Chapter 9: Linear Momentum and Collisions

PHY0101/PHY101

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Outline (Serway 9th Edition)

9.1 Linear Momentum

9.2 Isolated System (Momentum)

9.3 Nonisolated System (Momentum)

9.4 Collisions in One Dimension

9.5 Collisions in Two Dimensions

9.6 The Center of Mass

9.7 Systems of Many Particles

9.8 Deformable Systems

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Collisions and Impulse

- ***Briefly*** consider details of collision
  - Assume collision lasts a very small time $\Delta \mathbf{t}$
- During collision, net force **on one** of the objects (Newton's 2nd Law): $\sum \mathbf{F} = \Delta \mathbf{p}/\Delta \mathbf{t}$ $(= \mathbf{dp/dt})$

Or: $\Delta \mathbf{p} = (\sum \mathbf{F})\Delta \mathbf{t}$ (momentum change of the object)

$\Delta \mathbf{p} \equiv \mathbf{I} \equiv$ **Impulse** that collision gives the object

(change in momentum for the object!)

- Text writes this as integral over time of collision:

$\mathbf{dp} = (\sum \mathbf{F})\mathbf{dt}$; $\mathbf{I} = \int \mathbf{dp} = \int (\sum \mathbf{F})\mathbf{dt}$ (**p** limits: $\mathbf{p}_i$ to $\mathbf{p}_f$, **t** limits: $\mathbf{t}_i$ to $\mathbf{t}_f$)

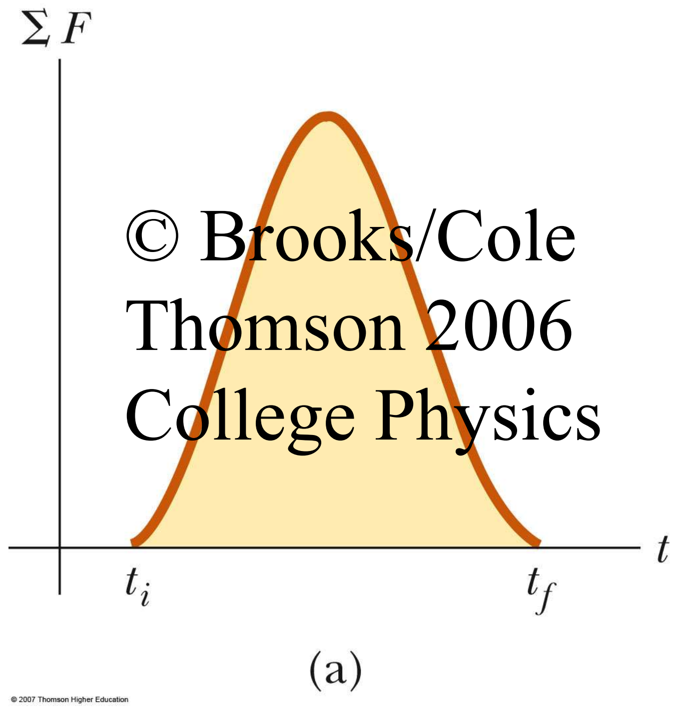
$$\mathbf{I} \equiv \Delta \mathbf{p} \equiv \mathbf{p}_f - \mathbf{p}_i = \int (\sum \mathbf{F})\mathbf{dt}$$

- **Usual case:** Replace time integral of net force by time average force: $[\int(\sum F)dt/(\Delta t)] \approx (\sum F)_{avg}$

$$\text{Impulse, } I = \Delta p = \int(\sum F)dt \approx (\sum F)_{avg}\,\Delta t$$

$\Delta t = t_f - t_i$ **= average collision time**

- **Math:** Time integral = area under the force vs. time curve: →



Impulse, $I = \Delta p$ = area under the curve.

