

Ankara University
Engineering Faculty
Department of Engineering Physics

PEN207

Circuit Design and Analysis

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Chapter-1

Electric Circuits and Circuit Elements (1/2)

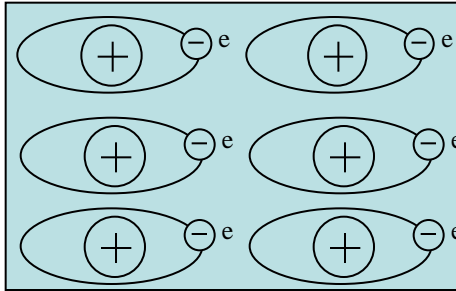
Chapter-1: Electric Circuit and Circuit Elements

- Electric Charge
- Current
- Potential Difference
- Electrical Energy, Work and Power
- Power Sources and Circuits Elements
- Resistance: Ohm's Law
- Inductance
- Capacitance
- Fundamental Circuit Laws: Kirchhoff's Laws

Chapter-1: Part One

- Electric Charge
- Current
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- Resistance: Ohm's Law
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- Capacitance
- Fundamental Circuit Laws: Kirchhoff's Laws

Conductor and Insulators



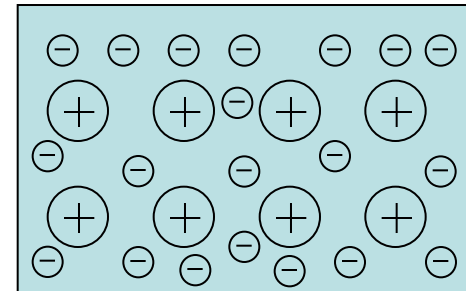
Dielectrics(Insulators)

Free carrier density (n): 0 cm^{-3}

$$q=0, \mathbf{I=0}$$

No net charge, no free carriers!

Glass
Ceramics
Plastic



Metal (Conductors)

Free carrier density (n): 10^{23} cm^{-3}

$$q=0, \mathbf{I \neq 0}$$

No net charge, but free carriers

Aluminium
Copper
Gold

Conductivity: $\sigma = nq\mu$

$$\text{Resistivity: } \rho = \frac{1}{\sigma} = \frac{1}{nq\mu}$$

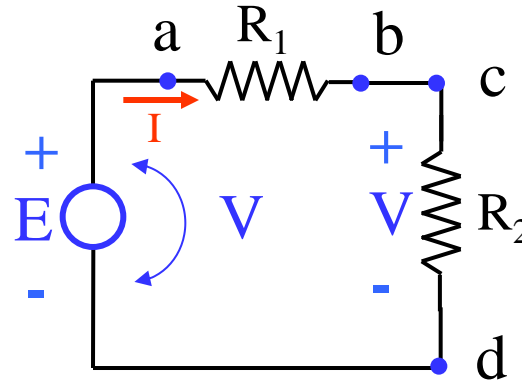
n: Free carrier density

q: electron's charge

μ : mobility

Definition: Current and Voltage

Current and Potential Difference (or Potential or Voltage)



I: Current (unit Ampere)

E: Electromotor Force (Power Supply) - (unit Volt)

V: Potential Difference (Measured btw two points in circuit) - (unit Volt)

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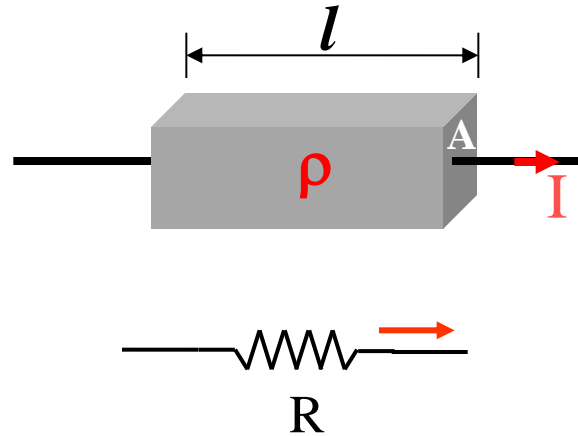
Current (I)

Potential Difference (E,V)

Uppercase letters: For constant **current (I)** and **Potential (E, V)**

Lowercase letters: For **current (i(t))** and **potential (e(t), v(t))** changing in time

Definitions



Resistivity is resistance of a specific material (specific electrical resistance, or volume **resistivity**)

Resistivity:

$$\rho = \frac{1}{\sigma} = \frac{1}{nq\mu}$$

l = Length

A = Cross Sectional Area

Physics

Resistivity (ρ)

$$R = \rho \left(\frac{l}{A} \right)$$

Electric Field (\mathbf{E})

$$V = \mathbf{E}l$$

Current Density (\mathbf{J})

$$I = \mathbf{J}A$$

Circuit

Resistance (\mathbf{R})

Potential (\mathbf{V})

Current (\mathbf{I})

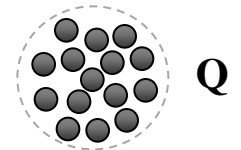
Electric Charge

Current is the movement of electric charges so we have to define charge first

Unit of charge (symbol Q) is **Coulomb (C)**

$$1\text{C} = 6.3 \times 10^{18} \text{ e}$$

Charge of an electron = $-1.6 \times 10^{-19} \text{ C}$



In order to have **1 Coulomb** charge we have to bring 6.3×10^{18} electrons together

Force between charges (F)

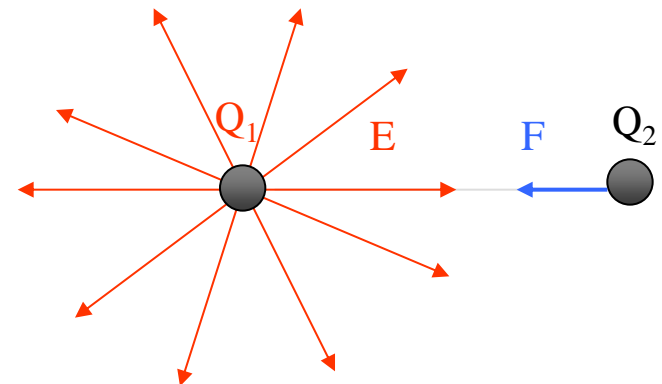
Coulomb's Law
$$F = k \frac{Q_1 \cdot Q_2}{d^2}$$



In physics forces can be expressed in terms of field:

$$F = \left(k \frac{Q_1}{d^2} \right) \cdot Q_2 = E \cdot Q_2$$

Electric Field (E)
$$E = k \frac{Q_1}{d^2}$$



Current

In engineering we interest in charges in motion because only moving charges transmit energy.

Current, is the flow of charges

Let's consider charge (Q) passing through a wire with a constant rate;



Current can be defined: $I = \frac{Q}{t}$

Capital letter **I** used for current from the French word **intensité**

If the number of charges changes with time then we can define *instantaneous current* $i(t)$:

Instantaneous current: $i(t) = \frac{dq}{dt}$

Number of charges: $q = \int i(t)dt$

Notation

I, Q (Constant rate)

i, q (changing with time) $i(t)$, $q(t)$

Current

Unit of electric current in SI unit system is *ampere* (symbol A)

[**one ampere** is the flow of electric charge at the rate of one coulomb per second]

$$[\text{ampere}] = \frac{[\text{coulomb}]}{[s]} \qquad 1\text{A} = \frac{1\text{C}}{1\text{s}}$$

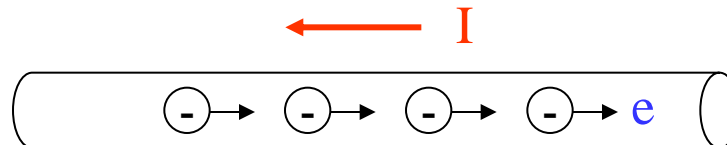
Homework-1.1:

If the current passing through a wire is 1 μA how many electrons are flowing in 4 sec?

