# Chapter 7: Energy of a System

PHY0101/PHY(PEN)101

Assoc. Prof. Dr. Fulya Bağcı

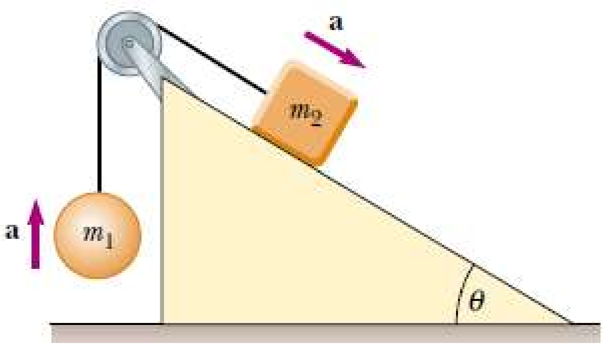
# Outline (from 9th Edition of Serway)

7.1 Systems and Environments
7.2 Work Done by a Constant Force
7.3 Work Done by a Varying Force
7.4 Kinetic Energy and the Work–Kinetic Energy Theorem
7.5 Potential Energy of a System
7.6 Conservative and Nonconservative Forces
7.7 Relationship Between Conservative Forces and the Potential Energy

- Every physical process that occurs in the Universe involves energy and energy transfers or transformations.
- The concept of energy can be applied to the dynamics of a mechanical system without resorting to Newton's laws, especially when the acceleration is not constant. There we can apply *conservation of energy*.

## 7.1 Systems and Environments

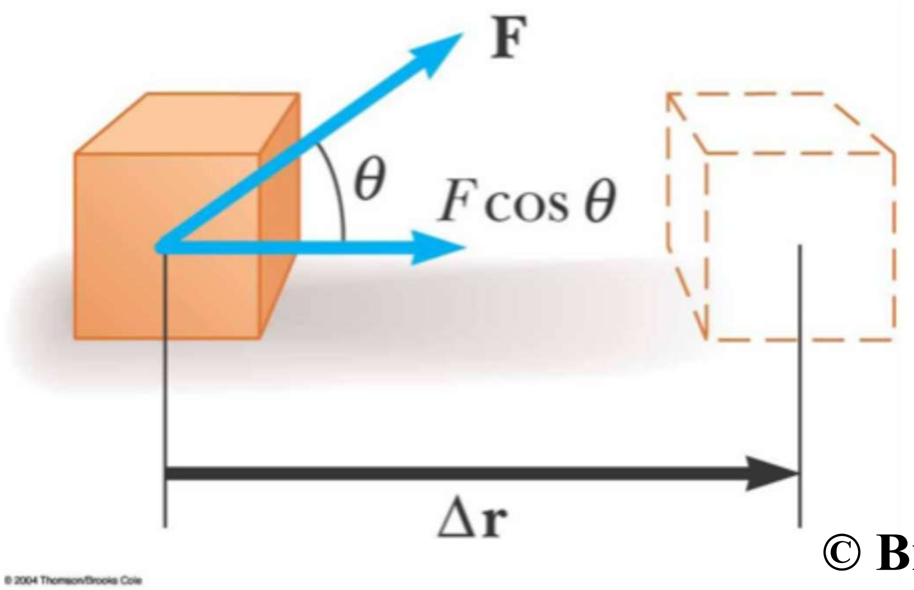
Our problem-solving treatment was particle model. Now, we will define a system. A valid system may be a single object or particle, be a collection of objects or particles, be a region of space and can vary in size and shape (such as a rubber ball, which deforms upon striking a wall).




The influence from the environment includes the gravitational forces on the ball and the cube, the normal and friction forces on the cube, and the force exerted by the pulley on the string.

# 7.2 Work Done by a Constant Force

The meaning of work is distinctly different in physics than its use in everday life.




$$\boxed{W = F\Delta r\cos\theta}$$

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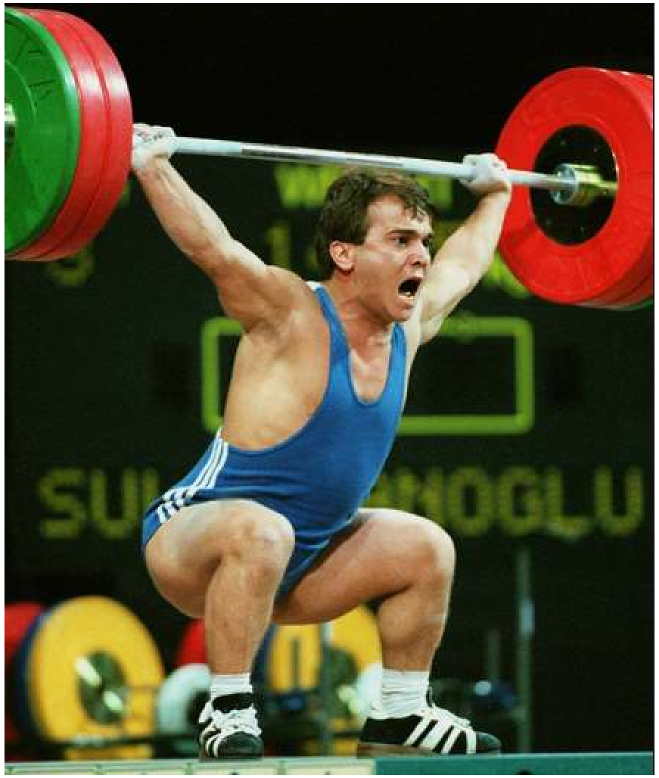
The work $W$ done on a system by an agent exerting a constant force on the system is the product of the magnitude $F$ of the force, the magnitude $\Delta r$ of the displacement of the point of application of the force, and $\cos\theta$ where $\theta$ is the angle between the force and displacement vectors.