Δt is usually very small

- The approximation of replacing

$$I = \Delta p = \int(\sum F)dt$$

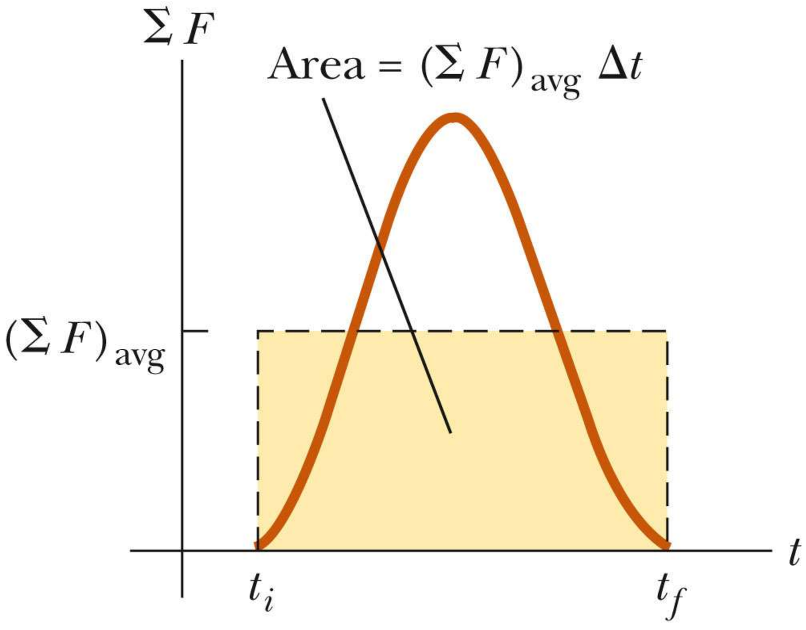
with

$$I = \Delta p \approx (\sum F)_{avg}\,\Delta t$$

is equivalent to replacing the true area under the curve by the rectangle shown.

- This is known as

the Impulse Approximation



(b)

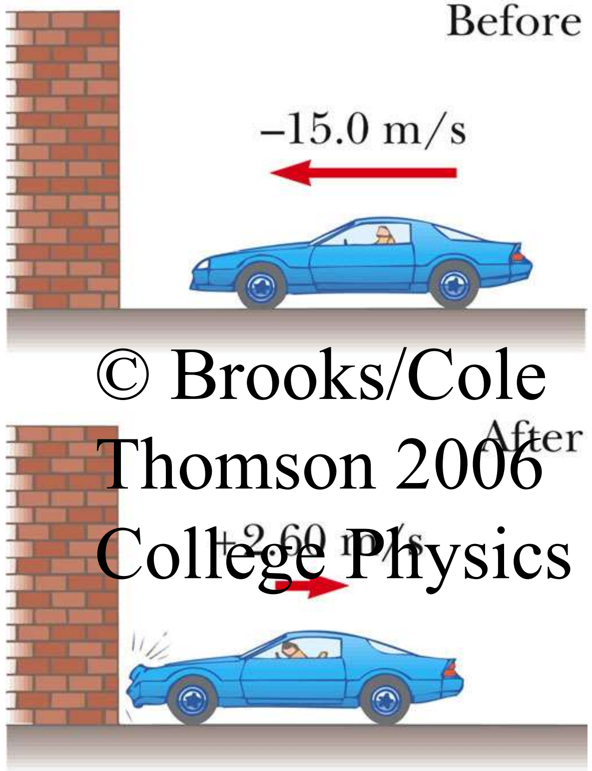
Example 9.3: Crash Test

- **Crash test:** Car, **m = 1500 kg**, hits wall. 1 dimensional collision. **+x** is to the right. Before crash, **v = -15 m/s**. After crash, **v = 2.6 m/s.** Collision lasts **Δt = 0.15 s**. **Find:** Impulse car receives & average force on car.