We have to also define the direction of current as well as its magnitude.

The direction of positive charges is assumed the **direction of current**.

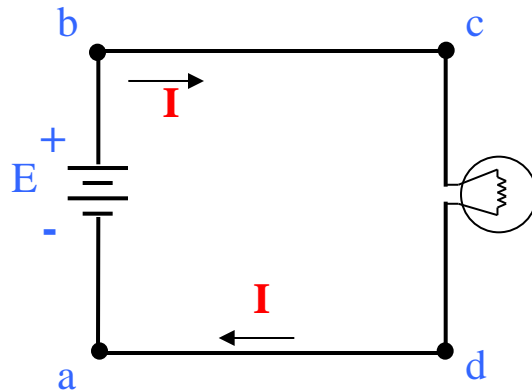
In fact in metals (or conductors) moving charges are negatively charged electrons, so current direction is the opposite direction of free carriers.



Measurement of Current

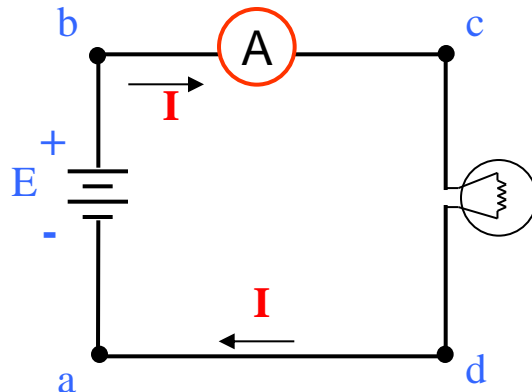
Current through a branch of a circuit does not change. Measuring instrument called **Ammeter** is used to measure current in a circuit.

Ammeters are connected **in series with the circuit**. The **resistance of an Ammeter is zero (ideally)**.



$$I_{ab} = I_{bc} = I_{cd} = I_{da} = I$$

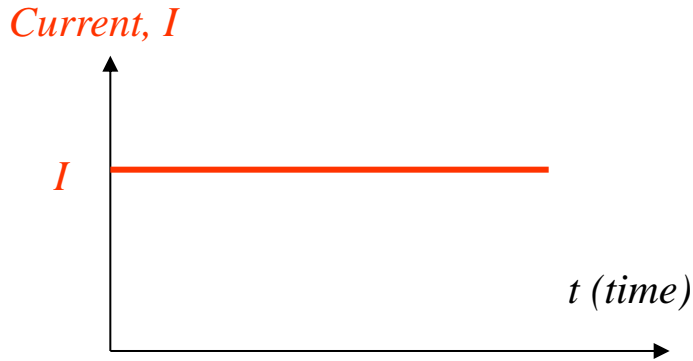
Ammeter



Measurement of current

Different Forms of Current

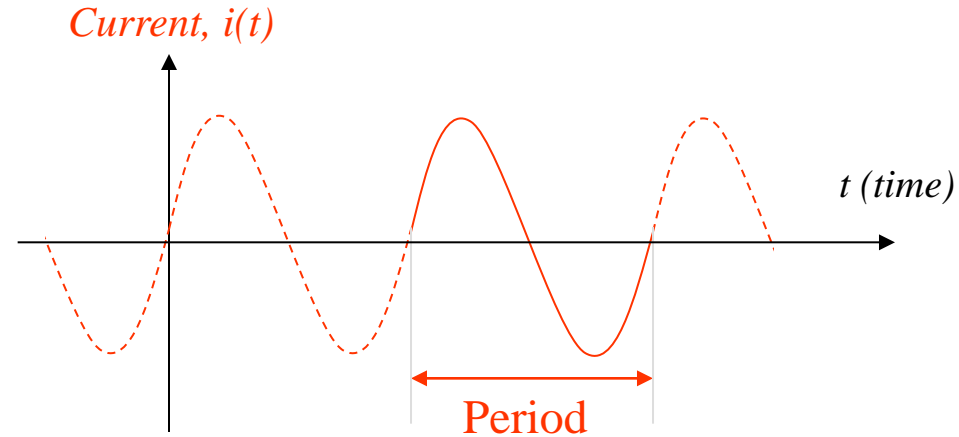
Direct Current (DC)



Direct current, flow of charges are in one direction number of charges does not change

Battery and accumulator are example of DC current

Alternating Current (AC)



Alternating Current, current that direction and value is changing periodically

$$\text{period} = 1 / \text{frequency}$$

Unit:

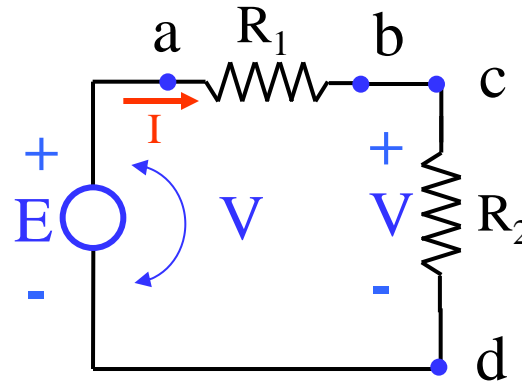
periyod [time]=second

frequency [1/time]=1 /second= hertz (Hz)

In Turkey mains voltage (grid power voltage) is 220V and frequency is 50 Hz

Definition of Potential Difference or Voltage

Potential Difference (or Potential or Voltage)



E: Electromotor Force (Power Supply) - (unit V)

V: Potential Difference (Measured btw two points in circuit) - (unit V)

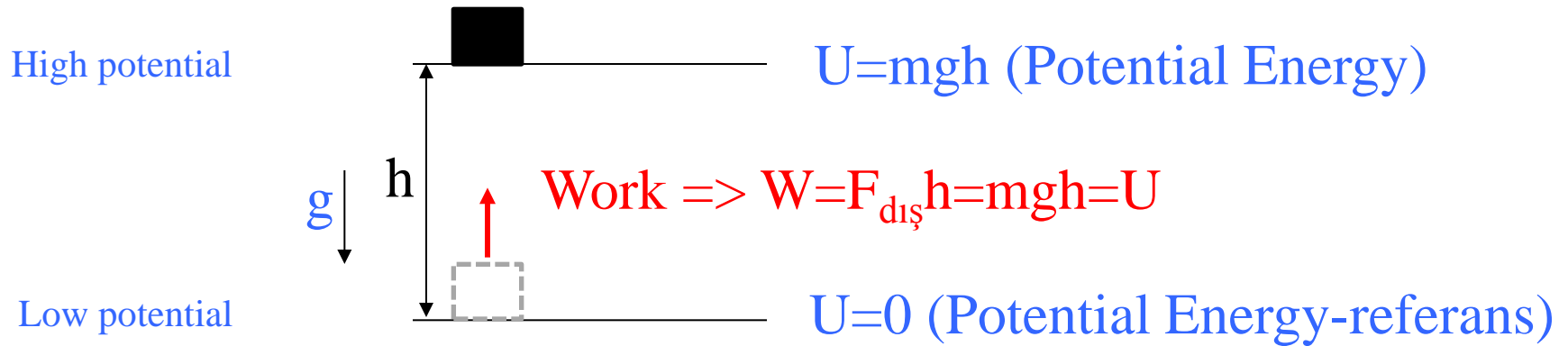
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Potential Difference (E, V)

Uppercase letters: For constant Potential (E, V)

Lowercase letters: For potential (e(t), v(t)) changing in time

Potential Energy and Potential Difference (Mech)