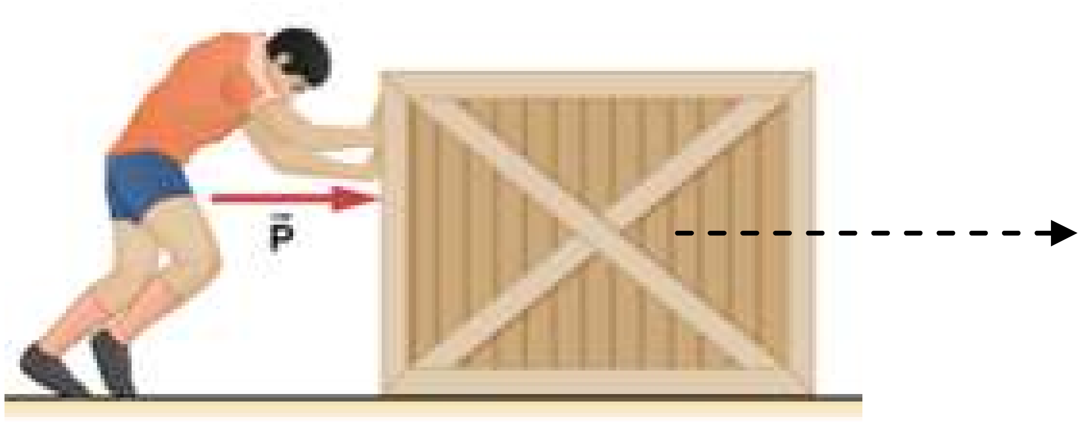


The weightlifter does no work on the weights as he holds them for a while.

Did he do any work when he raised the weights to this height? *Yes.*


Force is perpendicular to the displacement along $x$-axis.

If $\theta = 90°$, then $W = 0$ because $\cos 90° = 0$.

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Forces is parallel to the displacement along $x$-axis.

If $\theta = 0°$, then $\boldsymbol{W} = \boldsymbol{F}\Delta\boldsymbol{r}$ because $\cos 0° = 1$.

$$\boxed{W = F\Delta r\cos\theta}$$

Work is a scalar quantity, and its units are force multiplied by length. Therefore, the SI unit of work is the Newton.meter (N· m). This combination of units is used so frequently that it has been given a name of its own, the **joule** (**J**)

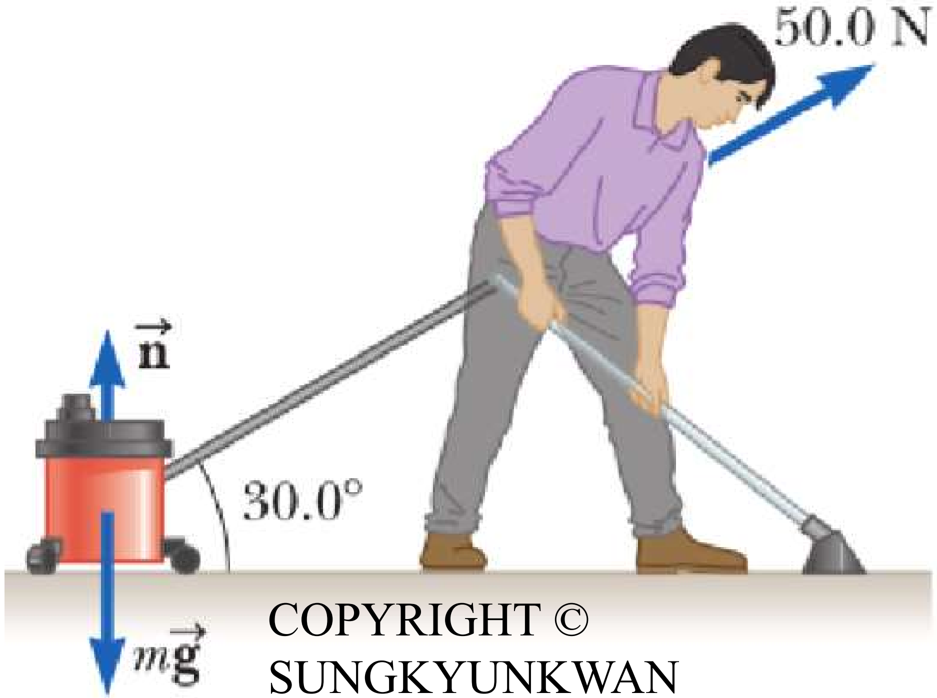
- Work is an energy transfer.
- If $W$ is the work done on a system and $W$ is positive, energy is transferred *to* the system.
- If $W$ is negative, energy is transferred *from* the system.