  Assume: Force exerted by wall is large compared to other forces

  Gravity & normal forces are perpendicular & don't effect the horizontal momentum

  ⇒ Use impulse approximation



(a)

$$p_i = mv_i = -2.25 \text{ kg m/s}, \; p_f = mv_f = 2.64 \text{ kg m/s}$$

$$I = \Delta p = p_f - p_i = 2.64 \times 10^4 \text{ kg m/s}$$

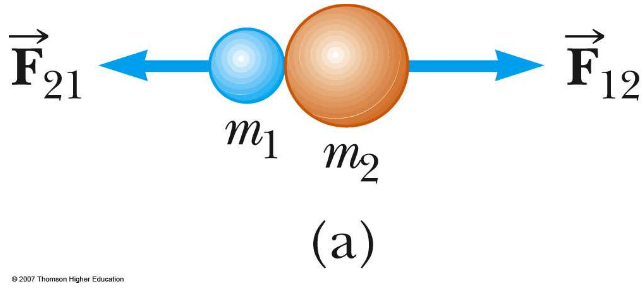
$$(\sum F)_{avg} = (\Delta p/\Delta t) = 1.76 \times 10^5 \text{ N}$$

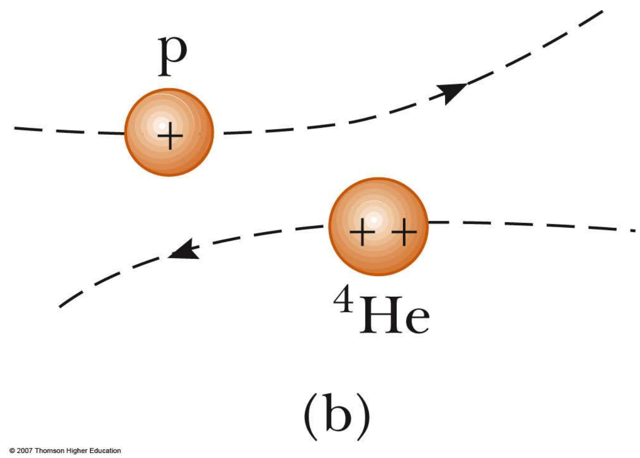
Collisions

- Use term **"collision"** to represent an event during which two particles come close to each other and interact by means of forces
  - May involve physical contact, but is generalized to include cases with interaction without physical contact
- Time interval during which velocity changes from its initial to final values is assumed to be short
- Interaction forces are assumed to be much greater than any external forces present

This means the impulse approximation can be used!

- Collisions may be result of direct contact →
- Impulsive forces may vary in time in complicated ways
  - This force is internal to system
- Collision needn't include physical contact between the objects →
- There are still forces between the particles
- This type of collision can be analyzed in the same way as those that include physical contact
- Momentum is ***ALWAYS*** conserved for **ALL COLLISIONS!!**





- **Perfectly Elastic collision**: ***BOTH*** momentum and kinetic energy are conserved
  - Perfectly elastic collisions occur on a microscopic level
  - In macroscopic collisions, only **approximately elastic** collisions actually occur
    - Generally some energy is lost to deformation, sound, etc.
- **Inelastic collision**: ***Kinetic energy is not conserved***, but **momentum is still conserved**
  - ***Perfectly inelastic collision***: Objects stick together after the collision.
- In an **inelastic collision**, some kinetic energy is lost, but the objects do not stick together
- Elastic and perfectly inelastic collisions are limiting cases, most actual collisions fall in between these two types

Momentum is *always conserved* in all collisions!!!!

9.3: Collisions in One Dimension

- Given some information, using conservation laws, we can determine a **LOT** about collisions without knowing the collision forces!
- To analyze ***ALL*** collisions:

Rule # 1:

Momentum is *ALWAYS* conserved in a collision!

$$\Rightarrow \qquad \mathbf{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

HOLDS for ALL collisions!