Potential Energy (U): $U=mgh$

Potential (V):
(Potential energy
per unit mass)

$V=U/m \Rightarrow V=U/m=mgh/m=gh$

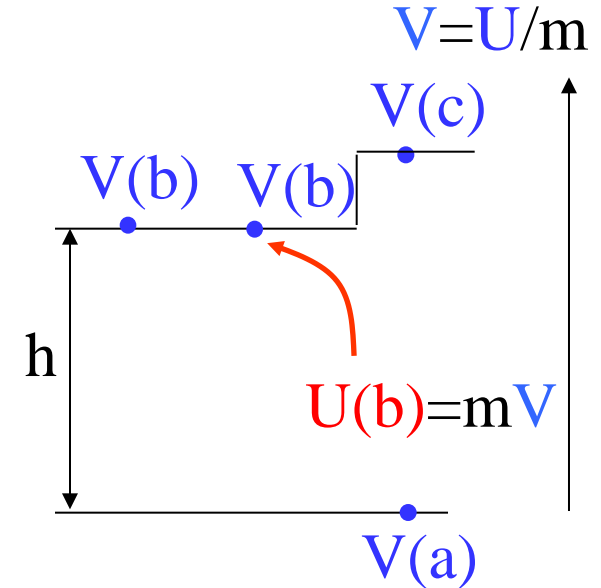
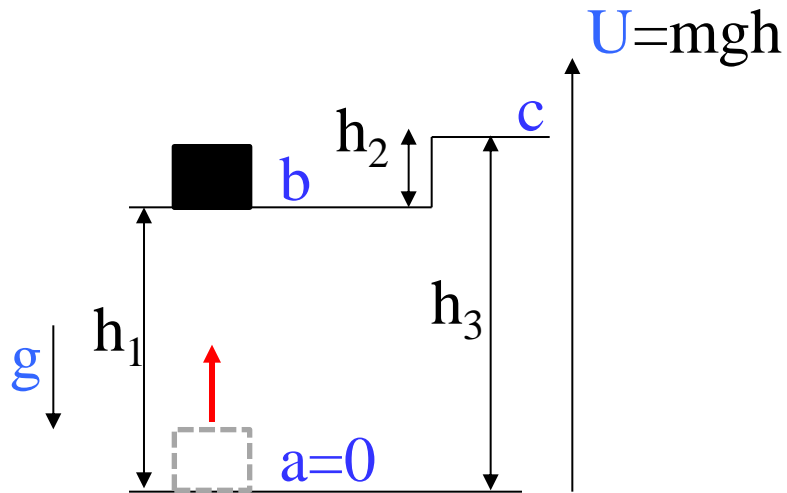
$$V=gh$$

Potential is proportional with height.

Relation btw **Potential Energy**
& **Potential difference**

$$U=mV$$

Potential Energy and Potential Difference



Potential Energy (U): $U=mgh$

Potential (V):
(Potential energy
per unit mass)

$$V=U/m \Rightarrow$$

$$V=U/m=mgh/m=gh$$

Potential difference is a
measure of the potential
energy independent of mass

$$U(h)=mV(h)$$

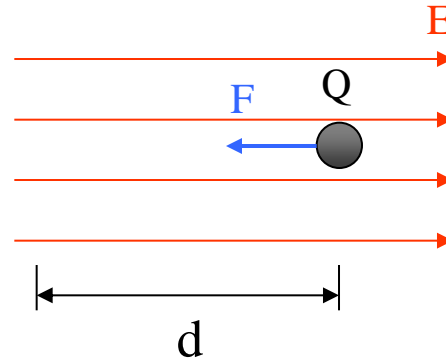
Potential Energy & Potential Difference (Elec)

Force acting on a charge Q in an electric field E

Coulomb Law: $F = k \frac{Q_1 \cdot Q_2}{d^2}$

Electric Field: $E = k \frac{Q_1}{d^2}$

$F = E \cdot Q$

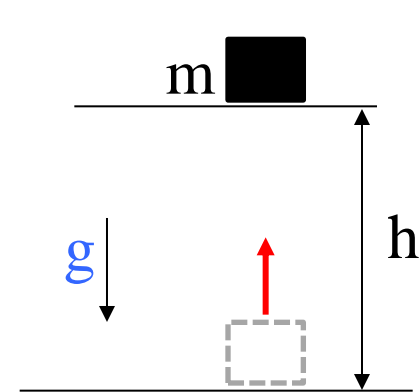


Work: $W = U = \vec{F} \cdot \vec{d} = \left(k \frac{Q_1 \cdot Q_2}{d^2} \right) \cdot d = (E \cdot Q) \cdot d = (E \cdot d) Q$
 (Potential Energy)

Potential: $V \equiv \frac{U}{Q} = Ed$

Relation between **Potential Energy** and **Potential Difference**:

$U = QV$



$W = U = mg \cdot h$

$V = \frac{U}{m} = gh$

$U = mV$

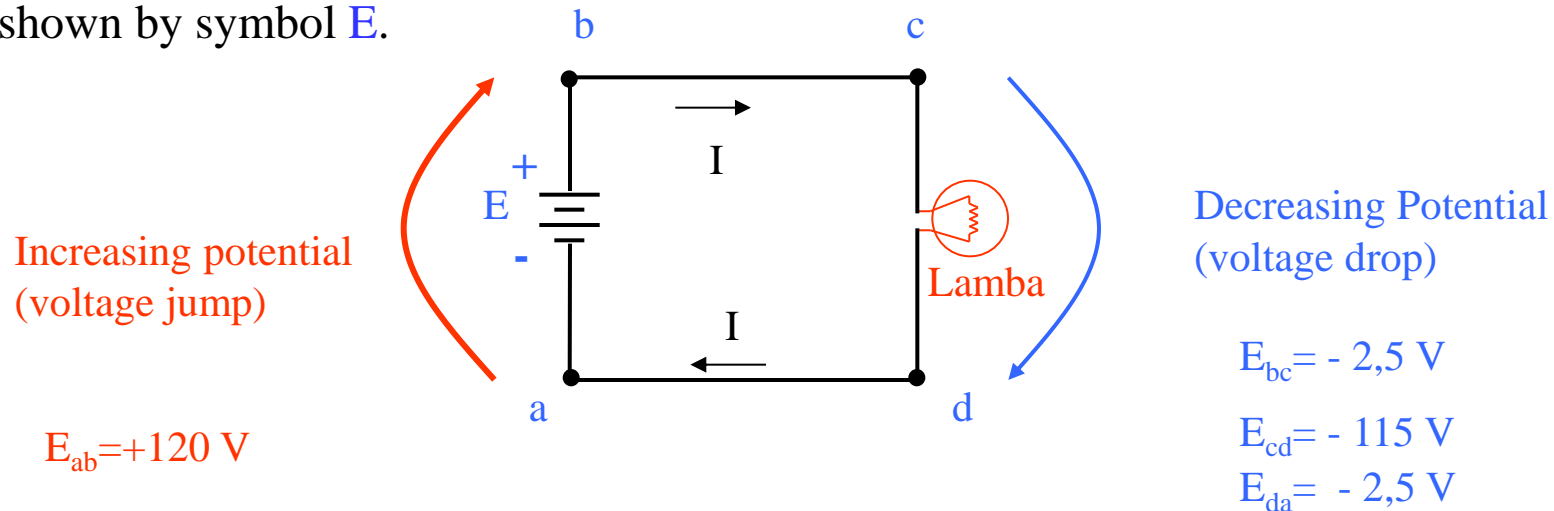
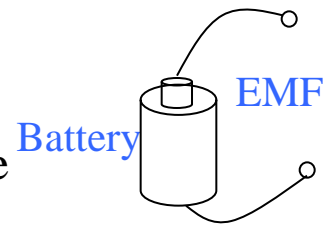
Potential Difference

In a circuit, work done to move a unit positive charge between two points is called *Potential Difference* or *Voltage*. In other words *Potential difference* or **Volt** is the work (energy) done per unit charge.