## Example 7.1 **Mr. Clean**

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F$ = 50.0 N at an angle of 30.0° with the horizontal. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

$$W = F\,\Delta r\cos\theta = (50.0\ \text{N})(3.00\ \text{m})(\cos 30.0°)$$

$$= 130\ \text{N}\cdot\text{m} = \boxed{130\ \text{J}}$$



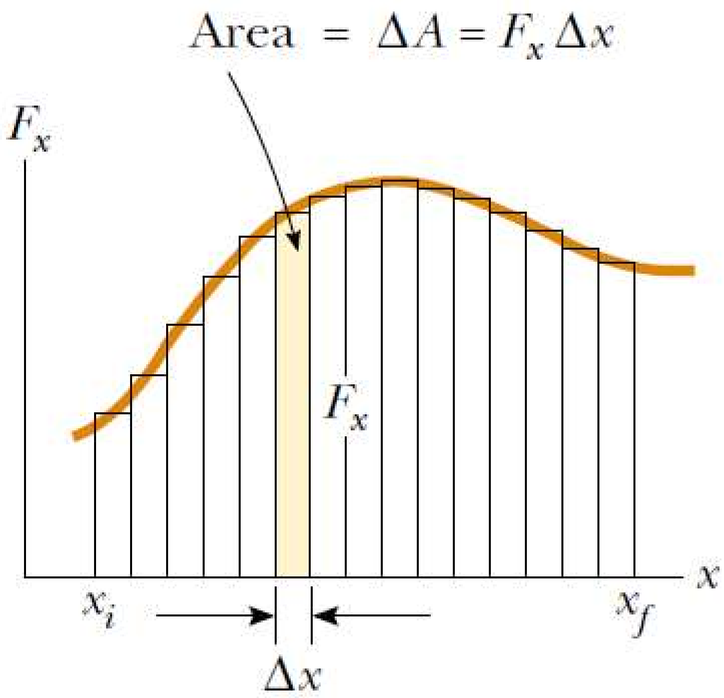
# 7.3 Work Done by a Constant Force

A particle moving in the *xy* plane undergoes a displacement given by $\Delta \boldsymbol{r} = (2.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}})$ m as a constant force $\boldsymbol{F} = (5.0\,\hat{\mathbf{i}} + 2.0\,\hat{\mathbf{j}})$ N acts on the particle. Calculate the work done by $\boldsymbol{F}$ on the particle.

$$
\begin{aligned}
W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} &= [(5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\ \mathrm{N}] \cdot [(2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})\ \mathrm{m}] \\
&= (5.0\hat{\mathbf{i}} \cdot 2.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{i}} \cdot 3.0\hat{\mathbf{j}} + 2.0\hat{\mathbf{j}} \cdot 2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}} \cdot 3.0\,\hat{\mathbf{j}})\ \mathrm{N} \cdot \mathrm{m} \\
&= [10 + 0 + 0 + 6]\ \mathrm{N} \cdot \mathrm{m} = 16\ \mathrm{J}
\end{aligned}
$$

# 7.4 Work Done by a Varying Force

Consider a particle being displaced along the $x$ axis under the action of a force that varies with position.




$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x \, dx$$

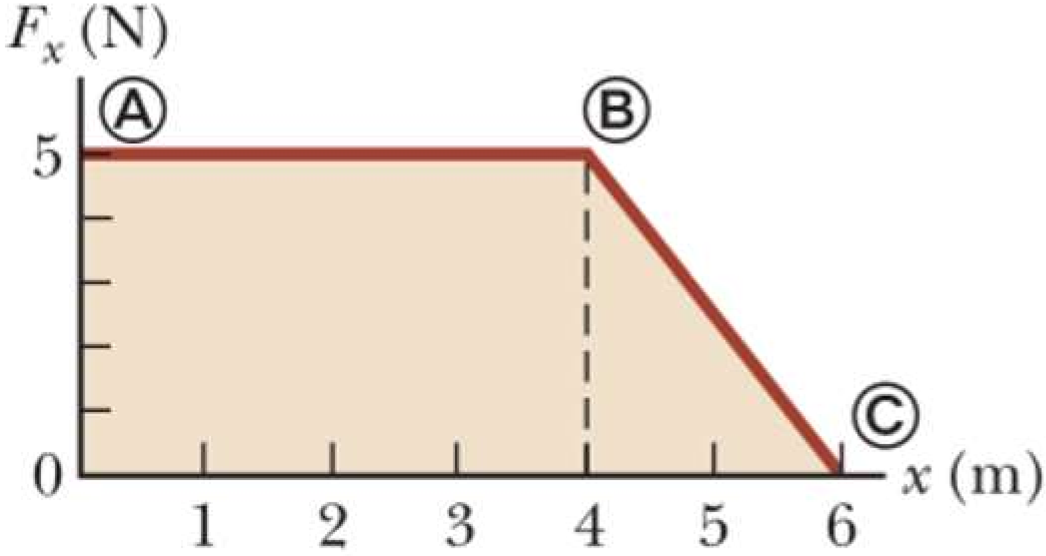
$$\boxed{W = \int_{x_i}^{x_f} F_x \, dx}$$

If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is just the work done by the net force.

$$\sum W = W_{\text{net}} = \int_{x_i}^{x_f} \left( \sum F_x \right) dx$$

# Example 7.4

A force acting on a particle varies with $x$ as shown in the figure. Calculate the work done by the force on the particle as it moves from $x = 0$ to $x = 6.0$ m.