Elastic Collisions

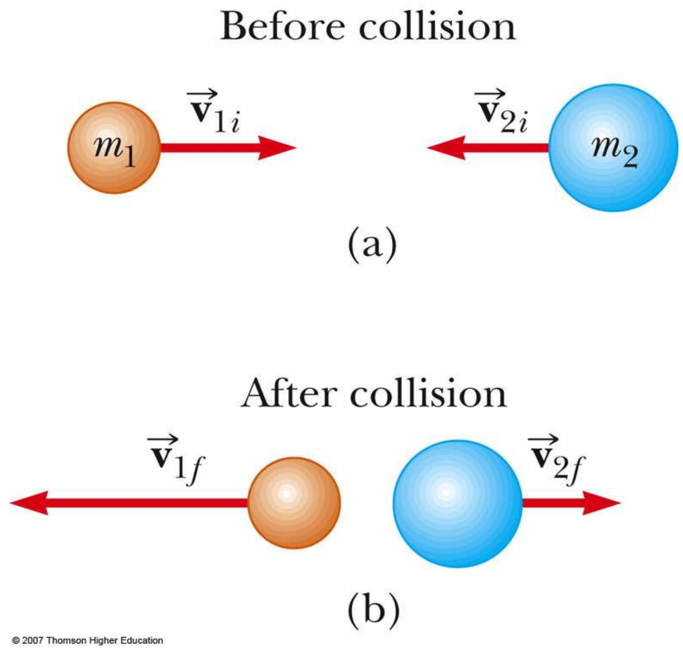
- ***Both momentum AND kinetic energy* are conserved!**

$\Rightarrow$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

AND

$$(\tfrac{1}{2})m_1(v_{1i})^2 + (\tfrac{1}{2})m_2(v_{2i})^2 = (\tfrac{1}{2})m_1(v_{1f})^2 + (\tfrac{1}{2})m_2(v_{2f})^2$$



↓ **Note!!!**

- ***Special case***: 2 **hard** objects (like billiard balls) collide (≡ "**Elastic Collision**")
- To analyze Elastic collisions:

Rule # 1: ***Still*** holds! **Momentum is conserved**!!

$$\Rightarrow \quad m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

Rule # 2: For Elastic Collisions ***ONLY*** (!!)

Total Kinetic energy is conserved!!

$$\Rightarrow \quad (\tfrac{1}{2})m_1(v_{1i})^2 + (\tfrac{1}{2})\, m_2(v_{2i})^2$$

$$= (\tfrac{1}{2})m_1(v_{1f})^2 + (\tfrac{1}{2})m_2(v_{2f})^2$$

- **Special case**:

Head-on Elastic Collisions.

  - Can analyze in one dimension.

- Types of head-on collisions: (Figure):