[If work done on a **1 Coulomb** charge to bring charge from point A to point B is **1 Joule** then the potential difference between these two points is **one volt** (symbol V)]

$$[volt] = \frac{[joule]}{[coulomb]} \quad 1V = \frac{1J}{1C}$$

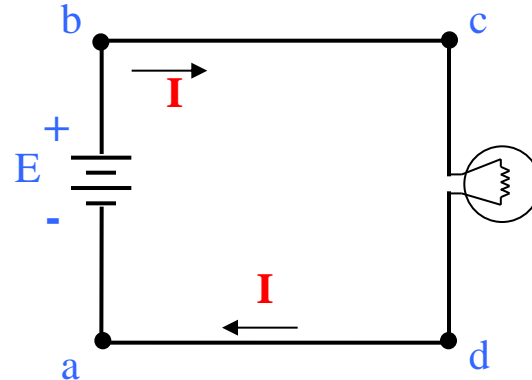
If the potential difference is the difference of power supply or battery it is commonly called *elektromotor force* or shortly *EMF* and potential difference is shown by symbol **E**.



The sign of voltage sign shows increase (+) or decrease (-).

Measurement of Voltage

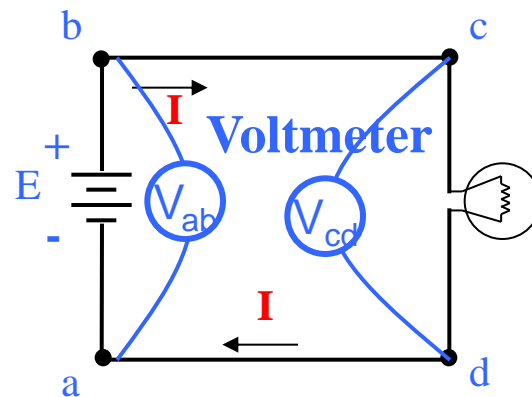
Potential difference in a circuit is measured with a measurement instrument called **Voltmeter**



$$V_{ab}=?$$

$$V_{cd}=?$$

Voltmeters are connected **in parallel with the circuit**. So the resistance of an **voltmeter is infinity**.

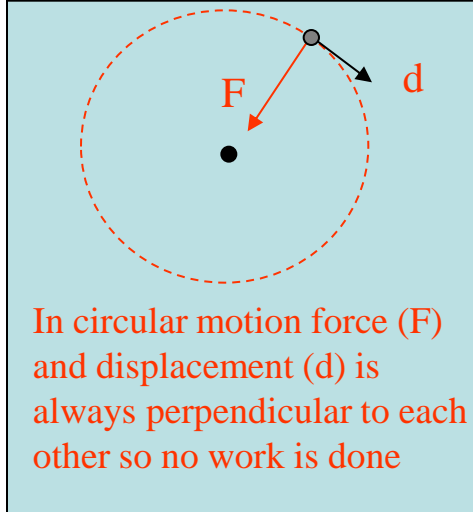


Potential Difference Measurement

$$V_{bc}=V_{da}=0$$

$$V_{ab}=V_{cd}$$

Energy, Work and Power



$$\text{Work} = (\text{Force}) \cdot (\text{Displacement}) \quad W = F \cdot d$$

Unit of work is (in SI unit system) **Joule (J)**
[joule]=[newton].[meter]

Energy is the ability to do work

$$\text{Power} = \text{Work} / \text{Time} \quad P = W/t$$

Measured in **Watt (W)** (in SI)
[watt]=[joule] / [second]

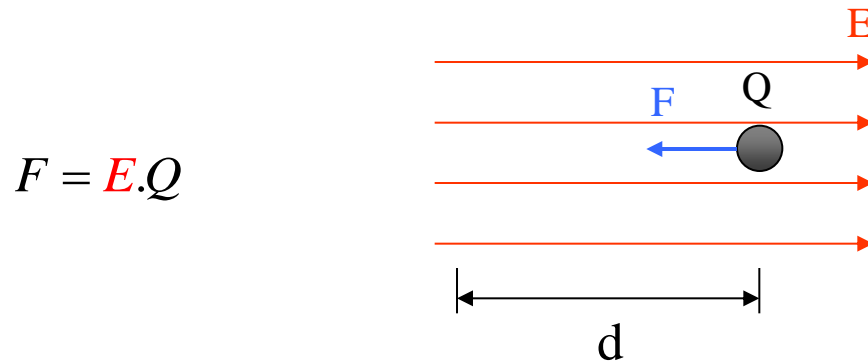
Power is the rate at which work is done

Since there is close relation between power and energy sometime energy unit is given in terms of power unit:

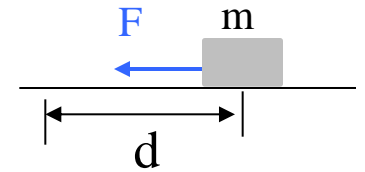
For example energy (joule), can be given as watt-sec or kilowatt-hour ($1000 \times 3600 = 3.6 \times 10^6$ watt-sec)

Work, Energy and Power in Circuits

Force acting on a charge (Q) in electric field (E),



In Mechanics



$$W = \vec{F} \cdot \vec{d}$$

Work: $W = \vec{F} \cdot \vec{d} = (E \cdot Q) \cdot d = (E \cdot d) Q = V \cdot Q = U$ (Potential energy)

We define potential:

$$V \equiv \frac{U}{Q} \equiv E \cdot d = \left(k \frac{Q}{d^2} \right) \cdot d = k \frac{Q}{d}$$

Power: $P = \frac{W}{t} = \frac{V \cdot Q}{t} = V \left(\frac{Q}{t} \right) = V \cdot I$

$$P = F \cdot v$$

Electrical Energy and Power

How can we express energy and power in terms of electrical quantities current and voltage?

Electrical energy can be given below in terms of , Voltage (V) and charge (Q)

Electric Energy: $W = (E.d).Q = V.Q$

İş, sabit bir hızda yapılırsa ve toplam Q yükü, t saniyede E voltluk bir gerilim altında hareket ederse, güç:

Electric Power: $P = \frac{W}{t} = \frac{V.Q}{t}$

Uygulamada, yükten ziyade akım ile ilgilendiğimizden güç ifadesi:

$$P = V.I \quad \text{Power}$$

Bu güç, uçlarındaki gerilim (V), üstünden geçen akım I olan devre elemanının birim zamanda soğurduğu veya dışarıya verdiği enerjidir.

If both current and voltage are changing with time $p(t)$;

$$p(t) = v(t).i(t) \quad \text{Instantaneous Power}$$

Example 1.1: Electrical energy is stored at a constant speed by transmitting it to a battery and converting it to 400W chemical energy. During the process, 20% of the power transmitted to the battery is lost in heat. If the cost of kW-hour of electricity is 1.25TL, find the energy value and cost spent to charge the battery for 10 hours.