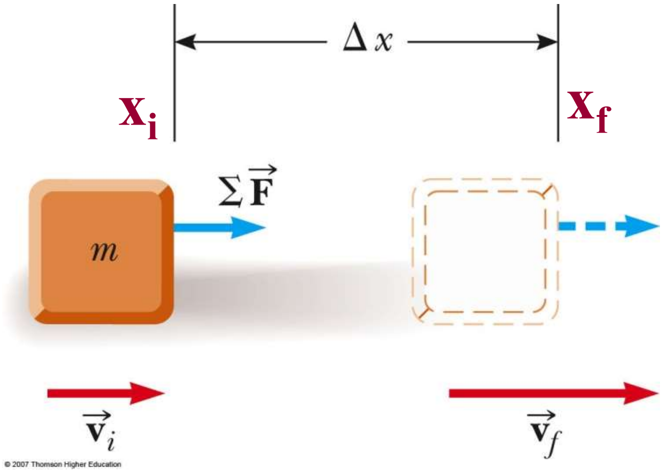

$$W_{A \text{ to } B} = (5.0\text{ N})(4.0\text{ m}) = 20\text{ J}$$

$$W_{B \text{ to } C} = \frac{1}{2}(5.0\text{ N})(2.0\text{ m}) = 5.0\text{ J}$$

$$W_{A \text{ to } C} = W_{A \text{ to } B} + W_{B \text{ to } C} = 20\text{ J} + 5.0\text{ J} = 25\text{ J}$$

# 7.5 Kinetic Energy and Work-Kinetic Energy Theorem

Consider a particle with mass *m* moving along the *x*-axis under the action of a constant net force with magnitude *F* directed along the positive *x*–axis.

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$$v_f^2 = v_i^2 + 2a\Delta x$$

$$a = \frac{v_f^2 - v_i^2}{2\Delta x}$$

$$F = ma = m\frac{v_f^2 - v_i^2}{2\Delta x}$$

$$F\Delta x = W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\boxed{W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = K_f - K_i = \Delta K}$$

## Work–kinetic energy theorem:

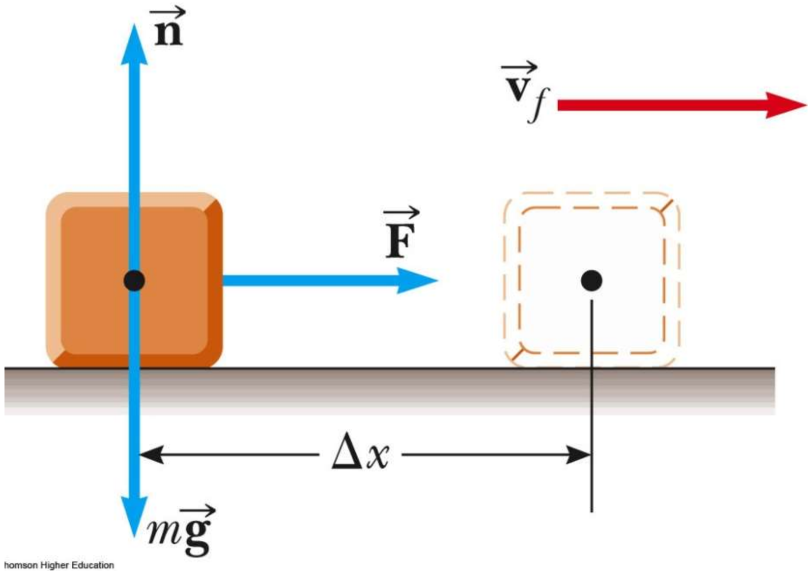
In the case in which work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.

The work–kinetic energy theorem indicates that the speed of a particle will *increase* if the net work done on it is *positive,* because the final kinetic energy will be greater than the initial kinetic energy. The speed will *decrease* if the net work is *negative,* because the final kinetic energy will be less than the initial kinetic energy.

The work–kinetic energy theorem is also valid for systems that undergo a change in the rotational speed due to work done on the system.

# Example 7.7 A Block Pulled on a Frictionless Surface

A block, mass $m = 6$ kg, is pulled from rest ($v_i = 0$) to the right by a constant horizontal force $F = 12$ N. After it has been pulled for $\Delta x = 3$ m, find it's final speed $v_f$.






**Solution:**

$$W_{net} = \Delta K \equiv (½)[m(v_f)^2 - m(v_i)^2] \quad (1)$$

If $F = 12$ N is the only horizontal force, we have

$$W_{net} = F\Delta x \quad (2)$$

Combine (1) and (2):

