Ankara University
Engineering Faculty
Department of Engineering Physics

PEN207

Circuit Design and Analysis

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Chapter-2

Methods of Circuit Analysis and Circuit Theorems (2/3)

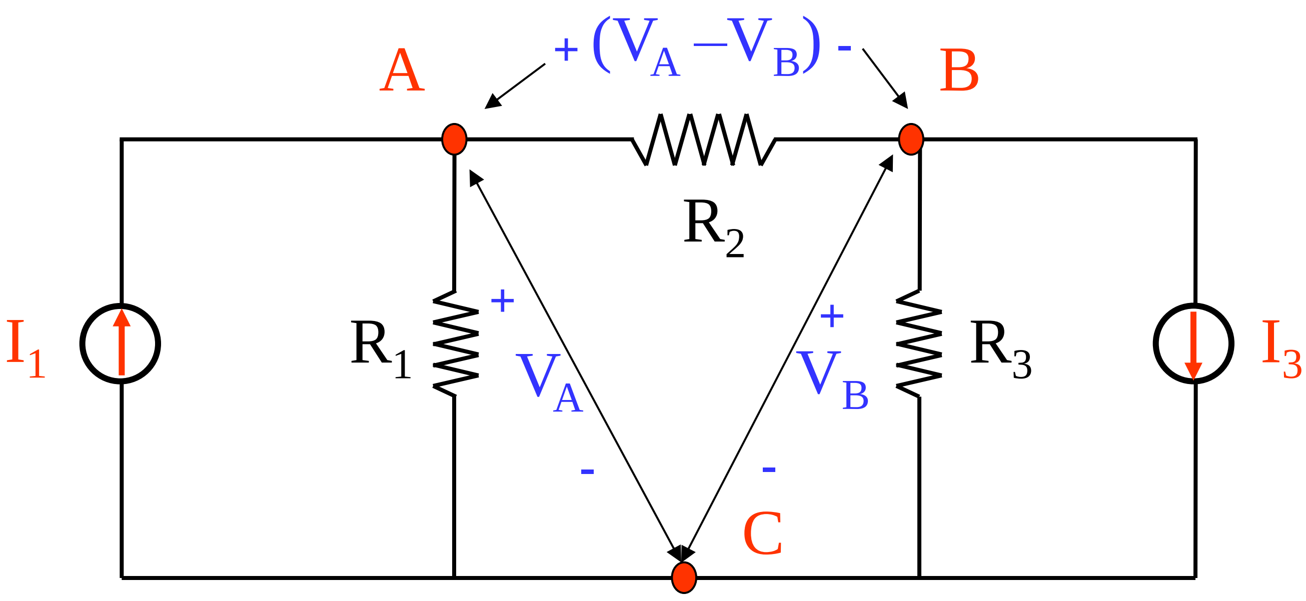
Nodal Voltage Analysis

In the circuit analysis, the **Nodal Analysis (Node Voltage Method)** is a method that requires the Kirchhoff's Voltage Law (KVL) equations for the circuit to be written in closed form, thus solving the Kirchhoff Current Law (KCL) equations only. In this method, voltages are defined for certain points (**Nodes**).

Nodal Voltage Analysis

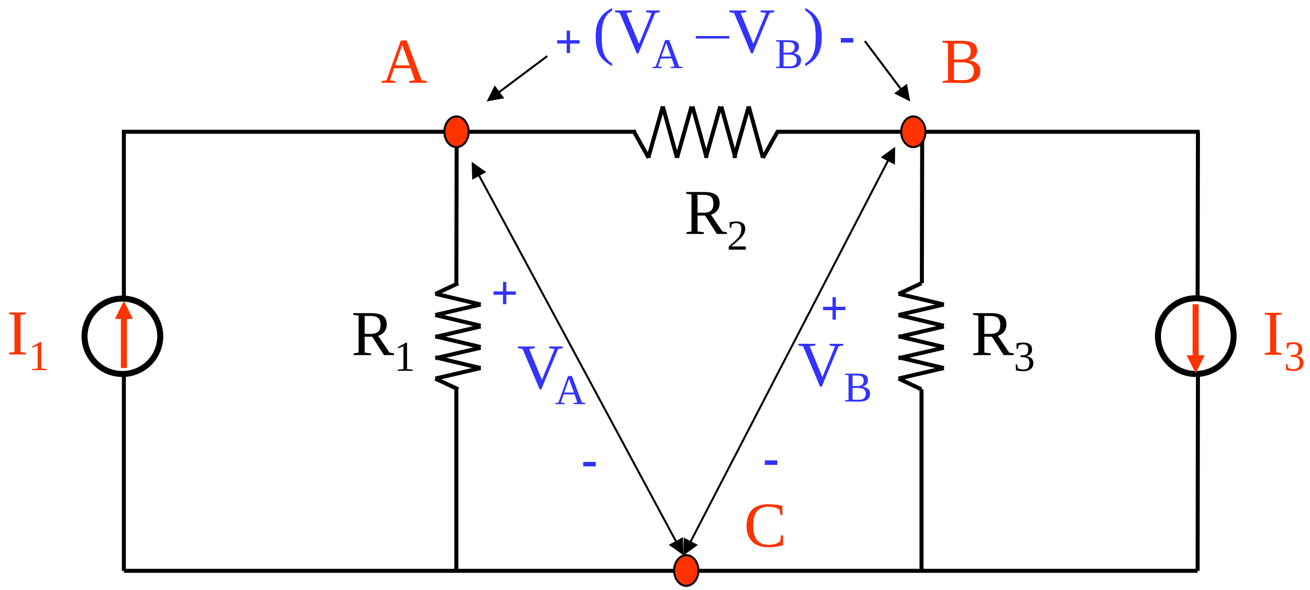
Node Voltage Method is a method that requires the Kirchhoff Voltage Law (KVL) equations to be written in a closed form for circuit, thus solving the Kirchhoff Current Law (KCL) equations only.

In this method, voltages are defined for certain points in circuit in a such a way that the circuit can be analyzed with the help of these voltages.

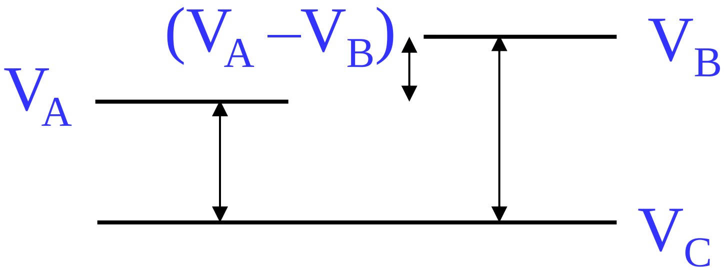


Nodal Voltage Analysis

Nodal Voltages

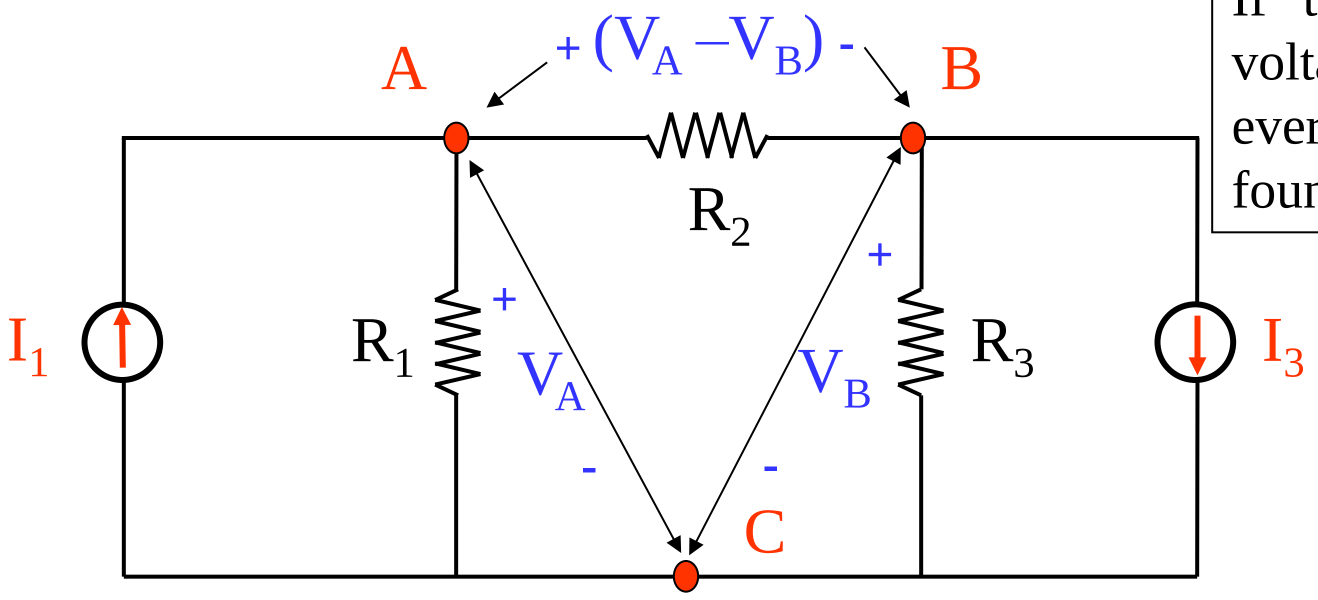


Mechanical Equivalent (If V_A and V_B potentials are known then all the potentials can be found



Nodal Voltage Analysis

To understand the Node Voltage Method, consider the following circuit:



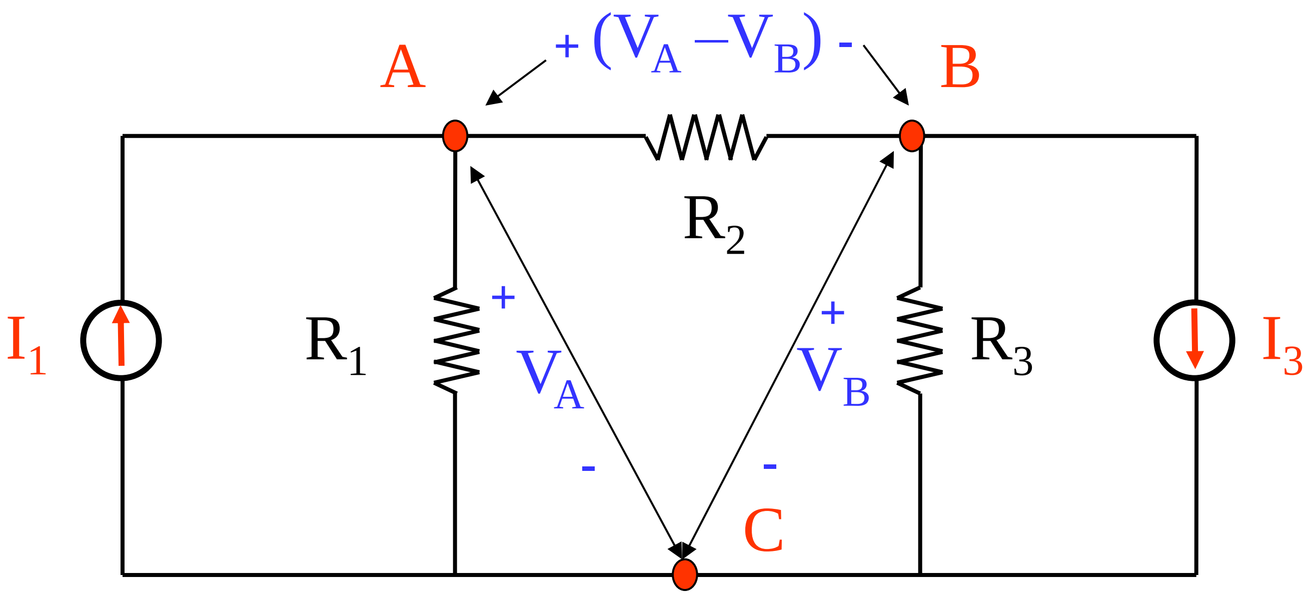
If the V_A and E_B voltages are known, everything can be found in the circuit.

- Two unknown voltages V_A and V_B were selected. E_A voltage is a voltage increase from node C to node A; likewise E_B is selected as a voltage increase from node C to node B. Since the unknown voltages are measured with respect to node C; C is called the **reference node.**

Nodal Voltage Analysis

The voltage increase from node B to node A is the third unknown voltage V_{AB} in the circuit, which is derived from the Kirchhoff's Voltage Law (KVL) equation.

$$V_{AB} = V_A - V_B$$



Nodal Voltage Analysis

KCL for node A: $-V_A(1/R_1)-(V_A-V_B)(1/R_2)+I_1=0$

KCL for node B: $-V_B(1/R_3)+(V_A-V_B)(1/R_2)-I_3=0$

$+I_1-I_{R_1}-I_{R_2}=0$

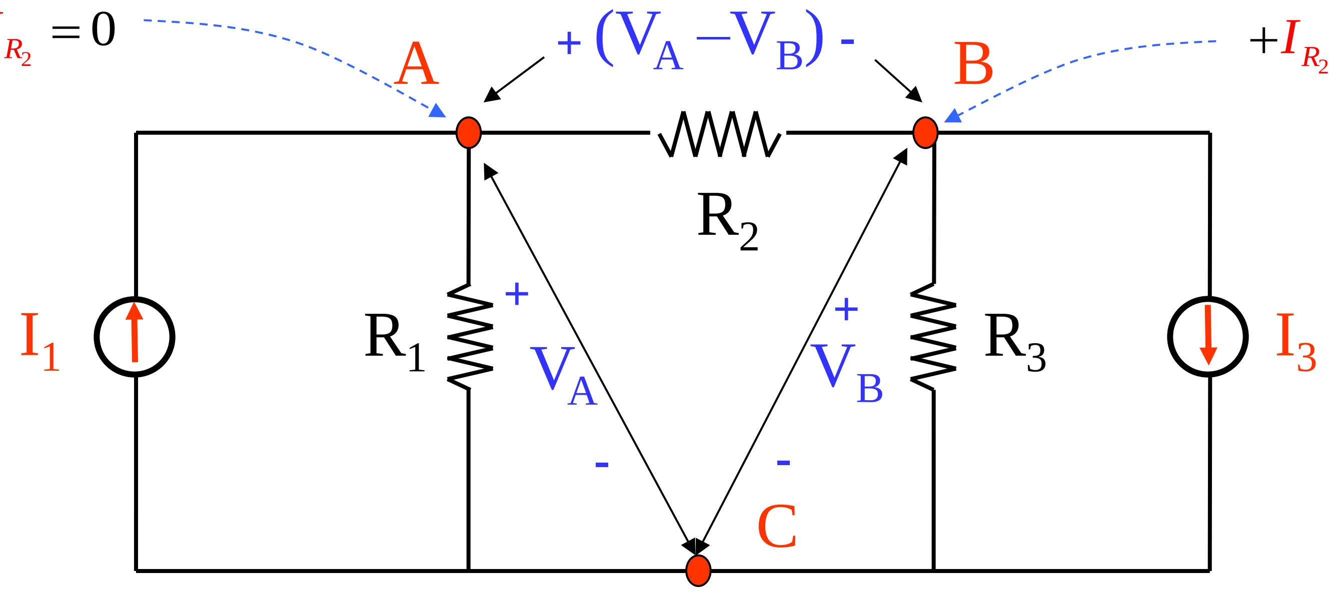
$+I_{R_2}-I_{R_3}-I_3=0$

$I_{R_1}=\frac{V_A}{R_1}$

$I_{R_2}=\frac{V_A-V_B}{R_2}$

$I_{R_2}=\frac{V_A-V_B}{R_2}$

$I_{R_3}=\frac{V_B}{R_3}$



If rearranged:

$$+(1/R_1+1/R_2)V_A-(1/R_2)V_B=I_1$$
$$-(1/R_2)V_A+(1/R_2+1/R_3)V_B=-I_3$$

In terms of conduction G :