Solution:

If the total power transmitted to the battery is P_b then:

$$400W + 0.2P_b = P_b \Rightarrow P_b = 500W$$

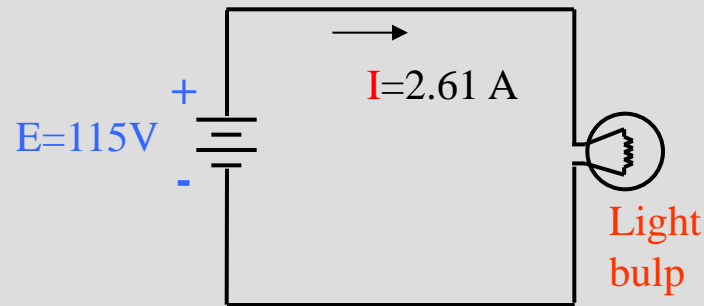
Total energy in 10 hours $W_b = (500W) \times (10 \text{ hours}) = 5000W\text{-hour} = 5 \text{ kW-hour}$

Cost of consumed energy = $(5\text{kW-hour}) \times (1.25\text{TL} / (\text{kW-hour})) = 6.25\text{TL}$

Efficiency %80 (loss %20)

Energy, Work and Power

Example-1.2: In the circuit below there is voltage supply of 115V . The current flowing in the circuit is 2.61A . (a) What is the power dissipated on the light bulb if the cost of kW-hour energy is 1.25TL . (b) How much we should pay if we use the bulb 10 hours?



Solution:

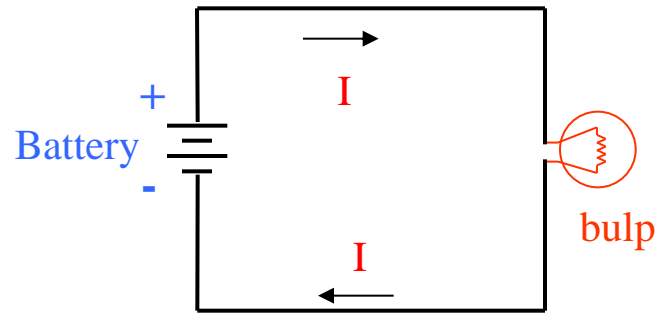
$$(a) P = E \cdot I = (115 \text{ V}) \cdot (2.61 \text{ A}) = 300 \text{ W}$$

$$(b) W = E \cdot I \cdot t = (300 \text{ W}) \cdot (10 \text{ hour}) = 3000 \text{ W-hour} = 3.0 \text{ kW-hour}$$

$$\text{Cost} = (3.0 \text{ kW-hour}) \cdot (1.25 \text{ TL/kW-hour}) = 3.75 \text{ TL}$$

Definition of Closed, Open and Short Circuits

CLOSED circuit is a circuit where power source and other circuit elements (such as resistor) are connected by wires so there will be **flow of current and voltage**.



CLOSED CIRCUIT

In Closed Circuit

$$R \neq \infty$$

$$\neq 0$$

(Considerable resistance)

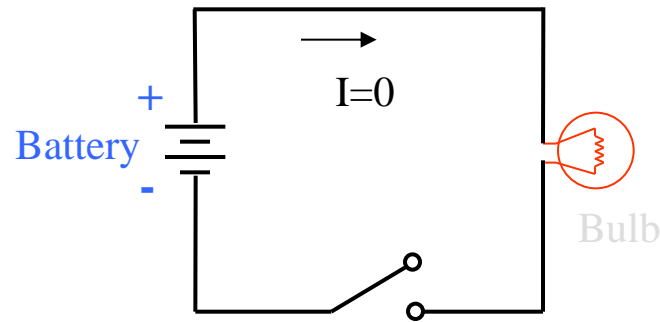
$$\text{Current} \neq 0$$

(no high current)

$$\text{Voltage} \neq 0$$

Open Circuit

In a circuit if wires are cut off so there will be no current flow, but there is **voltage**. Then, we have an **open circuit**.



OPEN CIRCUIT

In Open Circuit

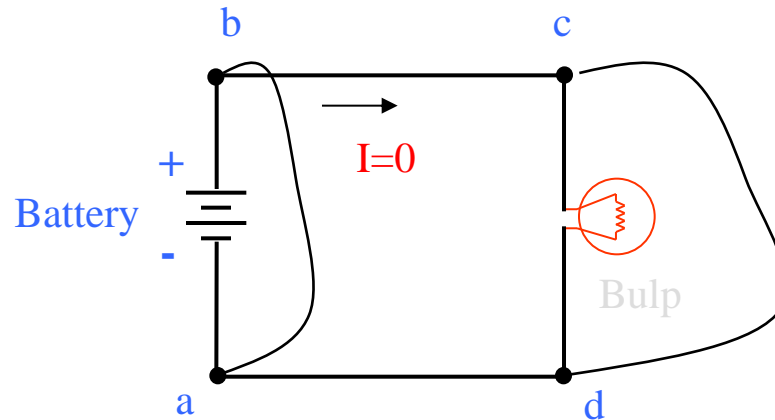
$R=\infty$

Current=0

Voltage \neq 0

Short Circuit

In Short circuit the poles of battery or power supply is connected directly



SHORT CIRCUIT

In Short Circuit

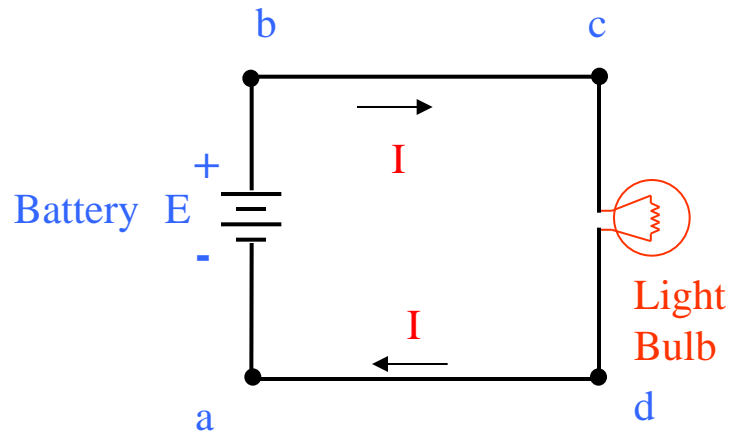
$$R=0$$

Current=Max (too high to damage the circuit)

Voltage=0 V (forced (by shorting) to be zero)

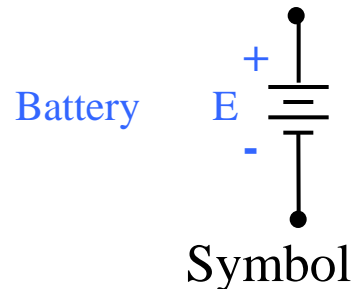
Battery

Consider a circuit below:



In order to keep current flowing through circuit an external electric energy must be supplied. The energy is supplied by the power sources. Battery is an example of voltage sources.

Battery gives necessary energy to electrons to flow in circuit.



Potansiyel (Referans)

Bir devrede en düşük nokta referans noktası olarak alınır ve ‘**toprak**’ olarak adlandırılır.