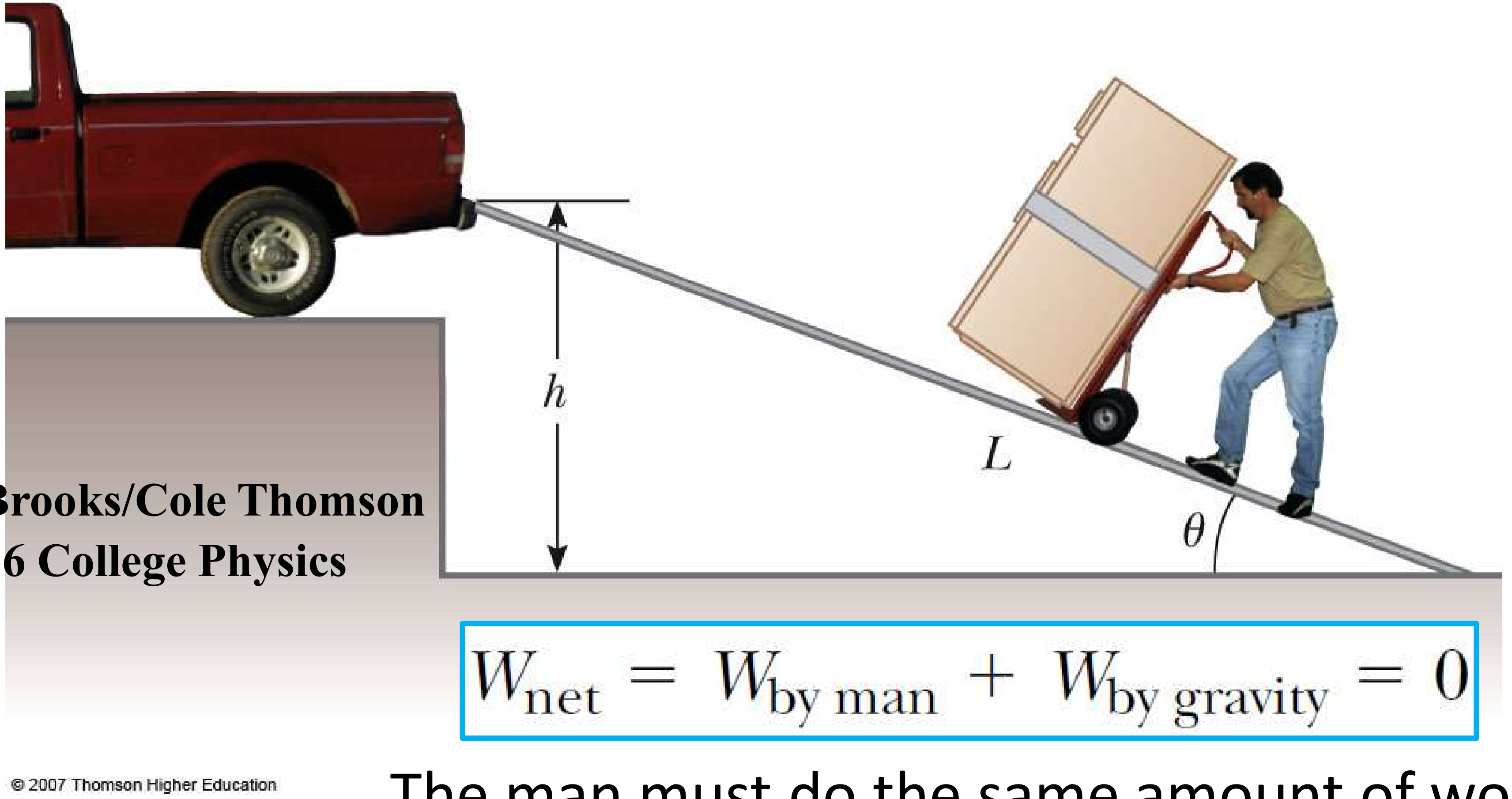
$$F\Delta x = (½)[m(v_f)^2 - 0]$$

Solve for $v_f$:

$$(v_f)^2 = [2\Delta x/m]$$

$$(v_f) = [2\Delta x/m]^{½} = 3.5 \text{ m/s}$$

# Conceptual Example 7.7 Does the Ramp Lessen the Work Required?





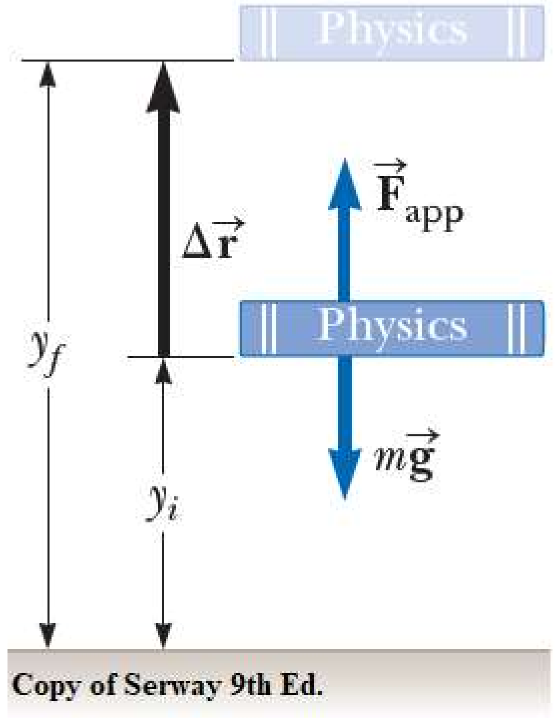

$$\boxed{W_{\text{net}} = W_{\text{by man}} + W_{\text{by gravity}} = 0}$$

The man must do the same amount of work *mgh* on the refrigerator, *regardless* of the length of the ramp.

