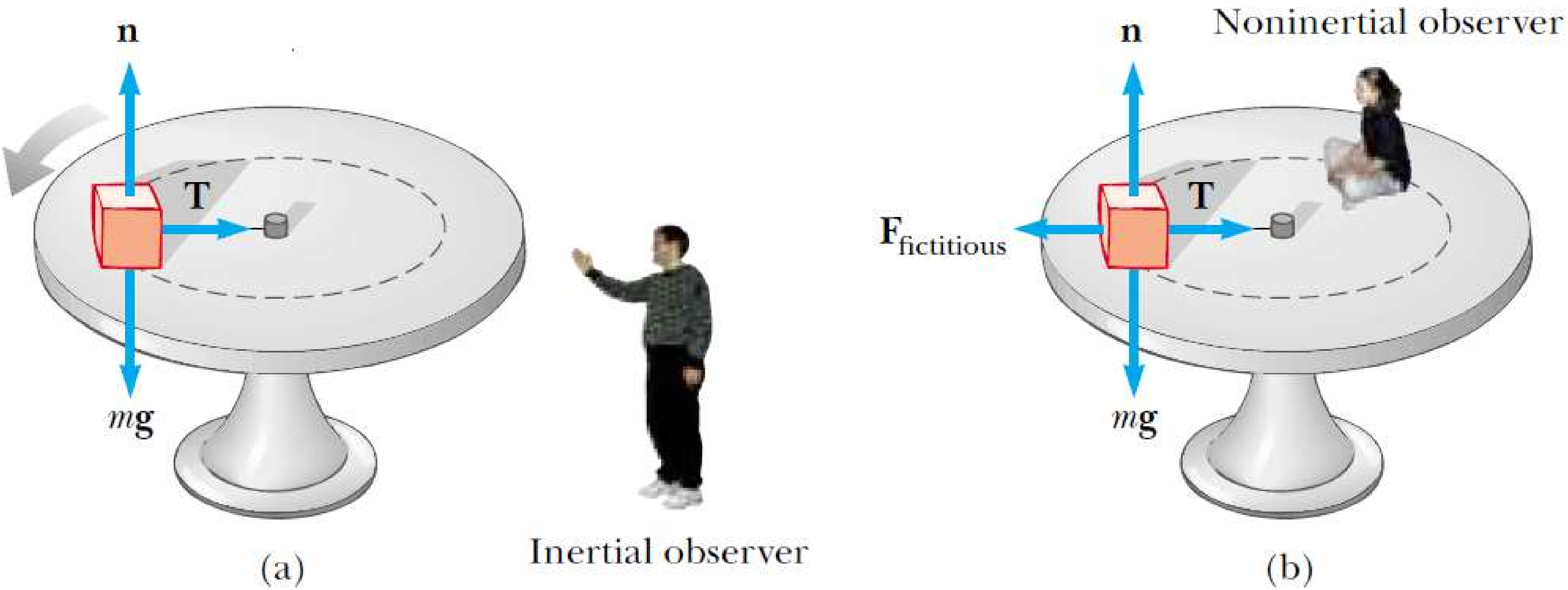
Example 6.9 **Fictitious Force in a Rotating System**



$$T = mv^2 / r$$

$$F_{fictitious} = m\vartheta^2 / r$$

$$T - mv^2 / r = 0$$

An observer in a noninertial (accelerating) frame of reference must introduce fictitious forces when applying Newton’s second law in that frame. If these fictitious forces are properly defined, the description of motion in the noninertial frame is equivalent to that made by an observer in an inertial frame.

- Net force causing the particle to undergo centripetal acceleration
- Nonuniform circular motion
- Motion in accelerated frames

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