V_b dediğimiz zaman anlamamız gereken V_{ba}

**Bu slide üzerinde
biraz daha çalış**

U_{bc} (veya V_{bc})=0

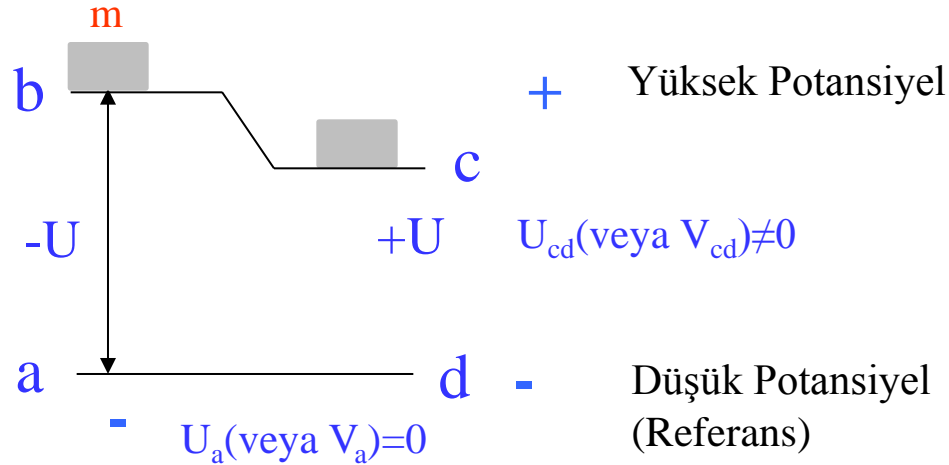
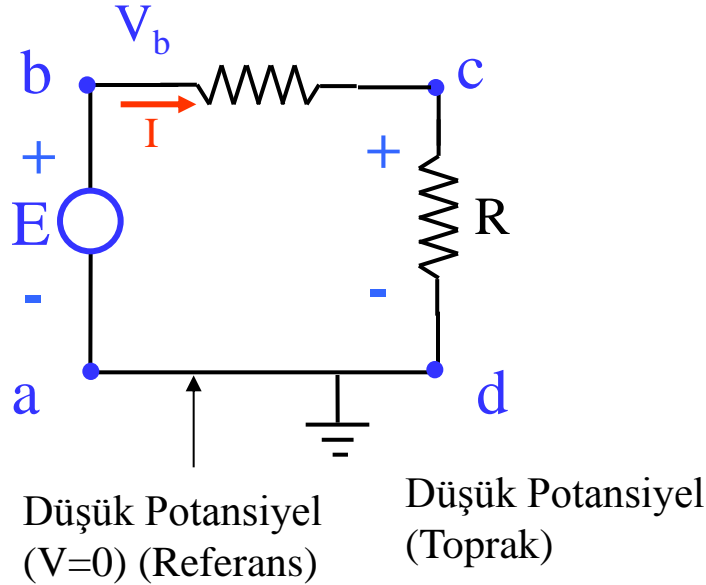


Table-Electrical Quantities and Mechanical Equivalences

Electric	Symbol	Unit (SI)	Equation	Mechanical Equivalence
Charge	q, Q	Coulomb (C)	-	Displacement
Curent	i, I	Amper (A)	$i=dq/dt$	Velocity
Potantial Difference or Voltage	e, E or v, V	Volt (V)	$e=dw/dq$	Force
Power Energy (Work)	p, P w, W	Watt (W) Joule (J)	$p=vi$ $w = \int v dq$ $w = \int v i dt$	Power Energy (Work)

Some Metric Prefixes Used in Physics

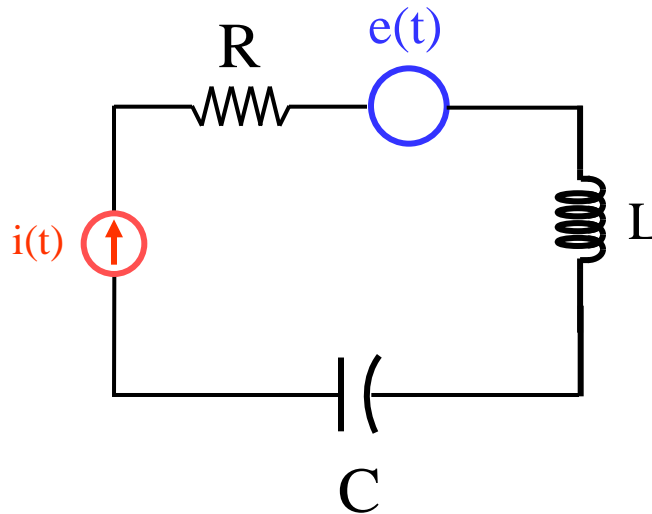
Large quantities		Small quantities	
Kilo (k)	10^3	Mili (m)	10^{-3}
Mega (M)	10^6	Micro (μ)	10^{-6}
Giga (G)	10^9	Nano (n)	10^{-9}
Tera (T)	10^{12}	Pico (p)	10^{-12}

Power Sources & Circuit Elements

Electric Circuit is consist of **electrical power sources** (voltage and current) and other **circuit elements (Receivers)** that absorb or store energy.

Sources

- Current
- Voltage

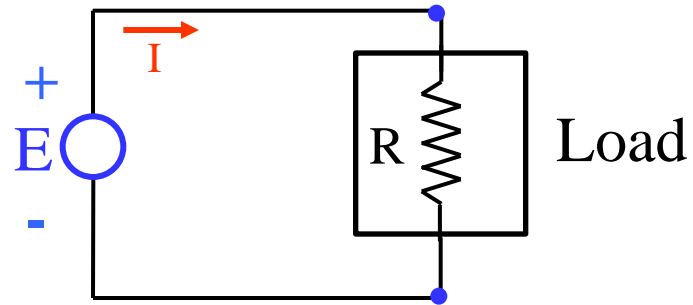


Receivers or Absorbers

- Resistor (R)
- Inductor (L)
- Capacitor (C)

Before we start to analyse circuits we have to know each individual circuit elements' voltage-current behaviour.

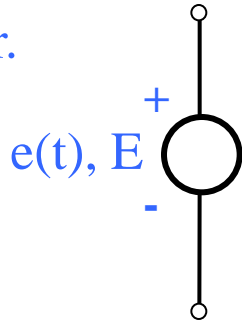
Definition of Load



Load can be resistors or any other combination of mix circuit elements

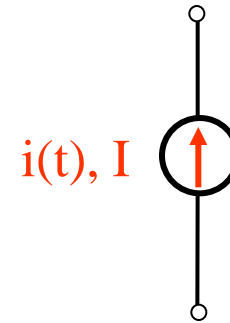
Ideal Power Sources-Independent

Ideal sources supply constant voltage (Voltage Source) or constant current (Current Source) ; the current and voltage values at the source terminal **does not changes** with the load (external resistor) it will always stay the same. Also ideal sources have no internal resistor.



Ideal Voltage Source

Voltage value of **an ideal voltage source** does not change with the load (external resistor) Voltage at the terminal is always constant! (But the drawn current can be changed)



Ideal Current Source

Current value of **an ideal current source** does not change with the load (external resistor) Current is always constant! (But the voltage at the terminal can be changed)

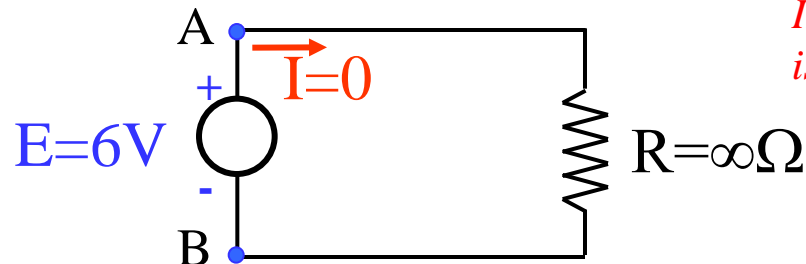
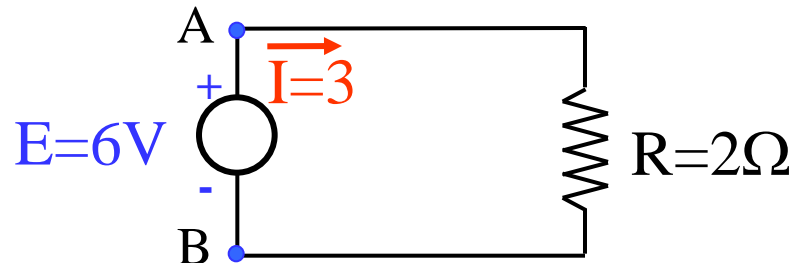
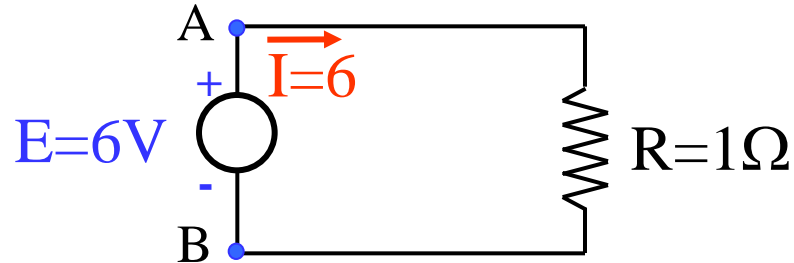
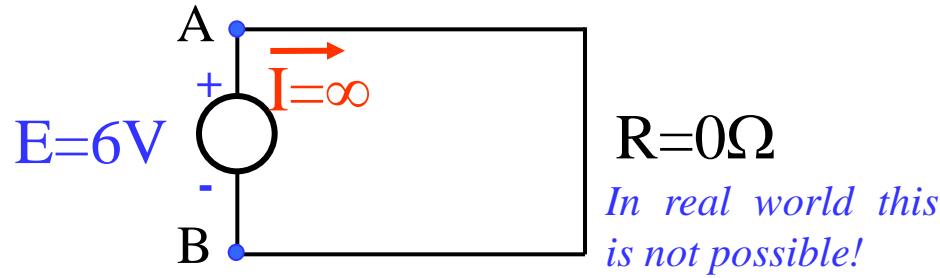
In reality no source is ideal and we will look at real sources in **Chapter-2**

Notation:

Uppercase letters: For constant **current (I)** and **Potential (E)** sources

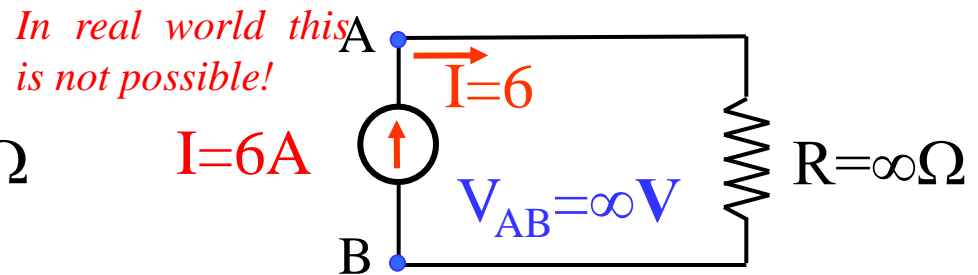
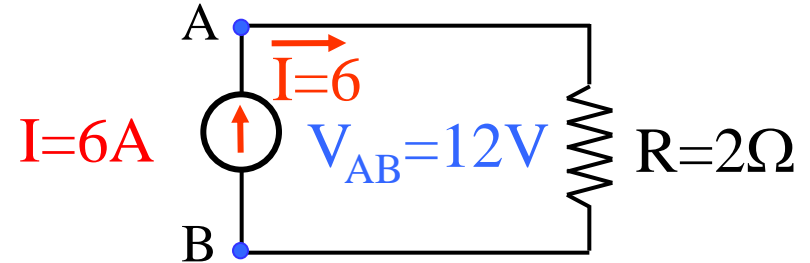
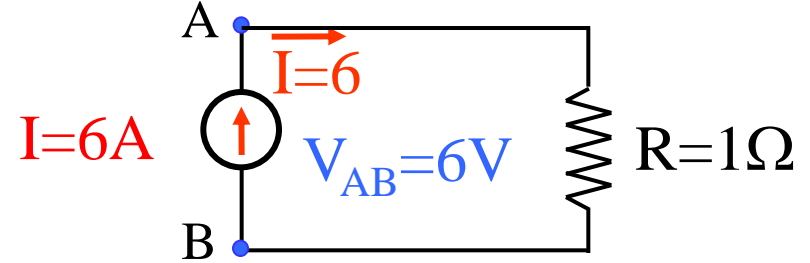
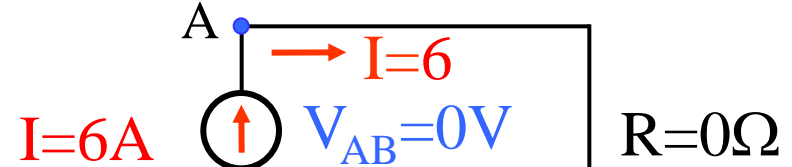
Lowercase letters: For **current (i(t))** and **potential (e(t), v(t))** sources changing in time

Ideal Voltage Source



Ideal Sources

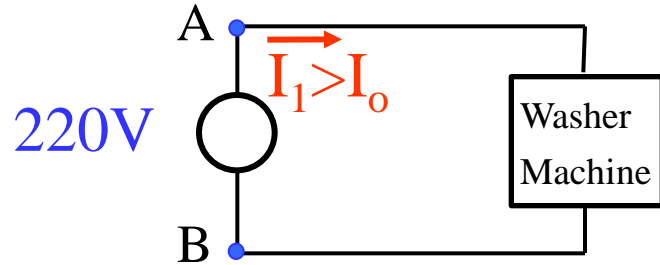
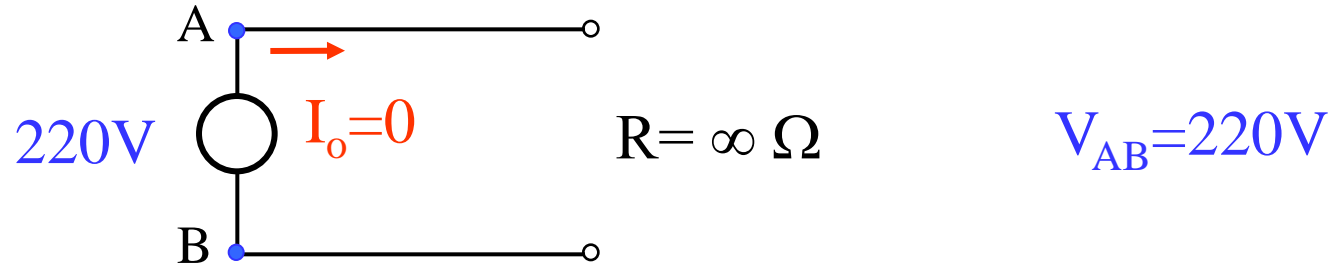
Ideal Current Source



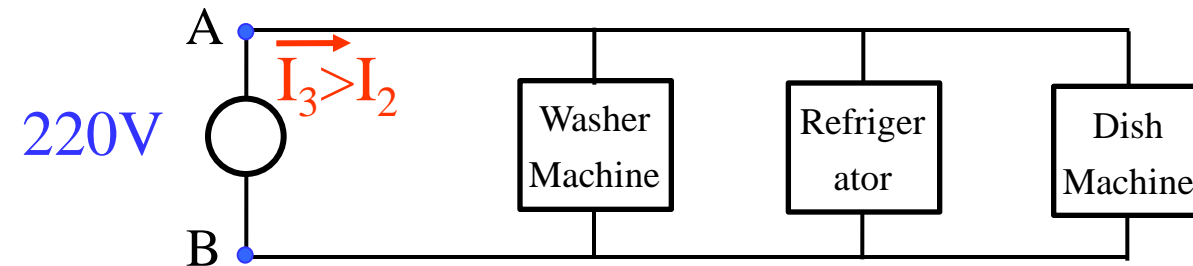
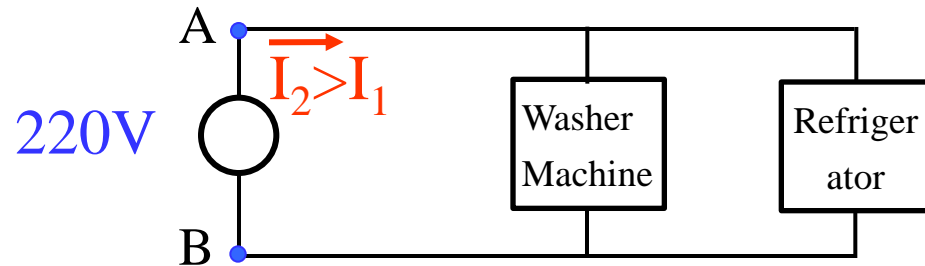
Voltage value of **an ideal voltage source** does not change with the load (external resistor) it is always constant! (But drawn current can be changed.)