## 7.6 Potential Energy of a System

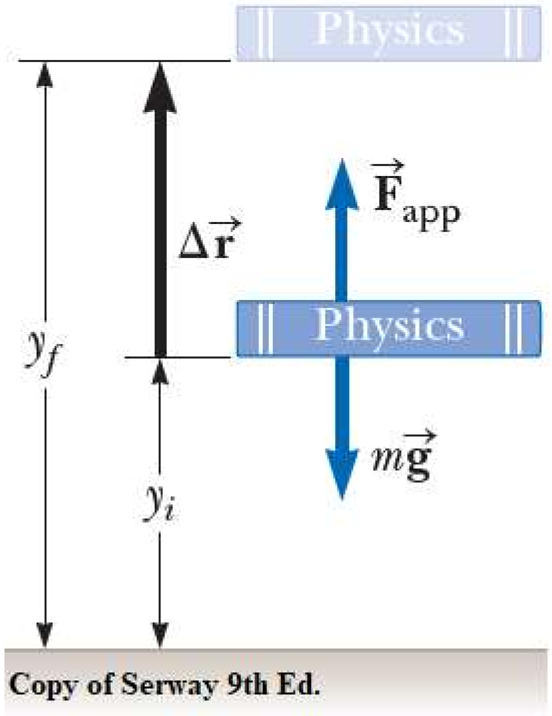
Imagine a system consisting of a book and the Earth, interacting via the gravitational force. We do some work on the system by lifting the book slowly from rest through a vertical displacement.

$$\Delta\vec{\mathbf{r}} = (y_f - y_i)\hat{\mathbf{j}}$$




The book is at rest before we perform the work and is at rest after we perform the work. We call the energy storage mechanism before the book is released potential energy.

Assume that lifting is done slowly without accelaration.




$$W_{ext} = (\vec{\mathbf{F}}_{app}) \cdot \Delta\vec{\mathbf{r}} = (mg\,\hat{\mathbf{j}}) \cdot [(y_f - y_i)\,\hat{\mathbf{j}}] = mgy_f - mgy_i$$

We can identify the quantity *mgy* as the gravitational potential energy $U_g$ of the system of an object of mass *m* and the Earth.

$$U_g \equiv mgy$$

The net external work done on the system in this situation appears as a change in the gravitational potential energy of the system.

$$W_{ext} = \Delta U_g$$

The important quantity is the difference in the potential energy and this difference is independent of the choice of reference configuration.

# Example 7.8 The Proud Athlete and the Sore Toe

A trophy being shown off by a careless athlete slips from the athlete's hands and drops on his toe. Choosing floor level as the $y = 0$ point of your coordinate system, estimate the change in gravitational potential energy of the trophy–Earth system as the trophy falls. Repeat the calculation, using the top of the athlete's head as the origin of coordinates.

Solution: Let's say the trophy has a mass of approximately 2 kg, and the top of a person's foot is about 0.05 m above the floor. Also, let's assume the trophy falls from a height of 1.4 m.

$$U_i = mgy_i = (2\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(1.4\ \mathrm{m}) = 27.4\ \mathrm{J}$$

$$U_f = mgy_f = (2\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(0.05\ \mathrm{m}) = 0.98\ \mathrm{J}$$

$$\Delta U_g = 0.98\ \mathrm{J} - 27.4\ \mathrm{J} = -26.4\ \mathrm{J}$$

**Assoc.Prof.Dr. Fulya Bağcı**

# Elastic Potential Energy

A block on a horizontal, frictionless surface is connected to a spring.
If the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force that can be expressed as,

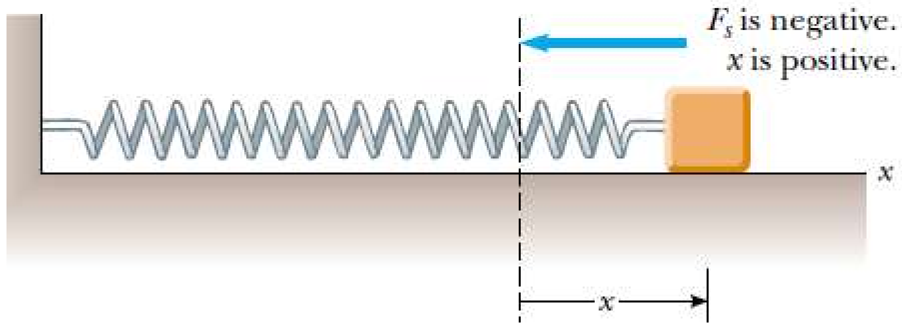
$$F_s = -kx$$

Hooke's law

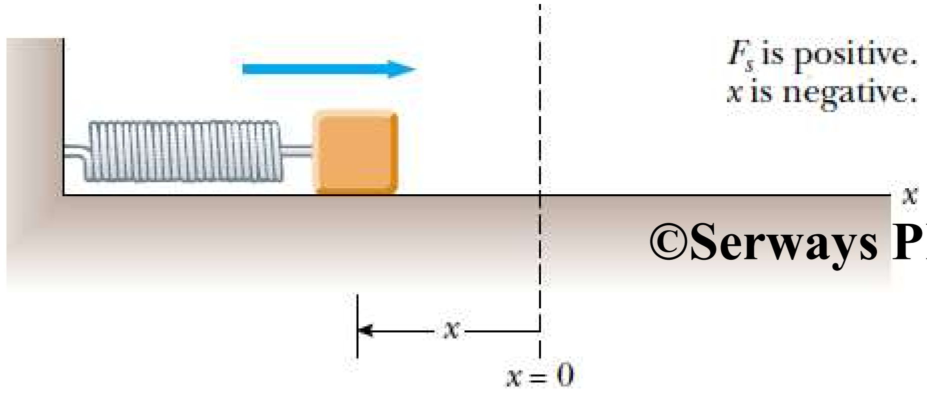
directed *opposite* to the displacement from equilibrium.