$G\equiv\frac{1}{R}$

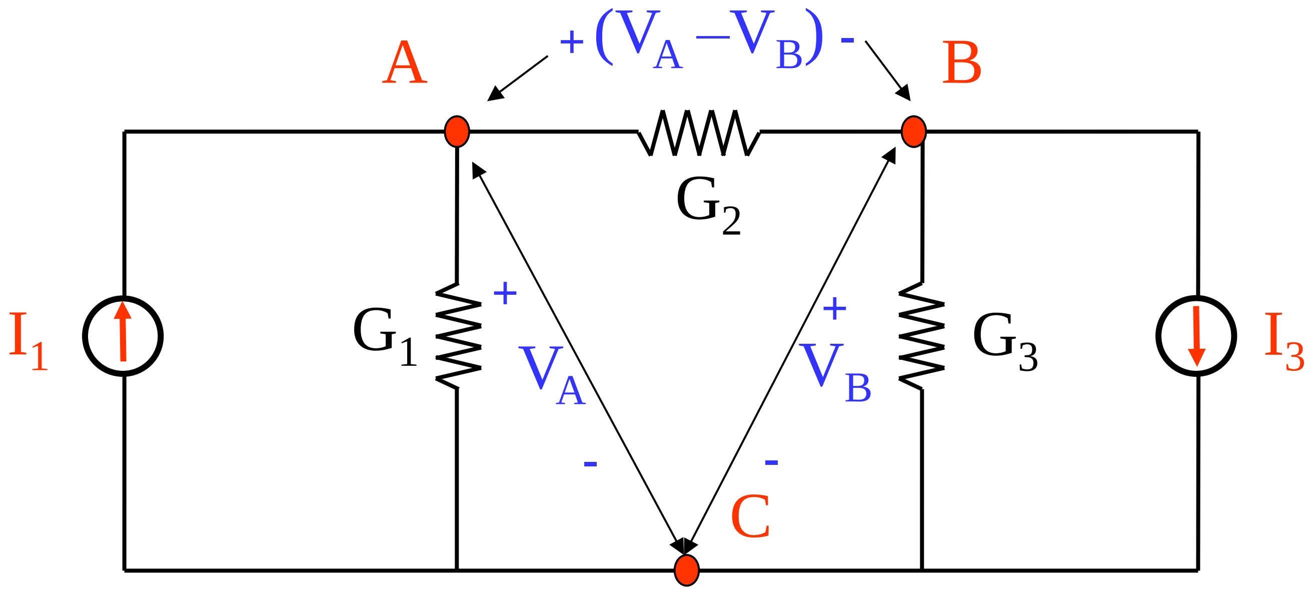
$$+(G_1+G_2)V_A-G_2V_B=I_1$$
$$-G_2V_A+(G_2+G_3)V_B=-I_3$$

Point A: $+(G_1+G_2)V_A - G_2V_B = +I_1$

Point B: $-G_2V_A + (G_2+G_3)V_B = -I_3$

The coefficient of the V_A is the positive sum of the conductors connected to node A, the negative sum of the conductors between node A and B, and the sum of the current sources connected to the node A on the right side of the equation.

Similarly for point B:
The coefficient of the V_B is the positive sum of the conductors connected to node B, the negative sum of the conductors between node B and A, and the sum of the current sources directly connected to the node B on the right side of the equation.



The order in the equations results from the equations of the current law and the way the voltage variable is selected. This rule is called the Node Voltage Method.

According to this method the steps to be followed:

1. Step: **All the power sources should be converted to** current source and the circuit must be redrawn according to the new configuration.

2. Step: **An arbitrary reference node is selected**, so let it be R (reference) point. The other nodes in the circuit are given the letters A, B,…., N, and unknown voltages V_A, V_B, …, V_N, R from A, B etc. points are selected as voltage increases.

3. Step: Node (current law) equations are written for A, B,…., N junctions, respectively.

$$A: \quad +G_{AA}V_A - G_{AB}V_B - G_{AC}V_C \ldots - G_{AN}V_N = I_A$$

$$B: \quad -G_{AB}V_A + G_{BB}V_B - G_{CC}V_C \ldots - G_{BN}V_N = I_B$$

$$C: \quad -G_{AC}V_A - G_{BC}V_B + G_{CC}V_C \ldots + G_{CN}V_N = I_C$$

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$$N: \quad -G_{AN}V_A - G_{BN}V_B - G_{CN}V_C \ldots + G_{NN}V_N = I_N$$

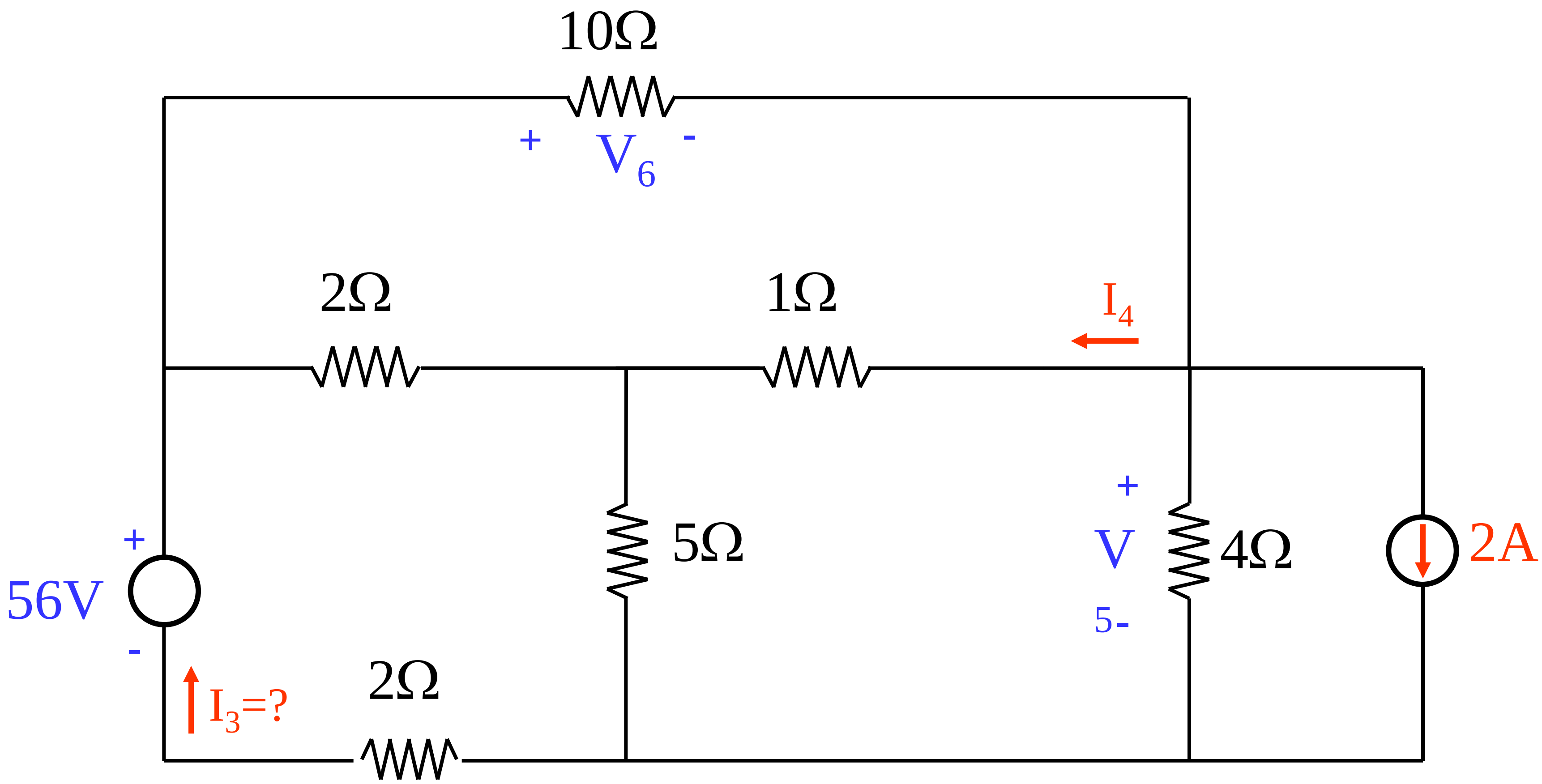
G_{XX}: is the sum of all conductors connected to the node X

G_{XY}: is the sum of all conductors connected between node X are Y

I_X: is the sum of current sources directly connected to the node X

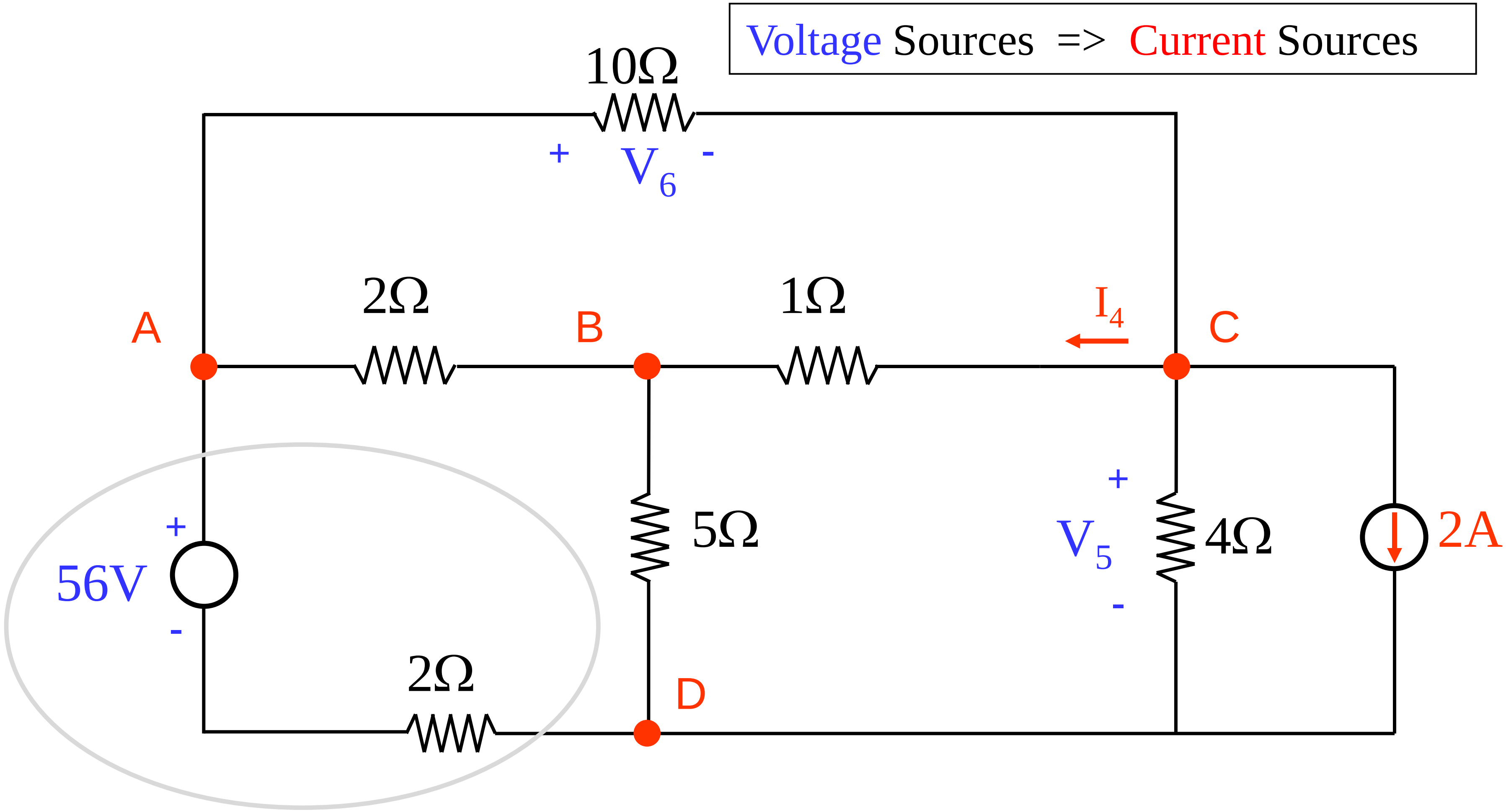
4. Step: Equations are solved to obtain the desired junction voltages. Other voltages and currents in the circuit can be found by applying Kirchhoff's Voltage Law and Ohm's Law.

Example-2.6: Find the voltages (V_5 ve V_6) in the following circuit using the Node Voltage Method. Also calculate the I_3 current.