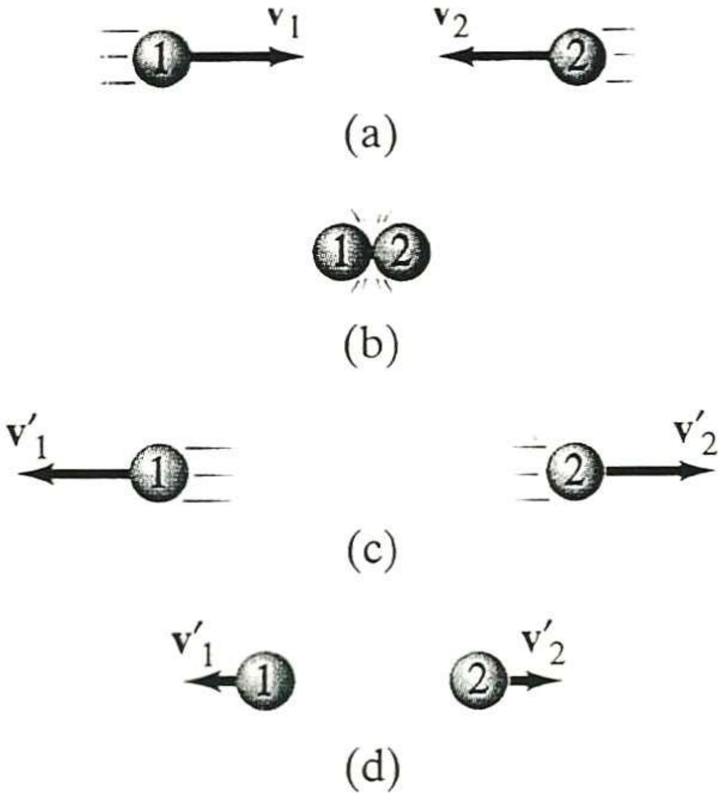
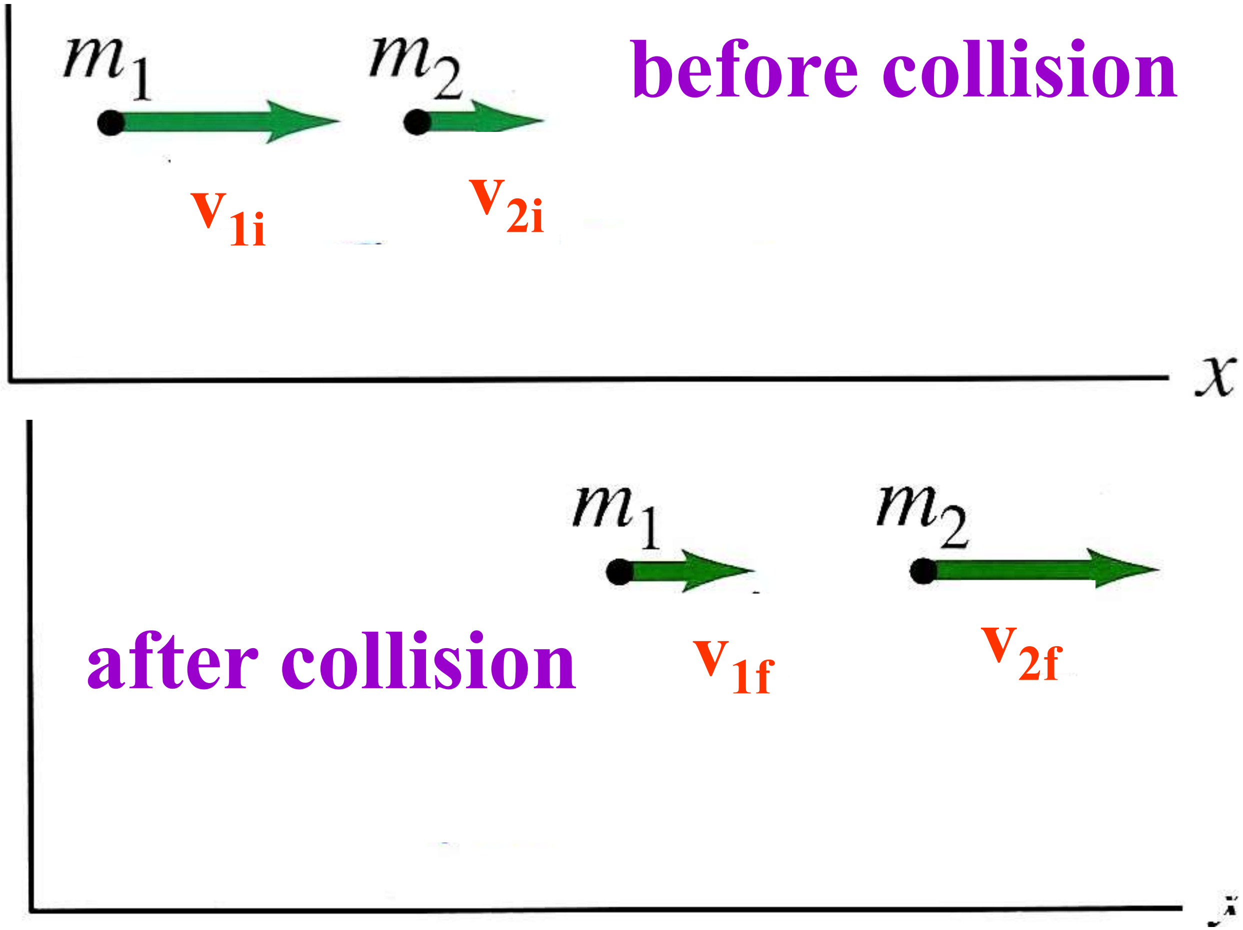


FIGURE 7–13 Two equal mass objects (a) approach each other with equal speeds, (b) collide, and then (c) bounce off with equal speeds in the opposite directions if the collision is elastic, or (d) bounce back much less or not at all, if the collision is inelastic.