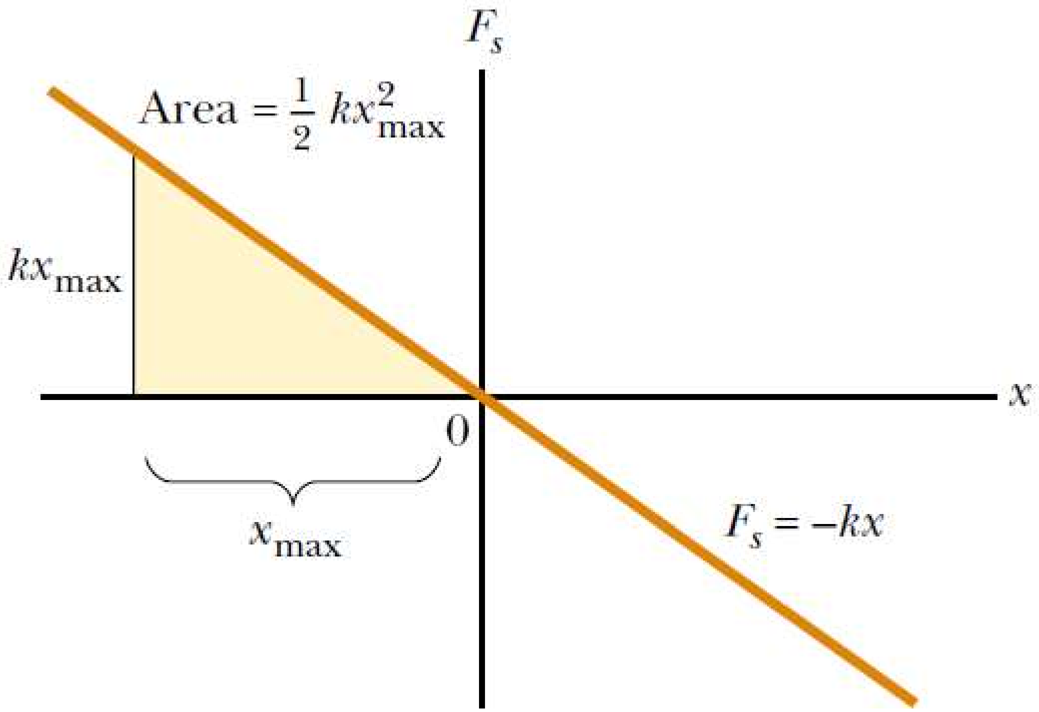
where $x$ is the position of the block relative to its equilibrium ($x = 0$) position and $k$ is a positive constant called the force constant or the spring constant of the spring.

When $x > 0$ the block is to the right of the equilibrium position, the spring force is directed to the left, in the negative $x$ direction.




When $x < 0$ the block is to the left of equilibrium and the spring force is directed to the right, in the positive $x$ direction.






The work done by the spring force as the block moves from $-x_{max}$ to 0 is the area of the shaded triangle $\frac{1}{2}kx^2_{max}$

If the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$, the work done by the spring force on the block is

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$W_{ext} = \Delta U_s \qquad \boxed{U_s \equiv \frac{1}{2}kx^2}$$

For example, if the spring has a force constant of 80 N/m and is compressed 3.0 cm from equilibrium, the work done by the spring force as the block moves from $x_i$=-3.0 cm to its unstretched position $x_f$=0 is $3.6.10^{-2}$J

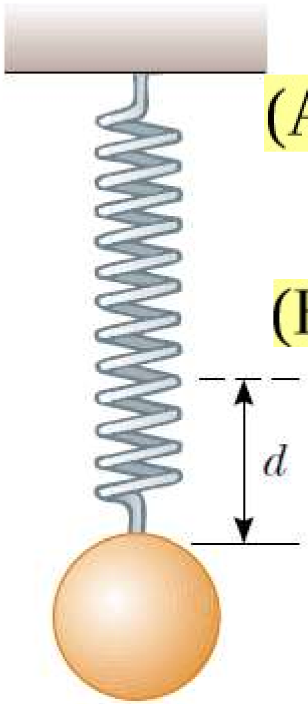
The work done by the spring force is zero for any motion that ends where it began ($x_i = x_f$).

# Example 7.5

A common technique used to measure the force constant of a spring is demonstrated by the setup in the figure. The spring is hung vertically, and an object of mass *m* is attached to its lower end. Under the action of the "load" *mg*, the spring stretches a distance *d* from its equilibrium position.

(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?