Solution:

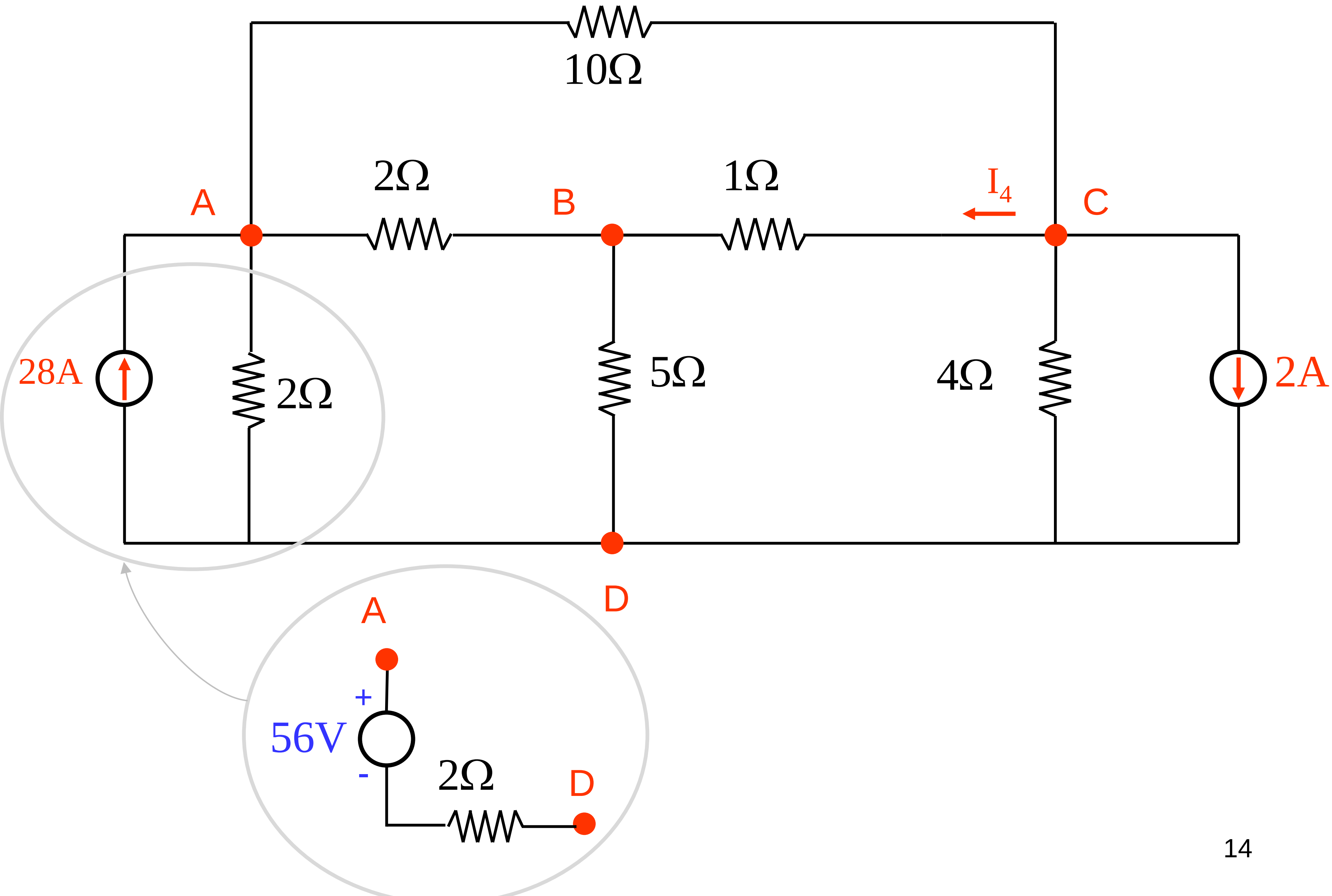
1. Step: First we need to convert all voltage sources in to current source.



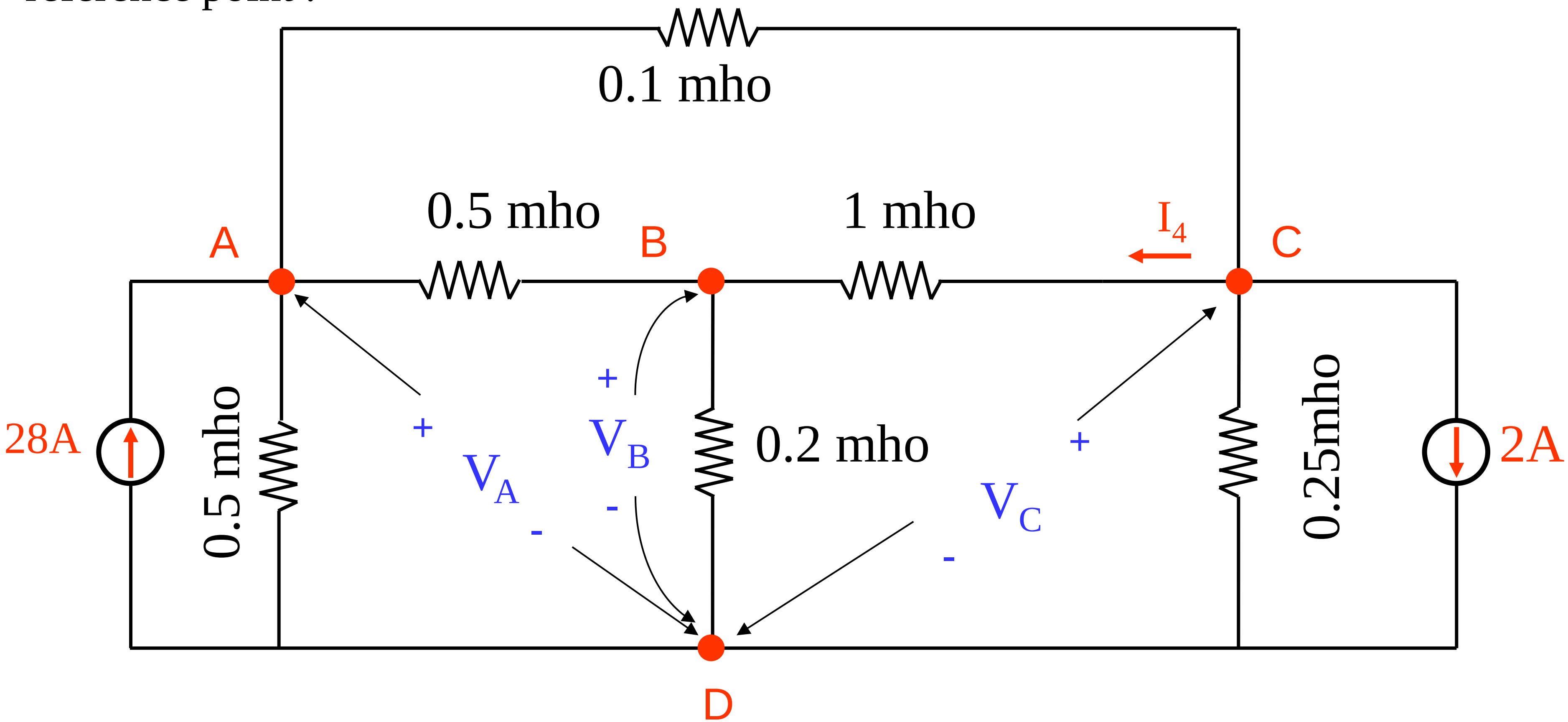
Let's find the equivalent current source of the 56V voltage source first: Voltage source 56V and 2 Ω resistor between **A** and **D** can be converted to a current source.

$$I = E_o / R_o \qquad R_o = 2\ \Omega \qquad I = 56V / 2\Omega = 28A$$

The circuit after source conversion:



2. Step: Let's define the equivalent nodes. Point D can be specified as the reference point :



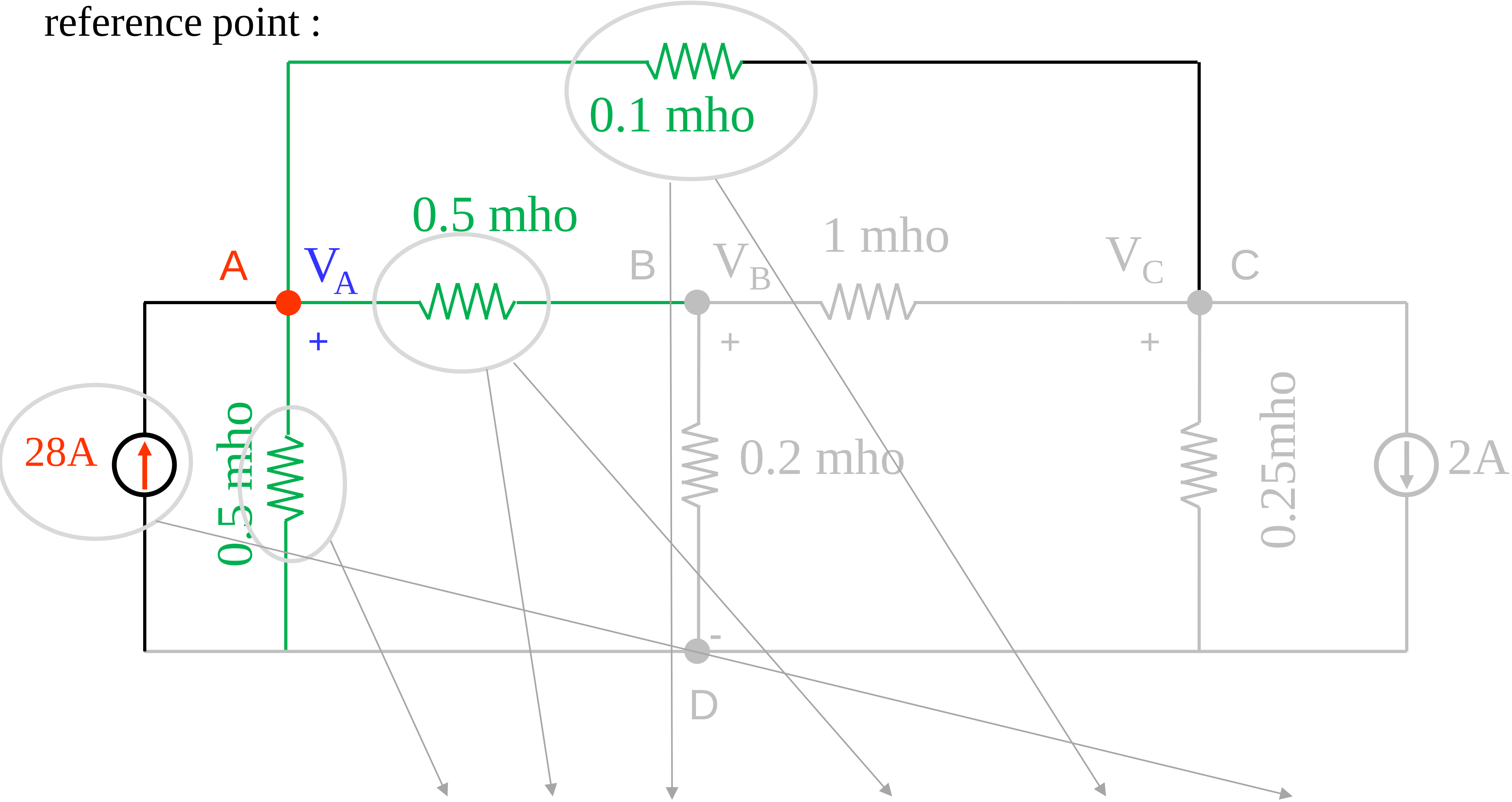
Current expressions for nodes:

$$A: \quad +(\qquad)V_A - (\qquad)V_B - (\qquad)V_C = I_A$$

$$B: \quad -(\qquad)V_A + (\qquad)V_B - (\qquad)V_C = I_B$$

$$C: \quad -(\qquad)V_A - (\qquad)V_B + (\qquad)V_C = I_C$$

2. Step: Let's define the equivalent nodes. Point D can be specified as the reference point :

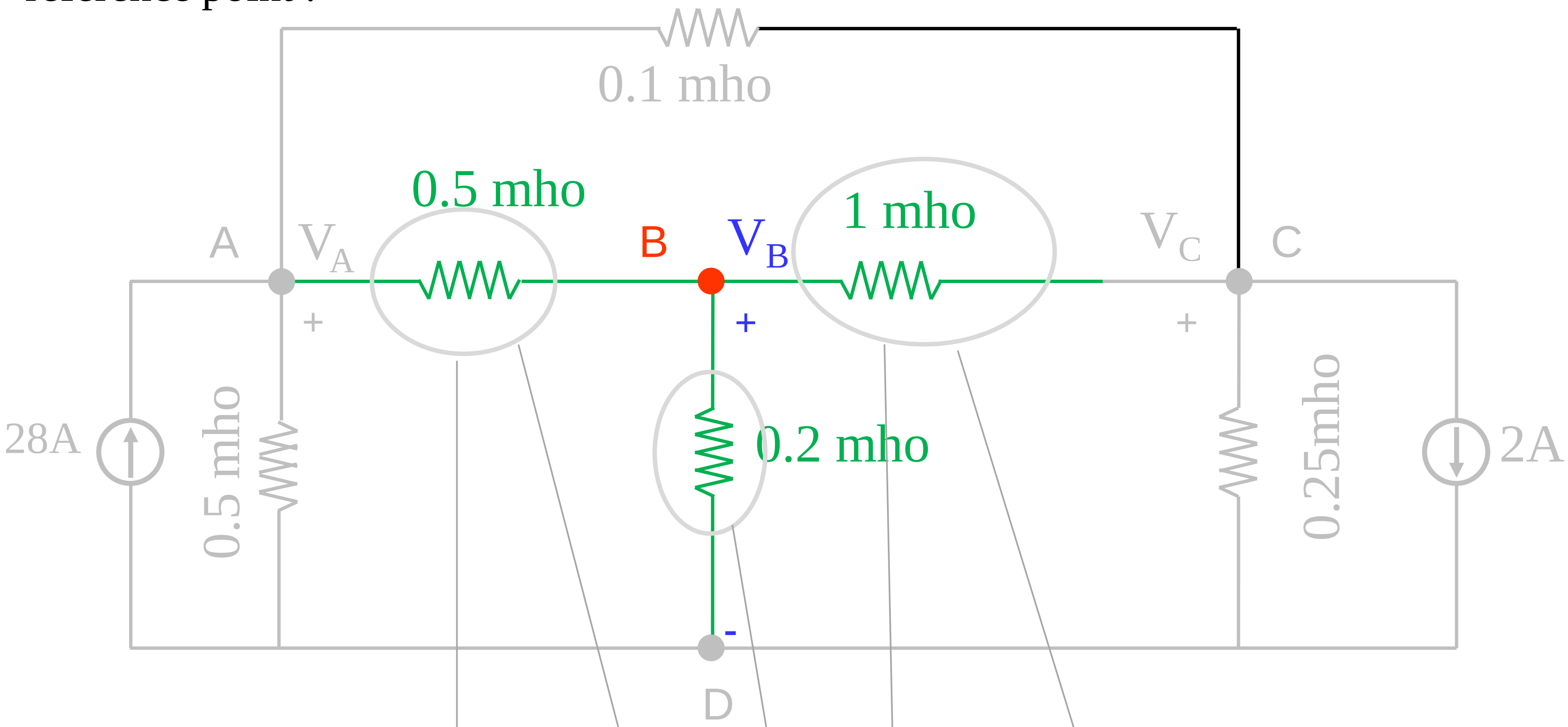


$$A: \quad +(0.5+0.5+0.1)V_A-(0.5)V_B-(0.1)V_C=+28$$

$$B: \quad -(0.5)V_A+(0.5+0.2+1.0)V_B-(1.0)V_C=0$$

$$C: \quad -(0.1)V_A-(1.0)V_B+(0.1+1.0+0.25)V_C=-2$$

2. Step: Let's define the equivalent nodes. Point D can be specified as the reference point :



$$A: \quad +(0.5+0.5+0.1)V_A-(0.5)V_B-(0.1)V_C=+28$$

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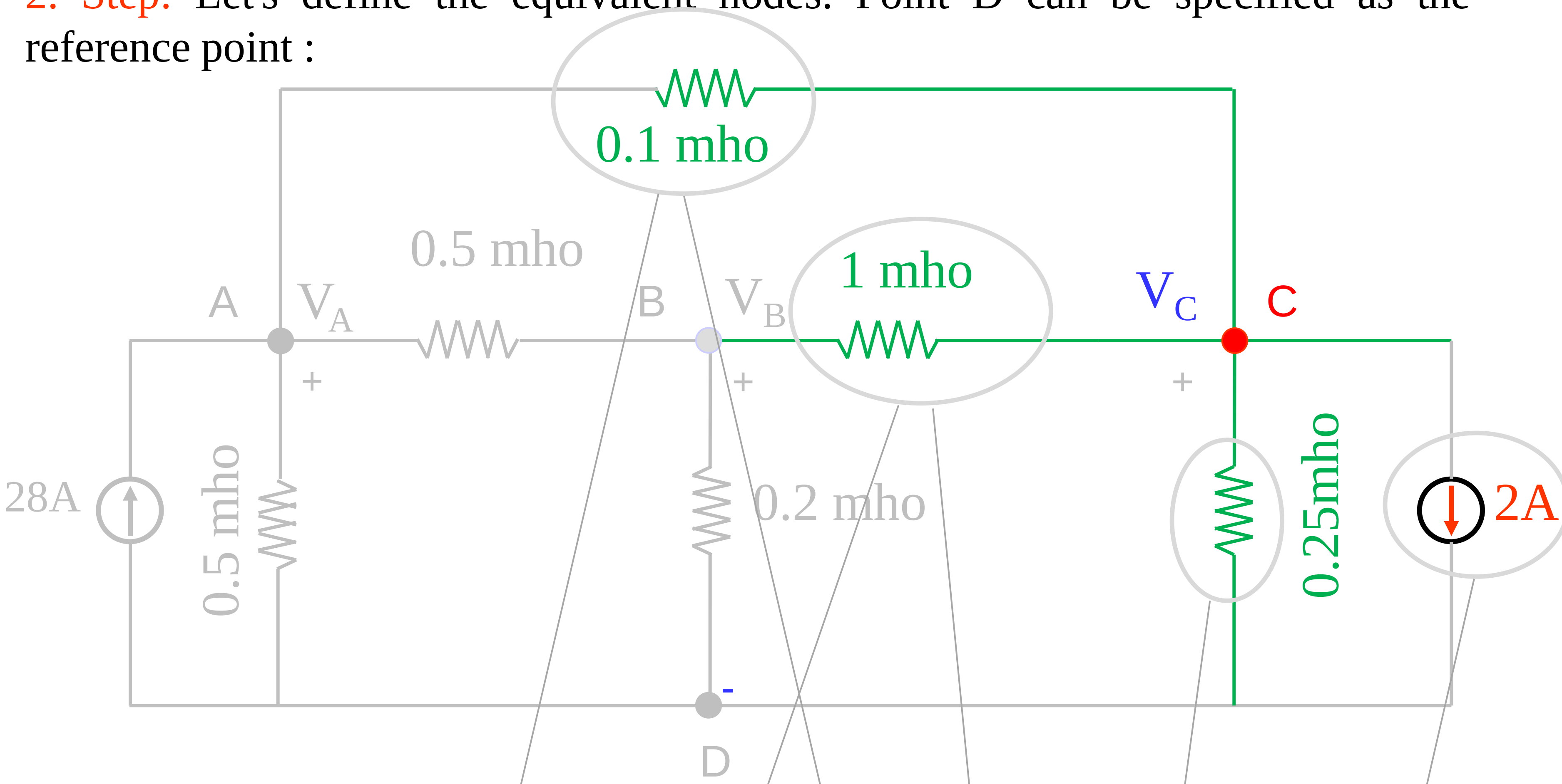
2. Step: Let's define the equivalent nodes. Point D can be specified as the reference point :



$$A: \quad +(0.5+0.5+0.1)V_A-(0.5)V_B-(0.1)V_C=+28$$

$$B: \quad -(0.5)V_A+(0.5+0.2+1.0)V_B-(1.0)V_C=0$$

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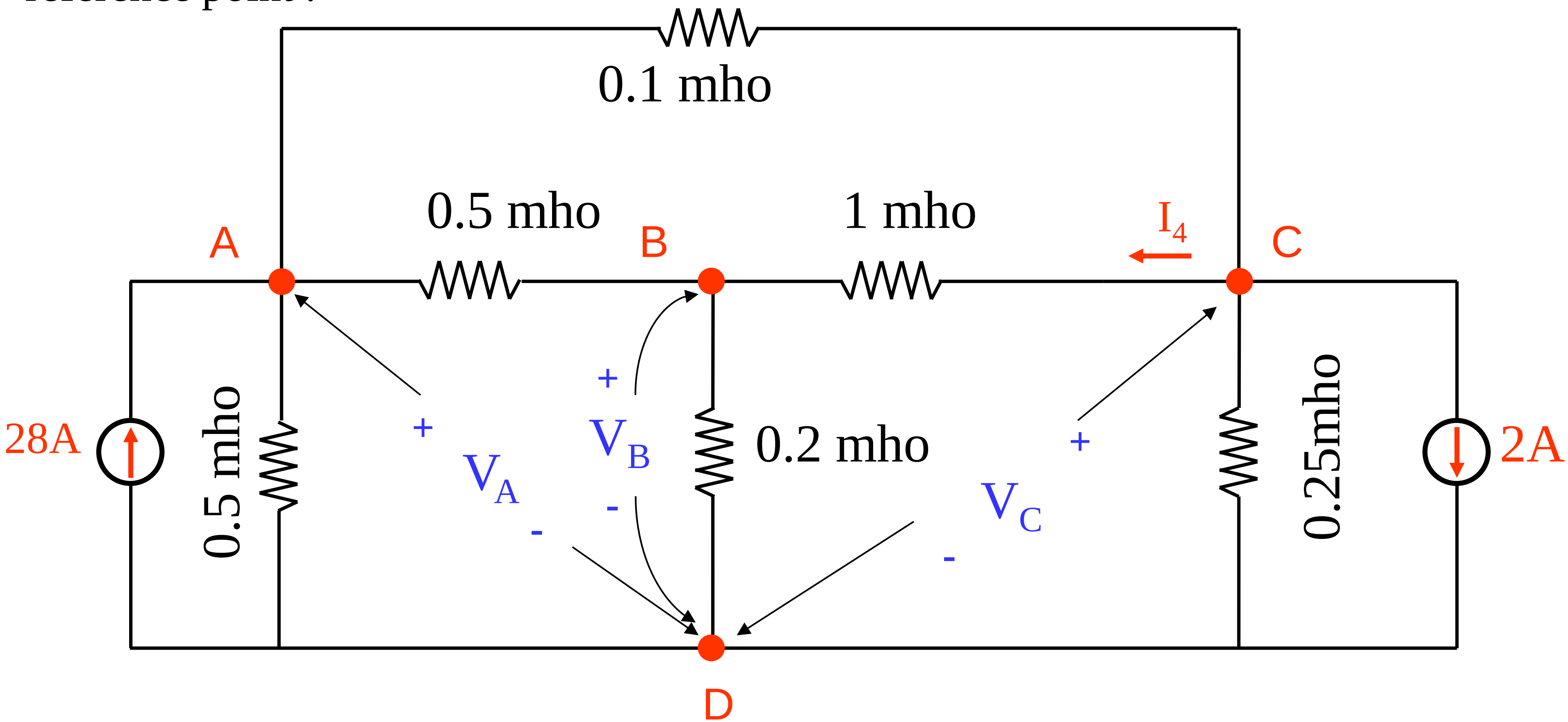
2. Step: Let's define the equivalent nodes. Point D can be specified as the reference point :



Current expressions for nodes:

$$A: \quad +(0.5+0.5+0.1)V_A-(0.5)V_B-(0.1)V_C=+28$$

$$B: \quad -(0.5)V_A+(0.5+0.2+1.0)V_B-(1.0)V_C=0$$

$$C: \quad -(0.1)V_A-(1.0)V_B+(0.1+1.0+0.25)V_C=-2$$

Rearranging the equations:

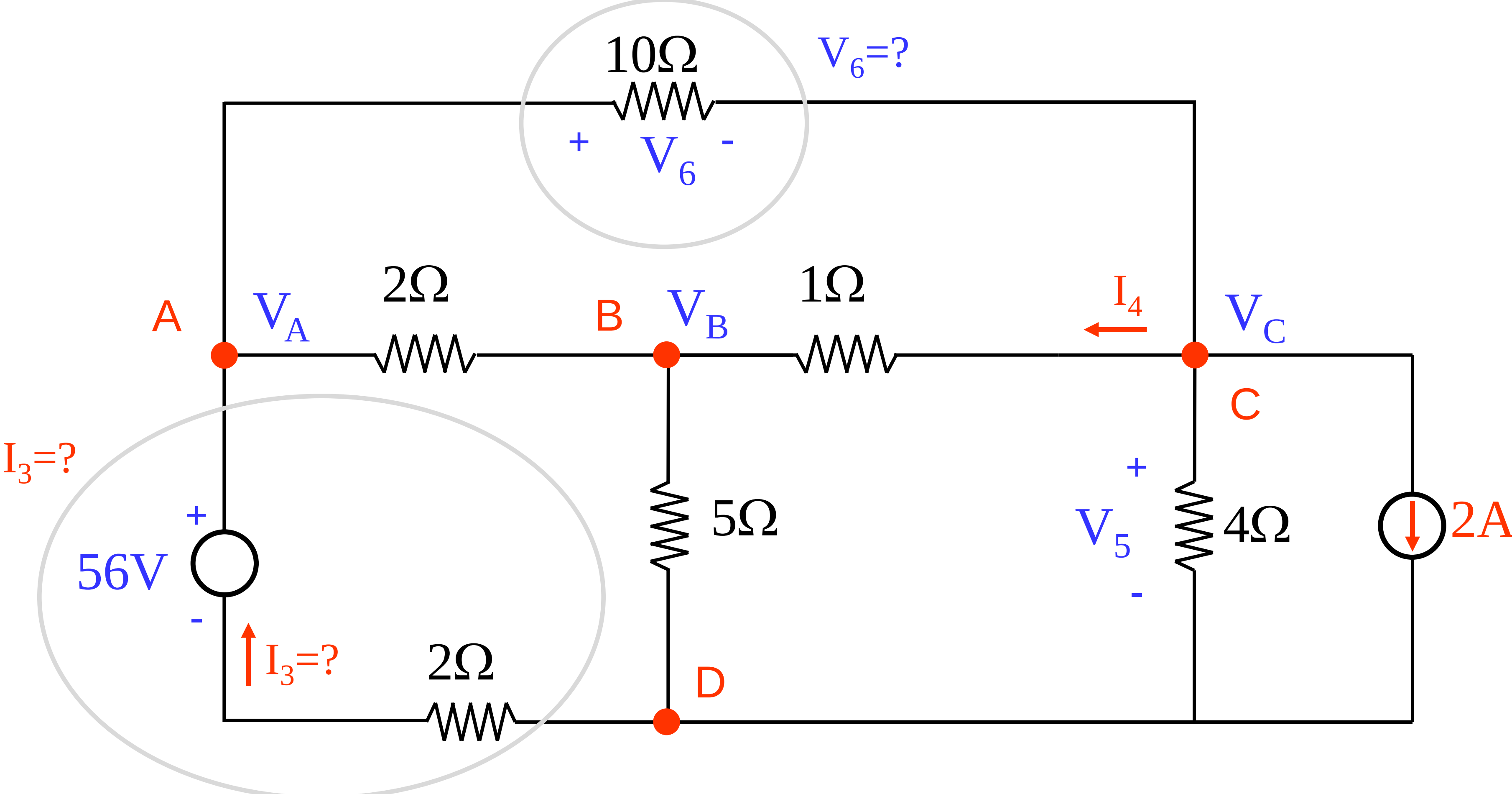
$$1.1V_A - 0.5V_B - 0.1V_C = +28$$

$$-0.5V_A + 1.7V_B - 0.1V_C = 0$$

$$0.1V_A - 1.0V_B + 1.35V_C = -2$$

Solution of above equation systems are:

$$V_A = 36V \quad V_B = 20V \quad V_C = 16V$$



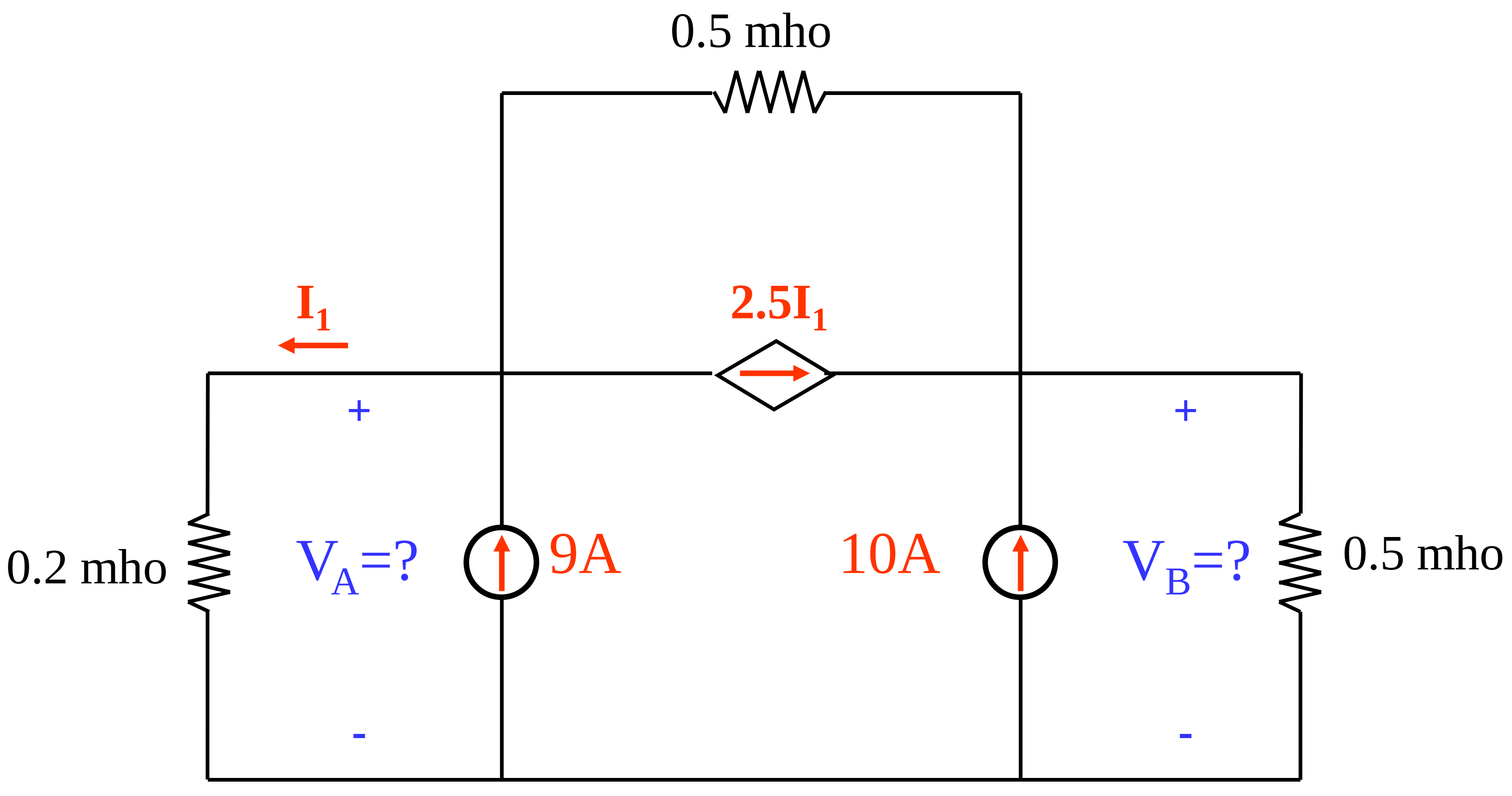
Knowing the node voltages makes it possible to find other voltages and currents in the circuit.

$$V_6 = V_A - V_C = 36\ V - 16\ V = 20\ V$$

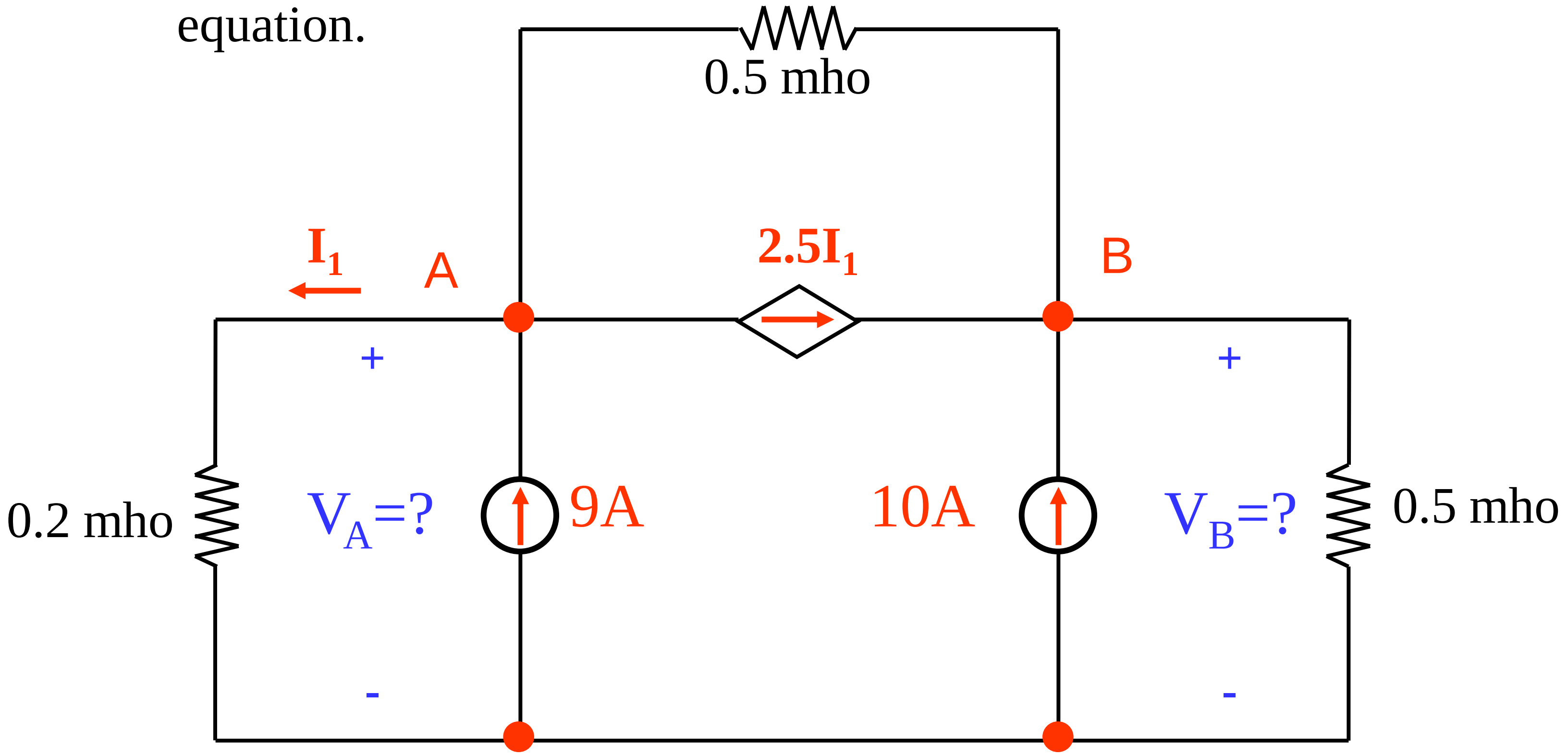
Current I_3 can be calculated by considering that the voltages from point **A** to point **D** must be the same in both circuits

$$56\,V - (2\Omega) I_3 = V_A = 36\ V \Rightarrow I_3 = 10\ A$$

Example-2.8: Find the V_A and V_B voltages in the circuit below.



Solution: Node Voltage Methods will be used. The **dependent** source will be considered as independent source first then we will find a binding equation.



Node A: $$(0.2+0,5)V_A - 0.5V_B = 9 - 2.5I_1$$

Node B: $$-0.5V_A + (0.5+0.5)V_B = 10 + 2.5I_1$$

Binding equation for dependent source: $$I_1 = 0.2V_A$$

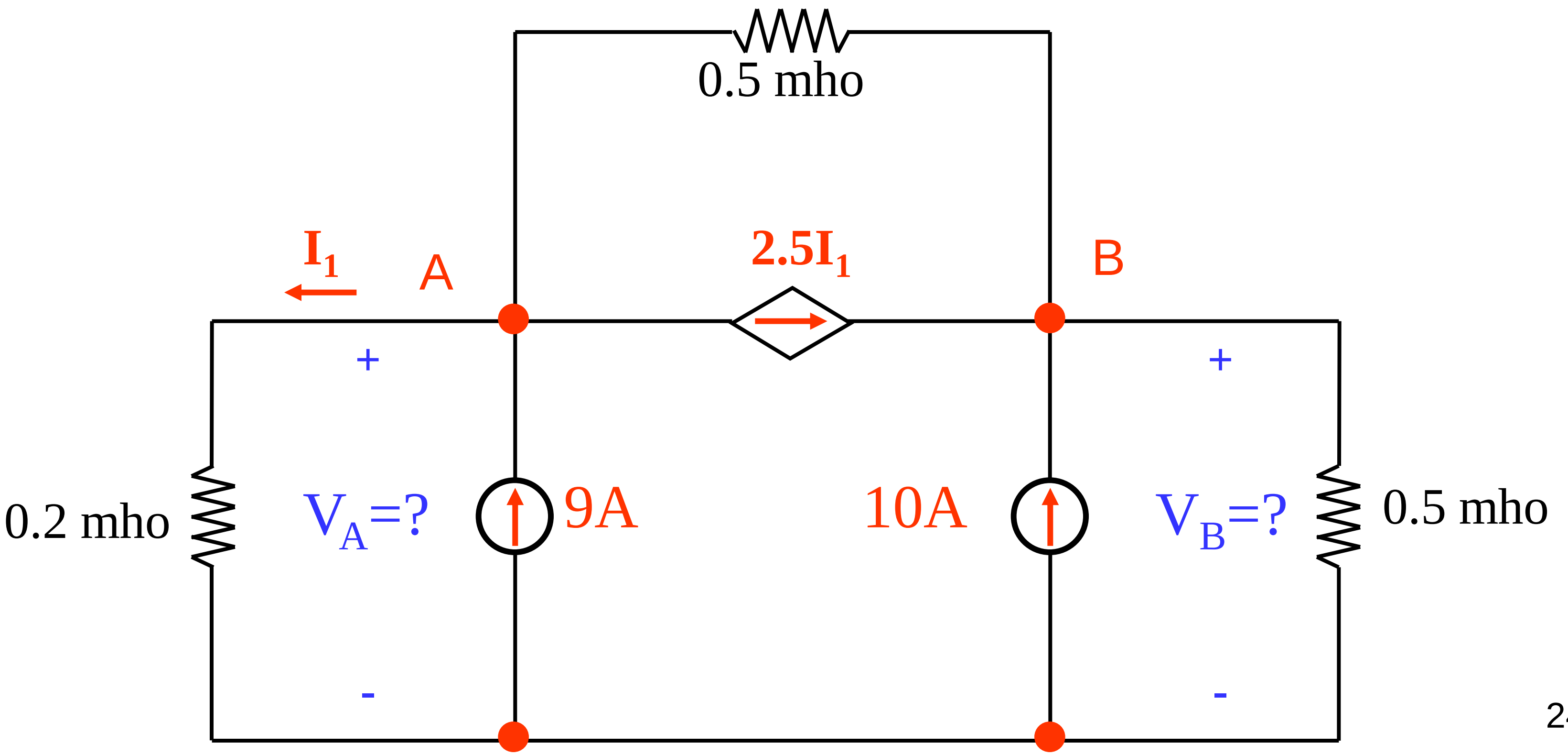
If equations are rearranged:

$$1.2V_A - 0.5V_B = 9$$

$$0.5V_A - 1.0V_B = 10$$

Solution:

$$V_A = 20\ V \qquad V_B = 30\ V$$



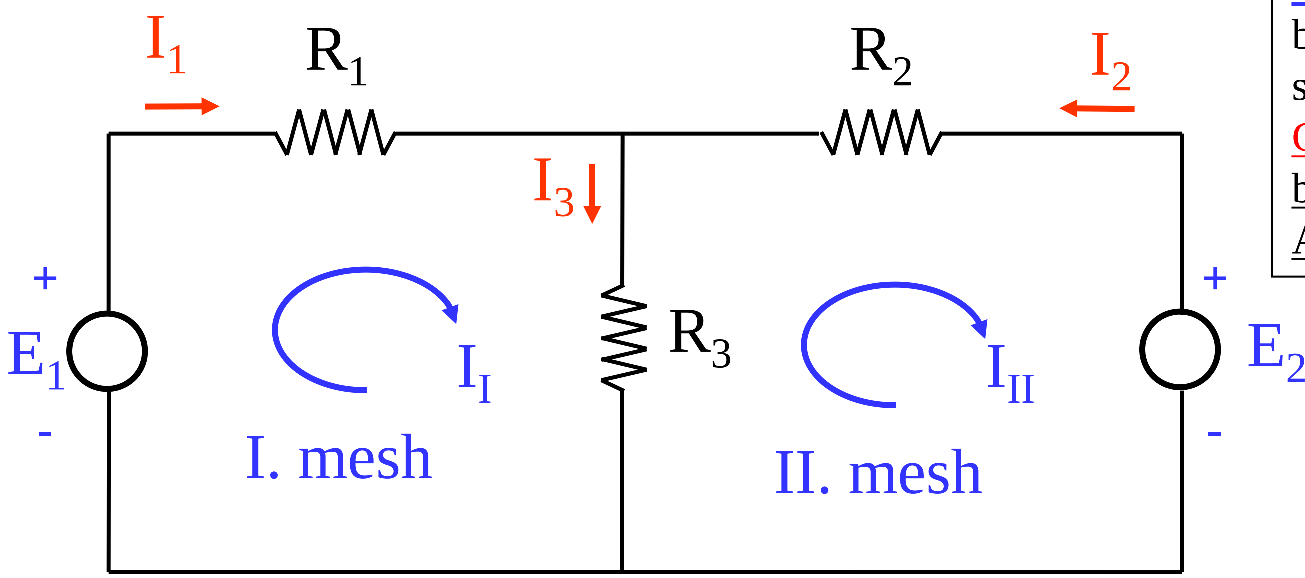
Mesh Current Analysis

Mesh Current Method is a method that requires Kirchhoff's Current Law (KCL) equations to be written in closed form in terms of the defined voltages so thus solves Kirchhoff's Voltage Law (KGY) equations.

Mesh Current Analysis

In this method, the currents circulating in the loops are defined (mesh currents). To understand this method, let's use the circuit below.

Notation:
Mesh currents will be shown as Roman subscript (I_{III});
Current on each branch will be shown as Arabic subscript (I_3).



- **Mesh Current Method**, the presence of unknown currents (mesh currents I_I and I_{II}) in the loop is considered. (remember, unknown voltage V_A and V_B were also defined in the node voltage method). The current I_I in the mesh I is present in all elements that form the loop (R_1, R_3 and E_1). Similarly, the current I_{II}, in the II. mesh all elements that make up the loop (R_2, R_3 and E_2).

Mesh Current Analysis

- In the Mesh Current Analysis, **<u>all the mesh currents</u>** are selected in the same direction (here clockwise). <u>Selecting currents in the same direction allows the resulting equations to be easily written in the form of a matrix</u>.

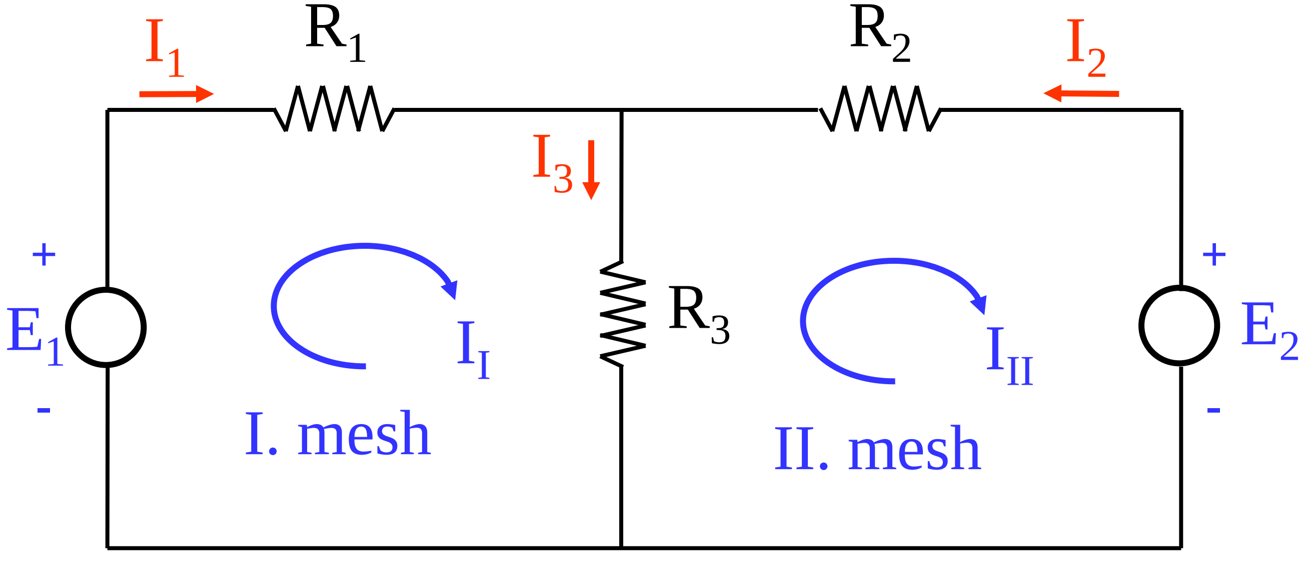
- The currents on each arm can be readily expressed in terms of the mesh currents (I_I and I_{II}), for example

$$I_1 = I_I, \qquad I_2 = -I_{II} \qquad ve \qquad I_3 = I_I - I_{II}$$

If Kirchhoff's Voltage Law (KVL) equations are written around loop I and II:

$$-R_1 I_I - R_3(I_I - I_{II}) + E_1 = 0 \qquad \text{I. Mesh}$$

$$+R_3(I_I - I_{II}) - R_2 I_{II} - E_2 = 0 \qquad \text{II. Mesh}$$



Mesh Current Analysis

Rearranged:

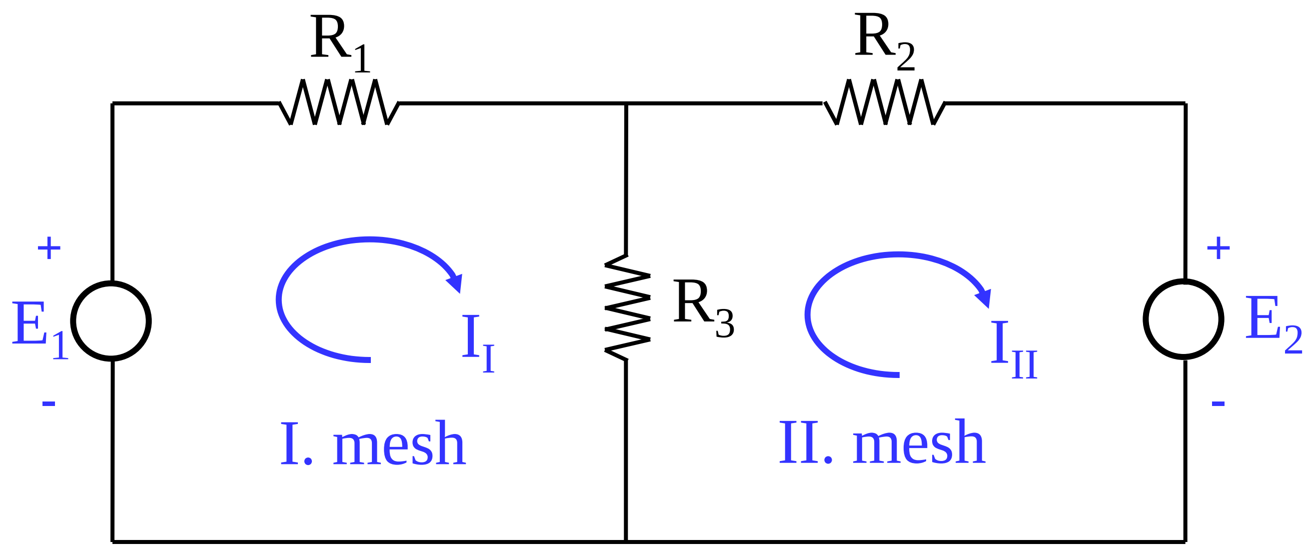
I. Mesh $+(R_1+R_3)I_I - R_3 I_{II} = E_1$

II. Mesh $-R_3 I_I + (R_2+R_3)I_{II} = -E_2$

Nodal Voltage Method:

$+(G_1+G_2)V_A - G_2 V_B = +I_1$

$-G_2 V_A + (G_2+G_3)V_B = -I_3$



The above equations show a similar pattern as those written for thr **Node Voltage Method**. The coefficient of the mesh current I_I written around the first mesh is the positive sum of the resistors forming the first loop. The coefficient of the second mesh current I_{II} is the negative sum of the common resistors betwen the 1st and 2nd mesh. The sum of the voltage source (clockwise if there voltage increase) in the right side of the equation. **Similar comments can be made for the equation written around the second mesh (Mesh** II.**).**

This order in the equations results from the voltage law equations and the way the current variable is selected. This method is known as Mesh Current Method

According to this method the stepps to be followed are:

1. Step: **All the power sources should be converted to** voltage source and the circuit must be redrawn according to the new configuration.

2. Step: **The meshes are identified** so that no other mesh is present in a selected mesh and the mesh currents are selected clockwise.

3. Step: If the mesh (voltage law) equations are written for I, II, III,…., N meshes, respectively

$$I: \quad +R_{I,I} I_I - R_{I,II} I_{II} - \ldots - R_{I,N} I_N = E_I$$

$$II: \quad -R_{I,II} I_I + R_{II,II} I_{II} - \ldots - R_{II,N} I_N = E_{II}$$

.

.

$$N: \quad -R_{I,N} I_I - R_{II,N} I_{II} - \ldots + R_{N,N} I_N = E_N$$

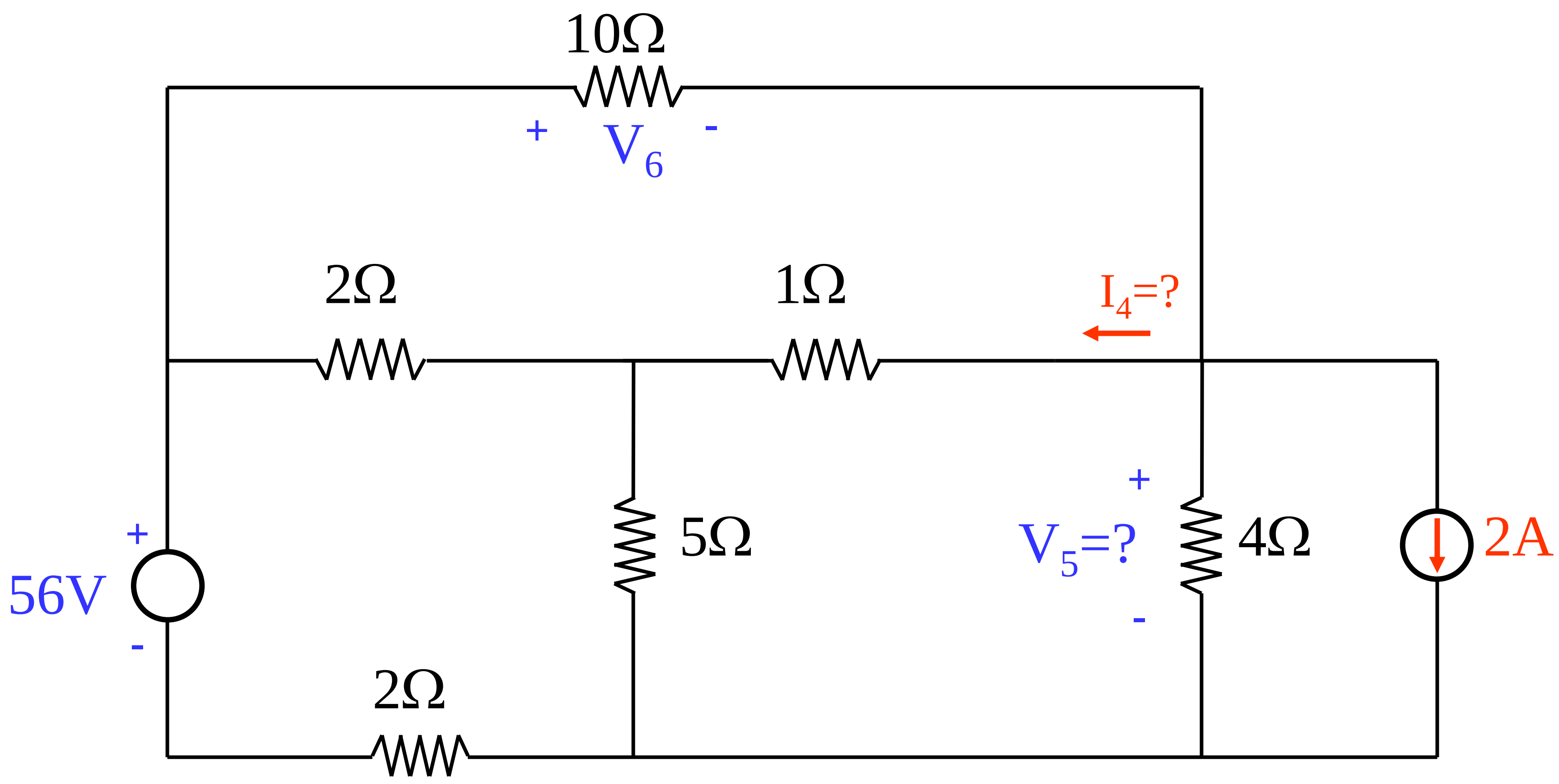
R_{XX}: Sum of the all resistors inside mesh X

R_{XY}: Sum of the all the resistors between X and Y meshes

E_X: Sum of the voltage sources inside mesh X

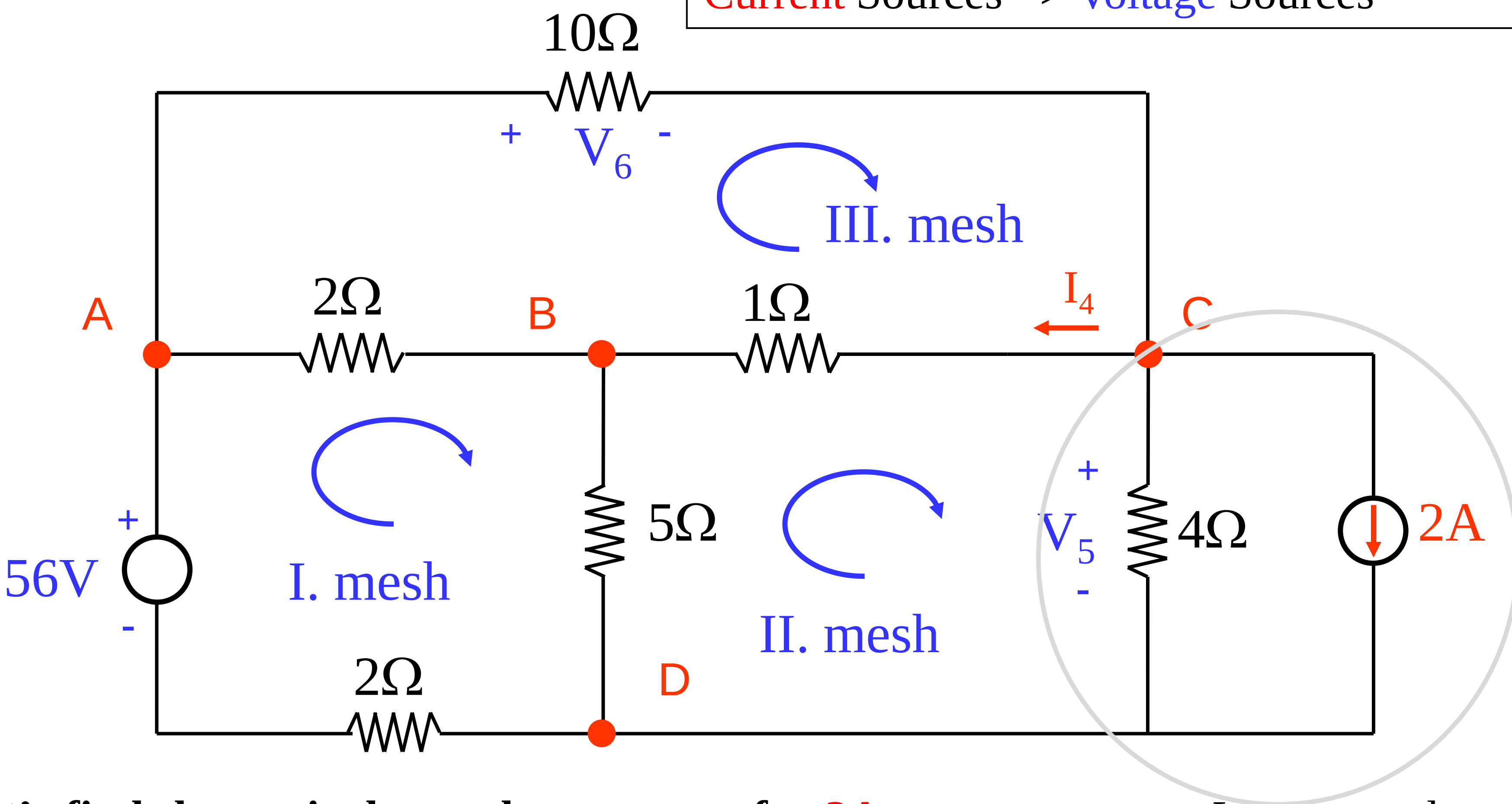
4. Step: The desired mesh currents are found from the common solution of the equations. Other currents and circuit voltages in the circuit can be found by applying Kirchhoff's Current Law and **Ohm's Law**.

Example-2.7: Using the Mesh Current Method, find the currents in the following circuit (mesh currents and I_4 current in the 1Ω resistor arm). Also calculate the V_5 voltage on 4Ω resistor.



Solution: First, we need to convert all the current sources in the circuit to a voltage source.

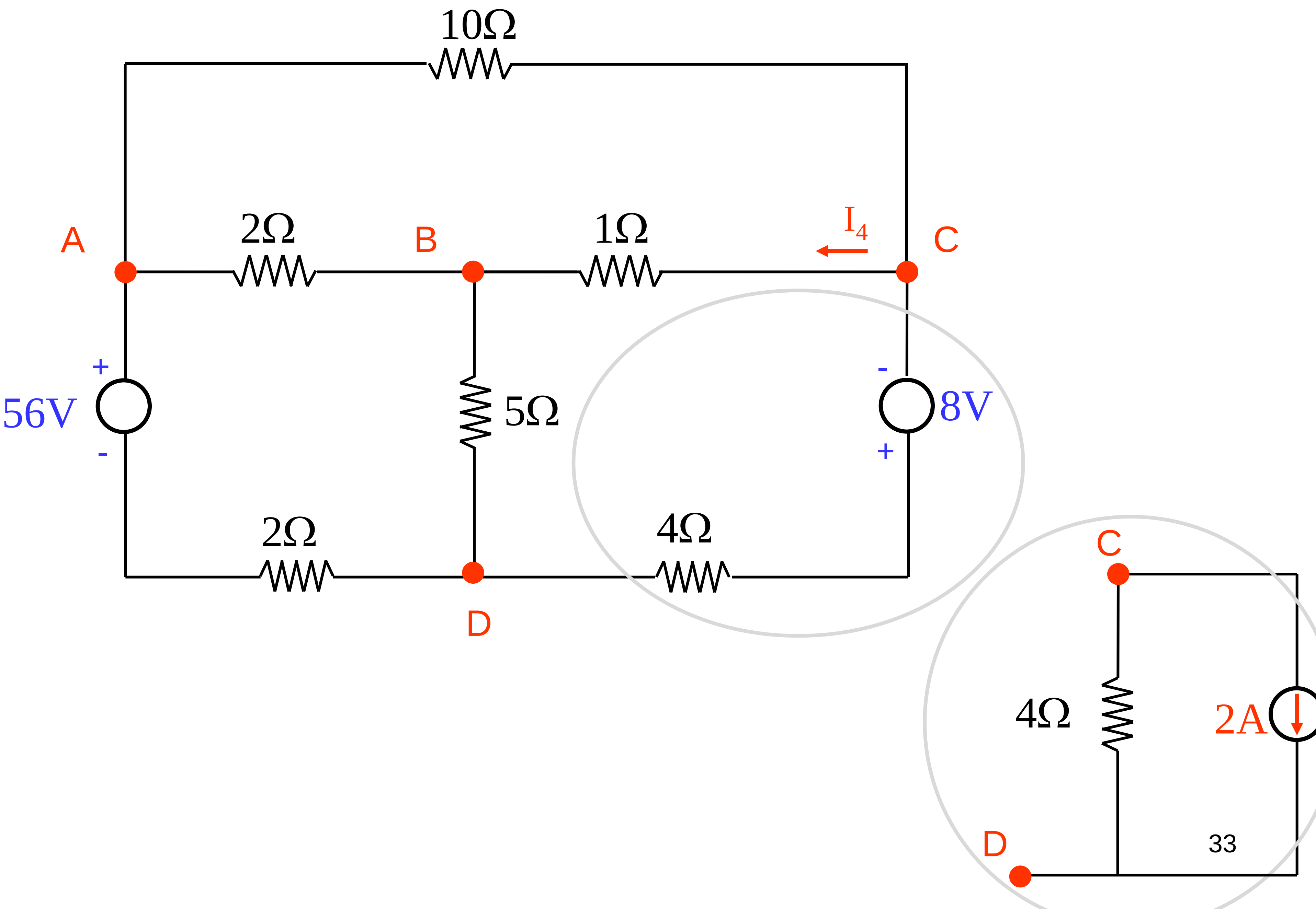
Current Sources => Voltage Sources



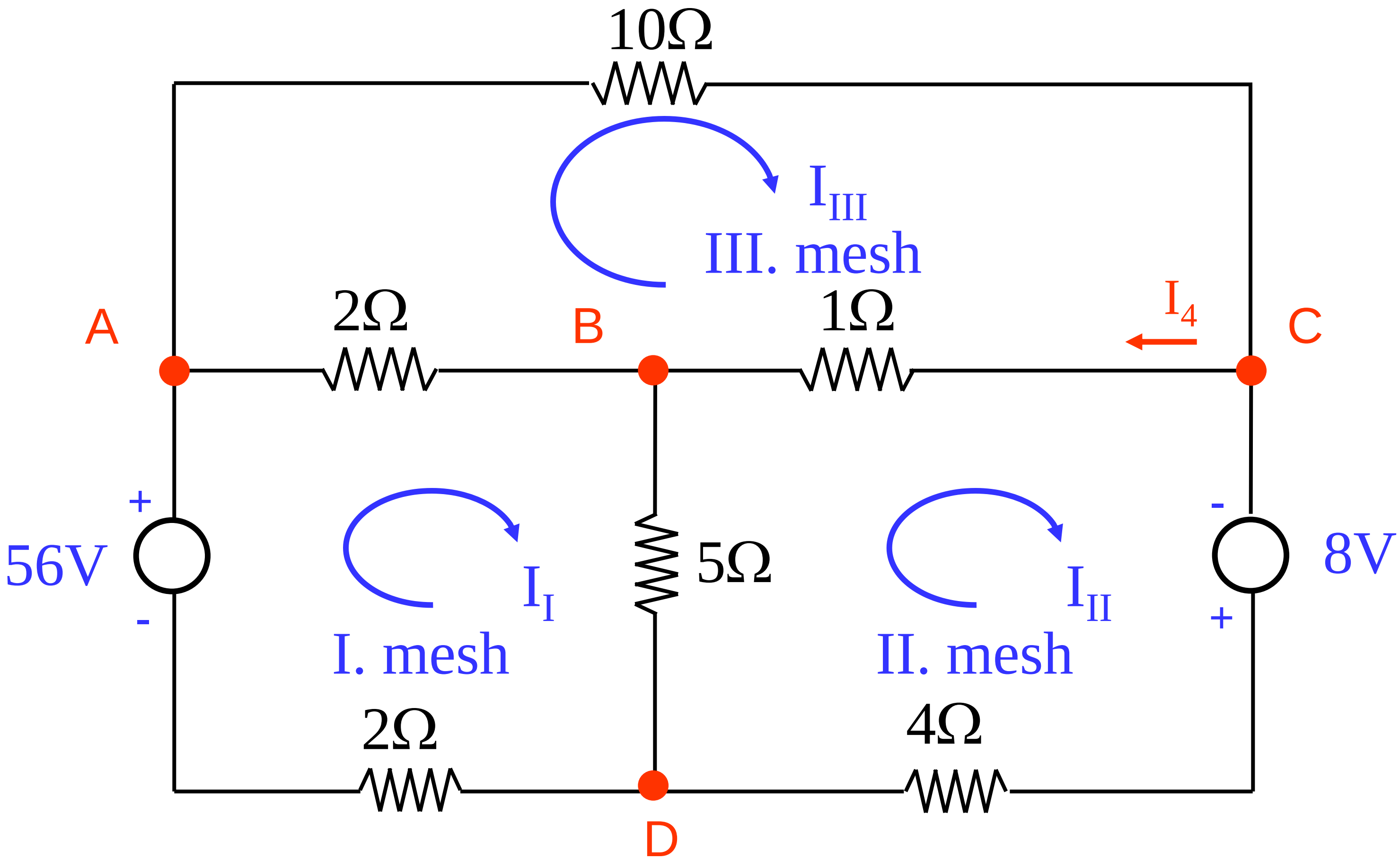
Let's find the equivalent voltage source for 2A current source: We can replace 2A current source and paralel 4Ω resistor with a voltage source and 4Ω serial resistor between **C** and **D** points.

$$E_o = R_o I \qquad R_o = 4\,\Omega \qquad E_o = (4\Omega)(2A) = 8V$$

The circuit after source conversion:



Next step is to define mesh ciurrents (**I**) :



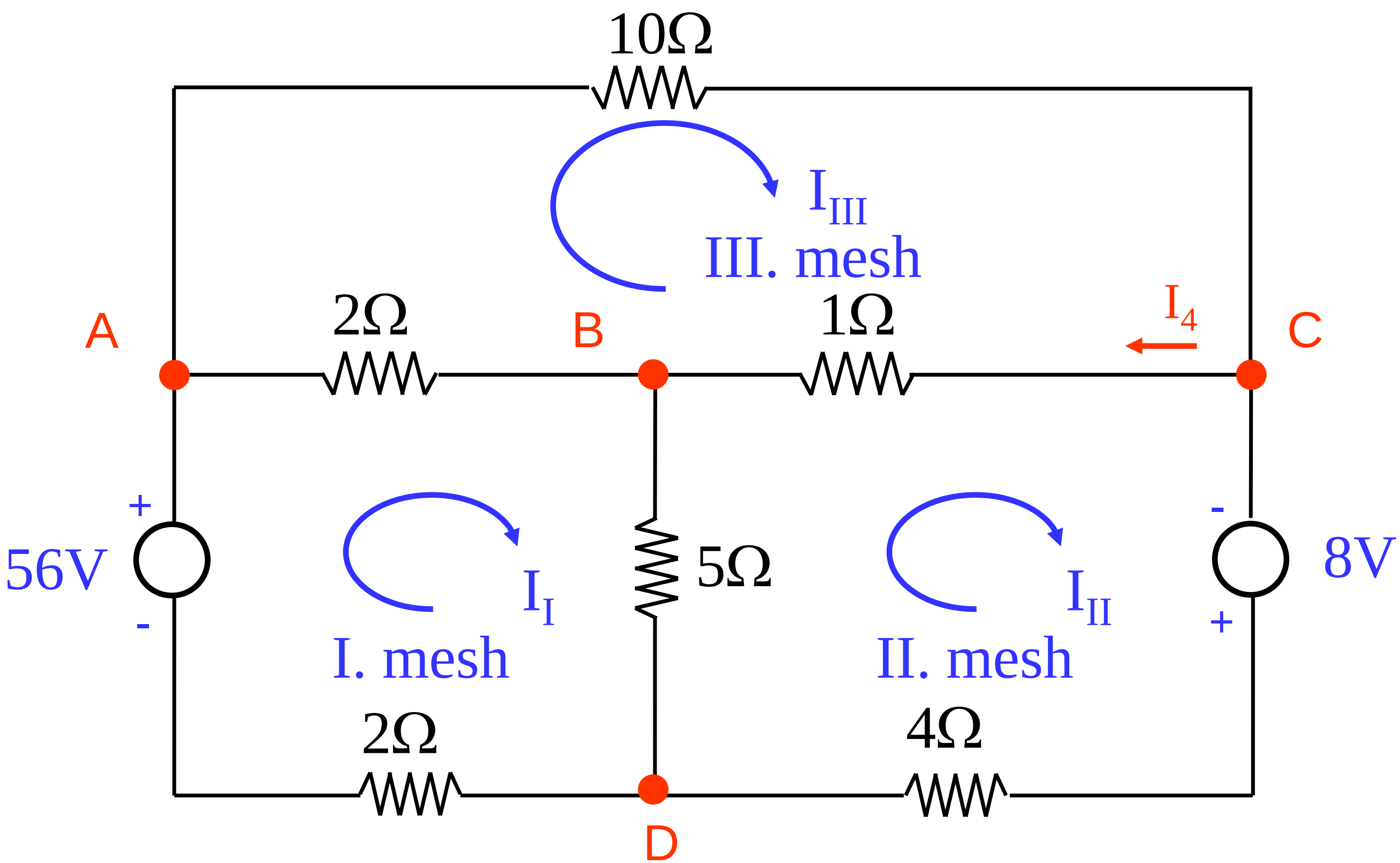
Kirchhoff's Voltage Law (KVL) for each mesh:

$$I: \quad +(\quad)I_I - (\quad)I_{II} - (\quad)I_{III} = E_I$$

$$II: \quad -(\quad)I_I + (\quad)I_{II} - (\quad)I_{III} = E_{II}$$

$$III: \quad -(\quad)I_I - (\quad)I_{II} + (\quad)I_{III} = E_{III}$$

Next step is to define mesh cıurrents (**I**) :



Kirchhoff's Voltage Law (KVL) for each mesh:

$$I: \quad +(2\Omega+5\Omega+2\Omega)I_I-(5\Omega)I_{II}-(2\Omega)I_{III}=56V$$

$$II: \quad -(5\Omega)I_I+(5\Omega+1\Omega+4\Omega)I_{II}-(1\Omega)I_{III}=8V$$

$$III: \quad -(2\Omega)I_I-(1\Omega)I_{II}+(2\Omega+1\Omega+10\Omega)I_{III}=0$$

Rearranging the equations:

$$I: \quad 9I_I - 5I_{II} - 2I_{III} = 56$$

$$II: \quad -5I_I + 10I_{II} - I_{III} = 8$$

$$III: \quad -2I_I - I_{II} + 13I_{III} = 0$$

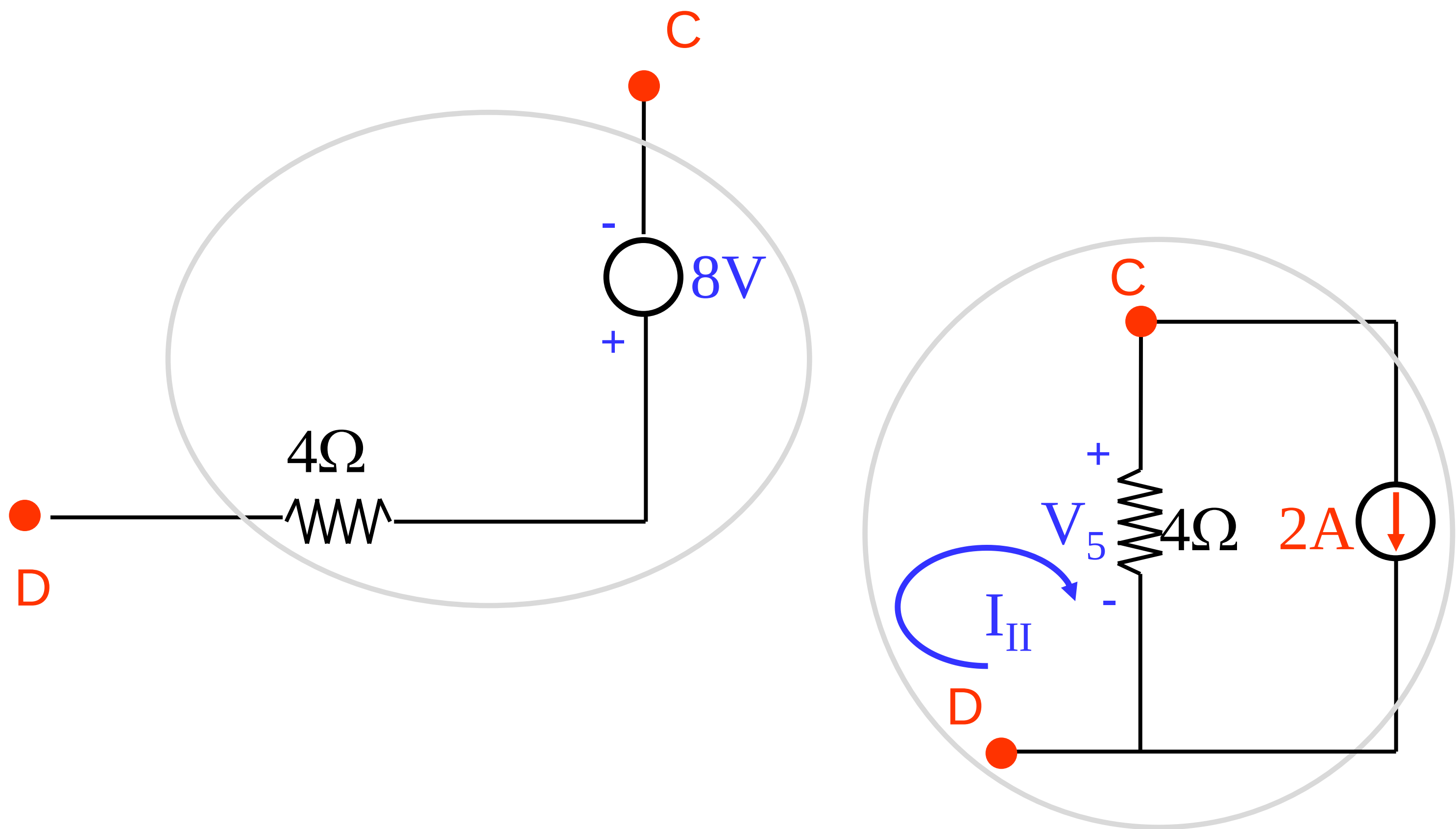
Solution for above equations:

$$I_I = 10A\,;\; I_{II} = 6A\;\;;\;\; I_{III} = 2A$$

We can find other currents on each arm using the mesh currents.

$$I_4 = I_{III} - I_{II} = 2A - 6A = -4A$$

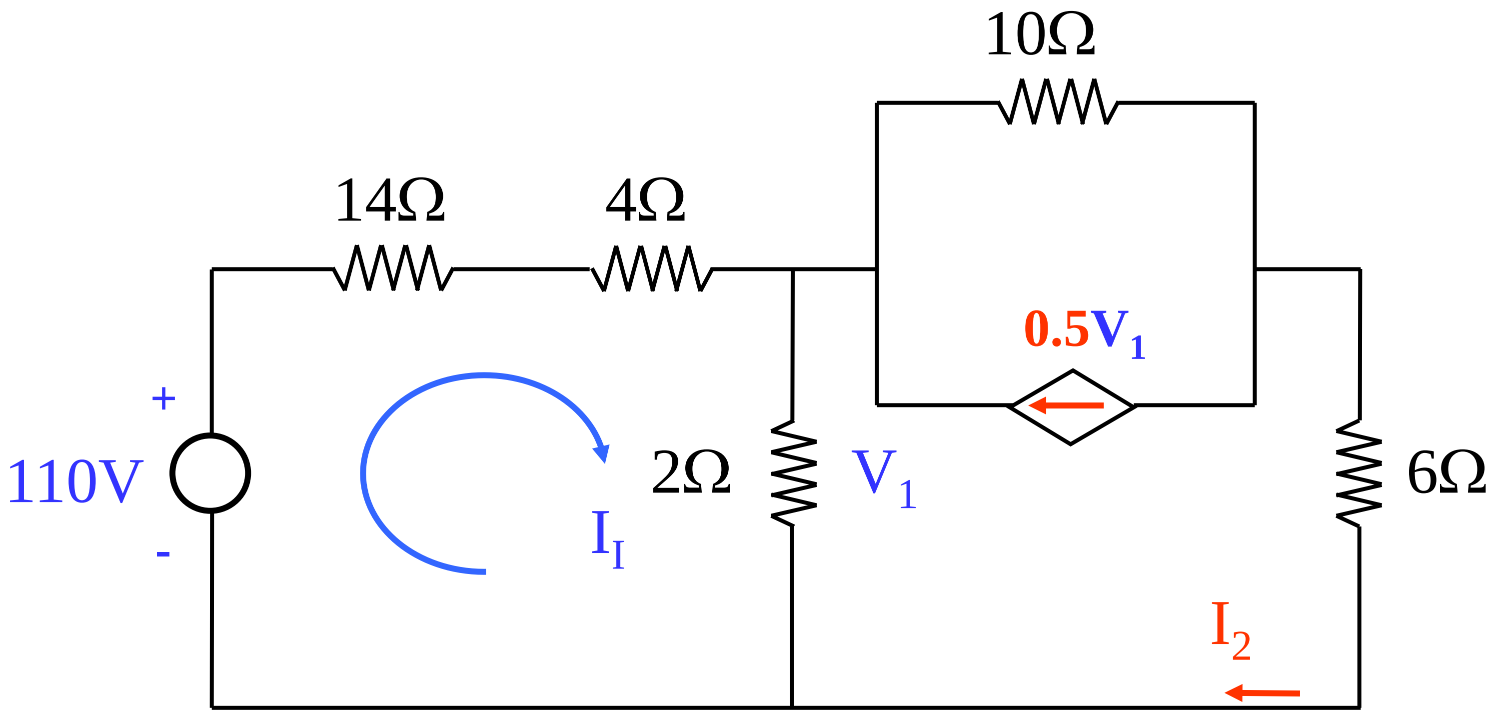
V_5 voltage is the voltage between point D and C.



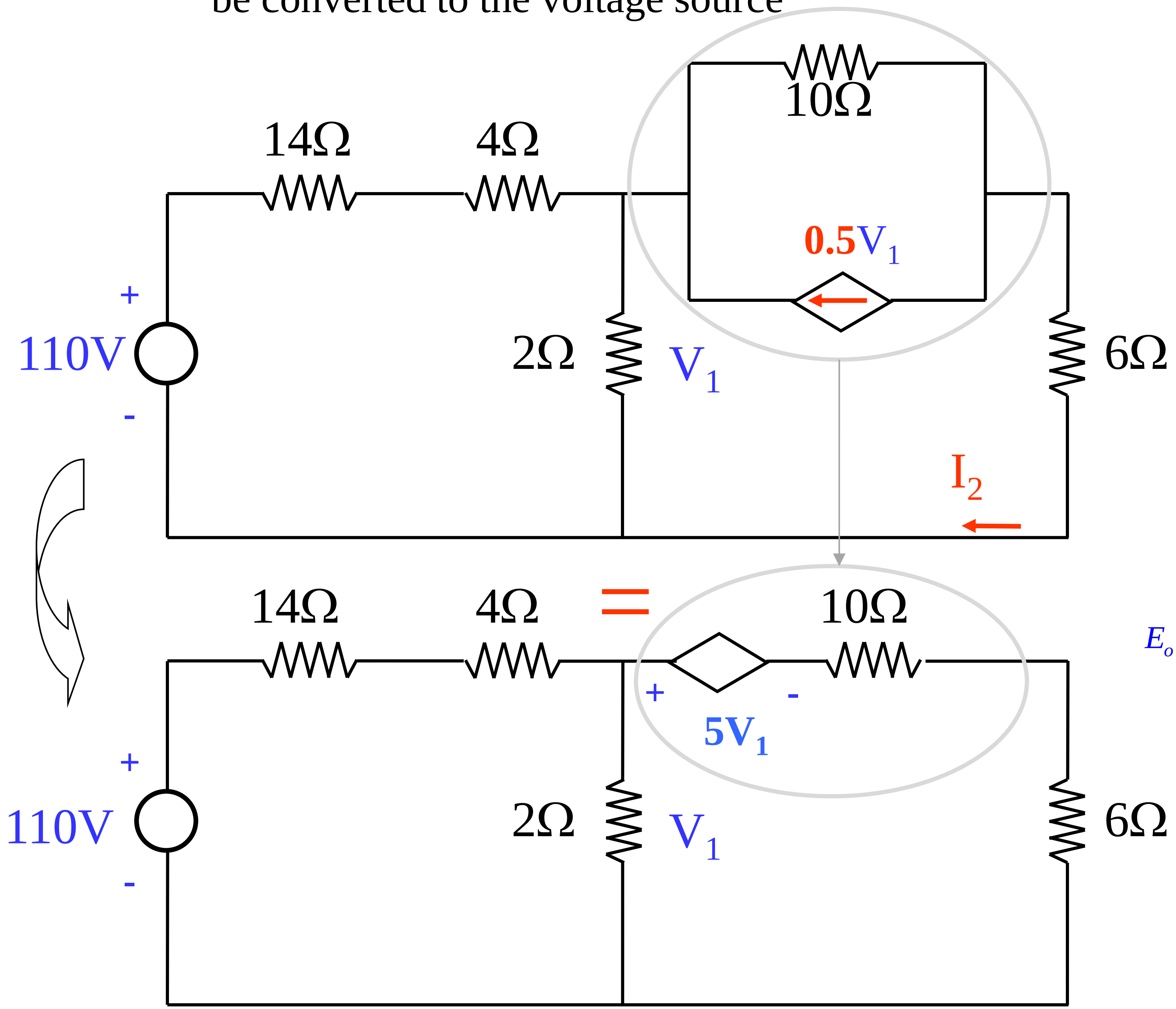
$$V_5 = (4\Omega) I_{II} - 8V = (4\Omega)(6A) - 8V = 16V$$

Voltage between DC node (after conversion to voltage source, voltage on 4Ω resistor and 8 V voltage source connected in series)

Example-2.9: Find the current I_I (mesh) and I_2 in the circuit below.



Solution: Mesh Current Method will be used. First the current source ($0.5V_1$) can be converted to the voltage source

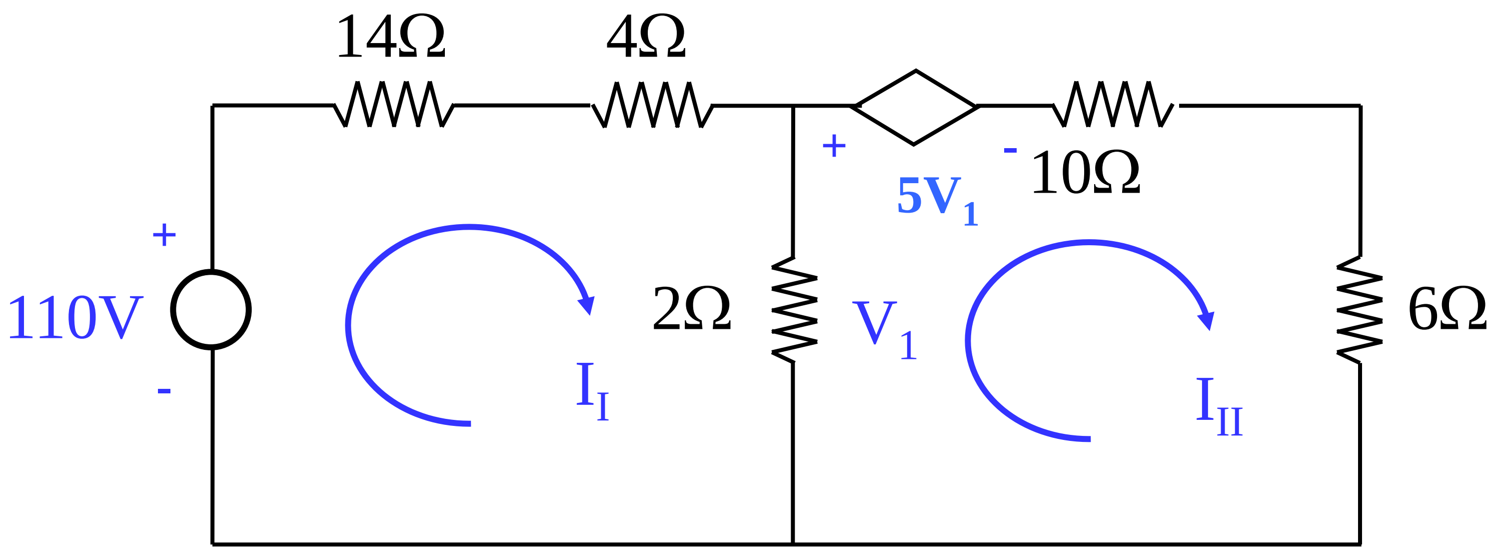


$$E_o = R_o I$$

$R_o = 10\Omega$

$$E_o = (10\Omega)(0.5V_1) = 5V_1$$

For meshes:



I. Mesh: $$(14\Omega + 4\Omega + 2\Omega) I_I - (2\Omega) I_{II} = 110V$$

II. Mesh: $$-(2\Omega) I_I + (2\Omega + 10\Omega + 6\Omega) I_{II} = -5V_1$$

Relation between V_1 and mesh current I_I and I_{II}: $$V_1 = (I_I - I_{II})(2\Omega)$$

If the eqs. are rearranged:

$$20I_I - 2I_{II} = 110$$

$$8I_I + 8I_{II} = 0$$

Solution of above equations:

$I_I = 5\ A$ $I_{II} = -5\ A$ found.

Since I_{II} is negative, it is understood that the current circulating in the second loop circulates in the opposite direction to the selected direction.