Current value of **an ideal current source** does not change with the load (external resistor) it is always constant! (But drawn current can be changed.)

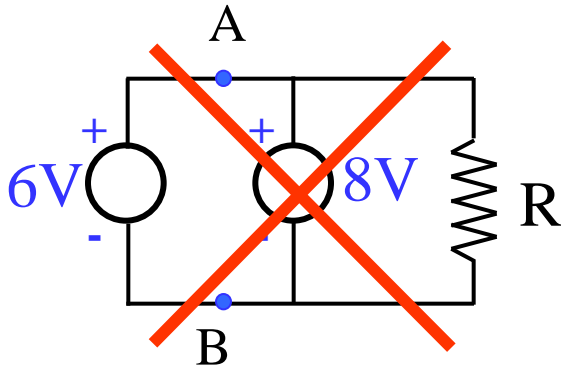
Ideal Voltage Source



Outlets in our homes are example of **an ideal voltage source**. The potential difference ($220V$) does not change with the addition of appliance (load) (Note: *current of an ideal voltage source is not defined*)

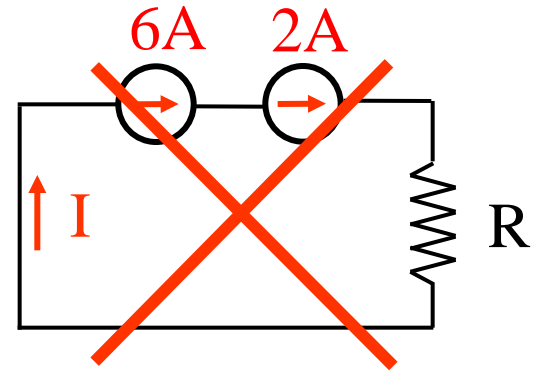


Ideal Power Sources



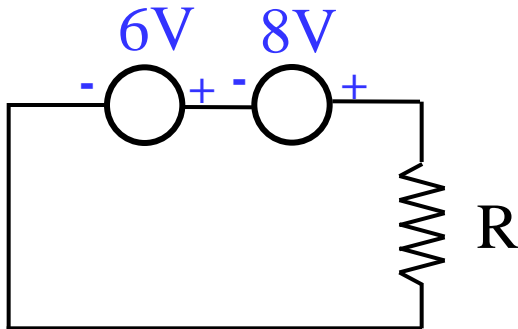
$$V_{AB}=?$$

This configuration does not apply because the voltage between A and B will be single value and same!

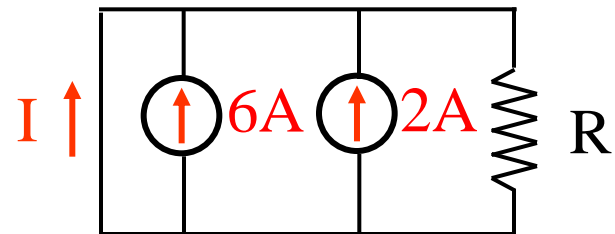


$$I=?$$

This configuration is not valid because the current through a branch should be constant!



$$V_{AB}=14V$$

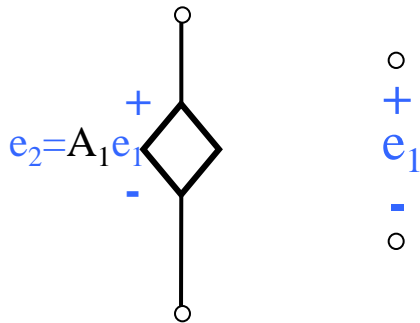


$$I=8A$$

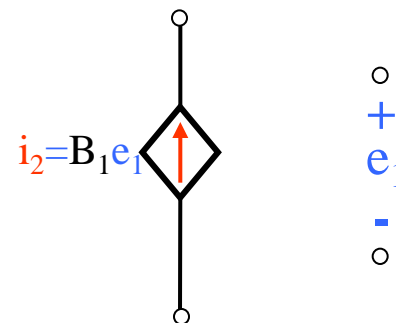
Dependent (Controlled) Source Types

In **Dependent Sources** the voltage or current at the terminals of the sources depends on the current or voltage value of any specific points at the circuit. Examples of such sources are transistors.

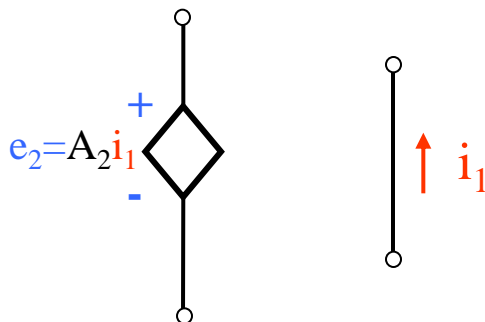
There are four possible case for the dependent sources



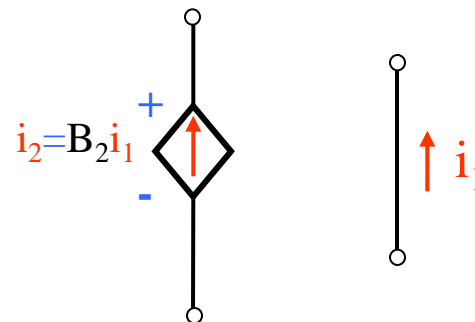
Voltage dependent **voltage source**



Voltage dependent **current source**



Current dependent **voltage source**



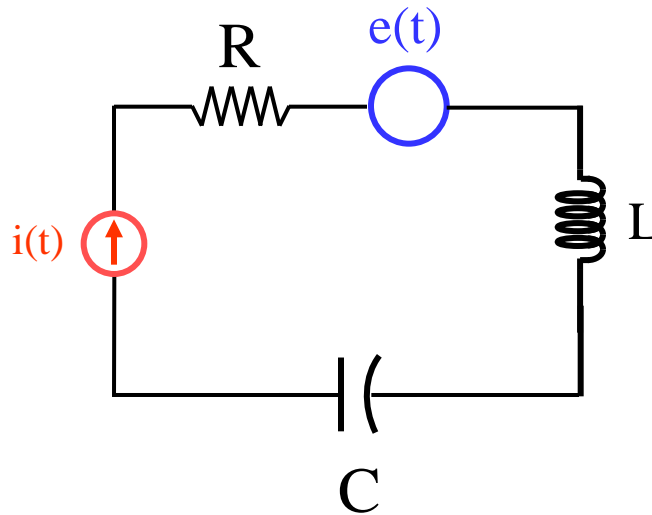
Current dependent **current source**

Power Sources & Circuit Elements

Electric Circuit is consist of **electrical power sources** (voltage and current) and other **circuit elements (Receivers)** that absorb or store energy.

Sources ✓

- Current
- Voltage



Receivers or Absorbers

- Resistor (R)
- Inductor (L)
- Capacitor (C)

Before we start to analyse circuits we have to know each individual circuit elements' voltage-current behaviour.