(B) How much work is done by the spring on the object as it stretches through this distance?




$$\vec{\mathbf{F}}_s + m\vec{\mathbf{g}} = 0 \;\rightarrow\; F_s - mg = 0 \;\rightarrow\; F_s = mg$$

(A)

$$k = \frac{mg}{d} = \frac{(0.55\text{ kg})(9.80\text{ m/s}^2)}{2.0 \times 10^{-2}\text{ m}} = \boxed{2.7 \times 10^2\text{ N/m}}$$

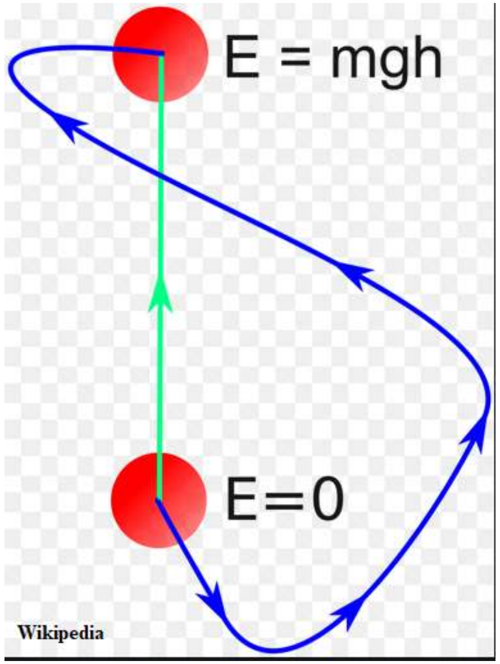
(B)

$$W_s = 0 - \tfrac{1}{2}kd^2 = -\tfrac{1}{2}(2.7 \times 10^2\text{ N/m})(2.0 \times 10^{-2}\text{ m})^2$$

$$= \boxed{-5.4 \times 10^{-2}\text{ J}}$$

# Conservative Forces

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.




2. The work done by a conservative force on a particle moving through any closed path is zero

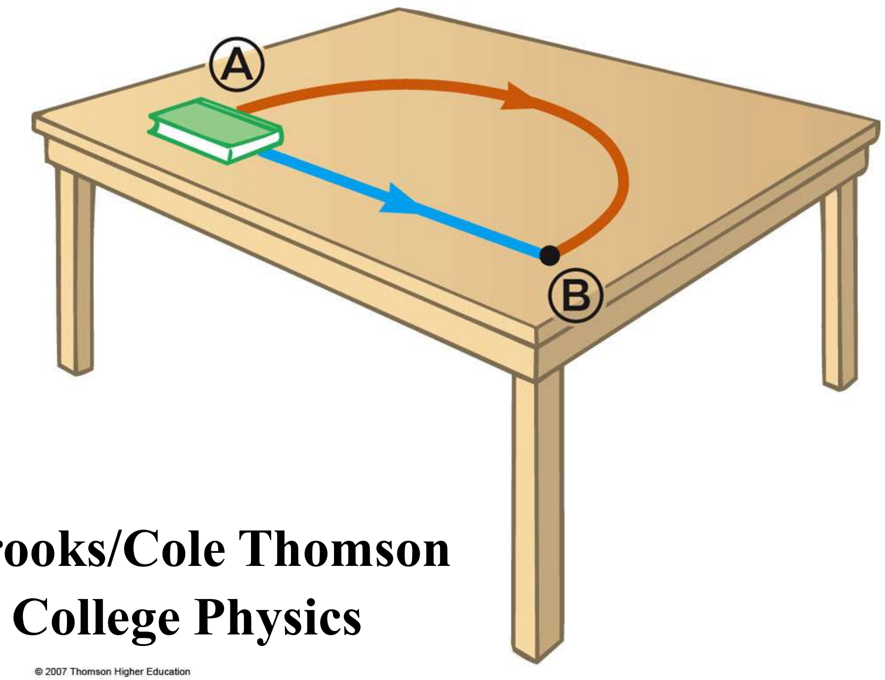
A closed path is one in which the beginning and ending points are the same.

# Nonconservative Forces

- A nonconservative force does not satisfy the conditions of conservative forces
- Nonconservative forces acting in a system cause a *change* in the mechanical energy of the system

# Nonconservative Forces

- The work done against friction is greater along the brown path than along the blue path
- Because the work done depends on the path, friction is a nonconservative force.



We define the sum of the kinetic and potential energies of a system as the **mechanical energy** of the system:

$$E_{\text{mech}} \equiv K + U$$

where *K* is kinetic and *U* is potential energy.

# Relationship Between Conservative and Nonconservative Forces

ΔU is negative when $F_x$ and $dx$ are in the same direction, as when an object is lowered in a gravitational field or when a spring pushes an object toward equilibrium.

$$W_{\text{int}} = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

$$F_x = -\frac{dU}{dx}$$

Let’s check:

1:

$$U_s = \tfrac{1}{2}kx^2$$

$$F_s = -\frac{dU_s}{dx} = -kx$$

2:

$$U_g = mgy$$

$$F_g = -mg$$


- What a system is
- The work under constant and varying force
- The kinetic energy of a particle
- Gravitational potential energy
- Elastic potential energy
- Conservative and nonconservative forces
- Total mechanical energy of a system

**Assoc.Prof.Dr. Fulya Bağcı**