- **Special case:** **Head-on Elastic Collisions**
  - 1 dimensional collisions: Some types:

v_{1i}, v_{2i}
v_{1f}, v_{2f}
are 1 d
vectors!



- **Special case: Head-on Elastic Collisions.**
  - Momentum is conserved (***ALWAYS***!)

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$v_{1i}, v_{2i}, v_{1f}, v_{2f}$ are one dimensional vectors!

  - Kinetic Energy is conserved (***ELASTIC***!)

$$(KE)_{before} = (KE)_{after}$$

$$(\tfrac{1}{2})m_1(v_{1i})^2 + (\tfrac{1}{2})m_2(v_{2i})^2 = (\tfrac{1}{2})m_1(v_{1f})^2 + (\tfrac{1}{2})\, m_2\, (v_{2f})^2$$

  - 2 equations, 6 quantities: $v_{1i}, v_{2i}, v_{1f}, v_{2f}, m_1, m_2$

⇒ Clearly, must be given 4 out of 6 to solve problems! Solve with ***CAREFUL*** algebra!!

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad (1)$$

$$(\tfrac{1}{2})m_1(v_{1i})^2 + (\tfrac{1}{2})m_2(v_{2i})^2 = (\tfrac{1}{2})m_1(v_{1f})^2 + (\tfrac{1}{2})\, m_2\, (v_{2f})^2 \qquad (2)$$

- Now, some algebra with **(1)** & **(2)**, the results of which **will help to simplify problem solving**:
  - Rewrite **(1)** as: $m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$ **(a)**
  - Rewrite **(2)** as:

$$m_1[(v_{1i})^2 - (v_{1f})^2] = m_2[(v_{2f})^2 - (v_{2i})^2] \qquad (b)$$

  - Divide **(b)** by **(a):**

$\Rightarrow$ $v_1 + v_{1f} = v_2 + v_{2f}$ or

$$v_1 - v_2 = v_{2f} - v_{1f} = -(v_{1f} - v_{2f}) \qquad (3)$$

Relative velocity before= - Relative velocity after

Elastic head-on (1d) collisions only!!

- **Summary:** ***1d Elastic collisions:*** Rather than directly use momentum conservation + KE conservation, often convenient to use:

Momentum conservation:

→ $$\mathbf{m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}} \qquad \mathbf{(1)}$$

use these! **along with:**

→ $$\mathbf{v_1 - v_2 = v_{2f} - v_{1f} = -(v_{1f} - v_{2f})} \qquad \mathbf{(3)}$$

- **(1)** & **(3)** are equivalent to momentum conservation + KE conservation, since **(3)** was derived from these conservation laws!

Example : Pool (Billiards)

Before:

$$m_1 = m_2 = m, \; v_{1i} = v, \; v_{2i} = 0, \; v_{1f} = ?, \; v_{2f} = ?$$

- <u>**Momentum Conservation:**</u> $mv + m(0) = mv_{1f} + mv_{2f}$

  Masses cancel $\Rightarrow$ $\quad v = v_{1f} + v_{2f} \qquad$ **(I)**

- Relative velocity results for elastic head on collision:

$$v - 0 = v_{2f} - v_{1f} \qquad \textbf{(II)}$$

Solve **(I)** & **(II)** simultaneously for v_{1f} & v_{2f} :

$$\Rightarrow \qquad v_{1f} = 0, \qquad v_{2f} = v$$

Ball 1: to rest. Ball 2 moves with original velocity of ball 1

Before: