Ankara University
Engineering Faculty
Department of Engineering Physics

PEN207

Circuit Design and Analysis

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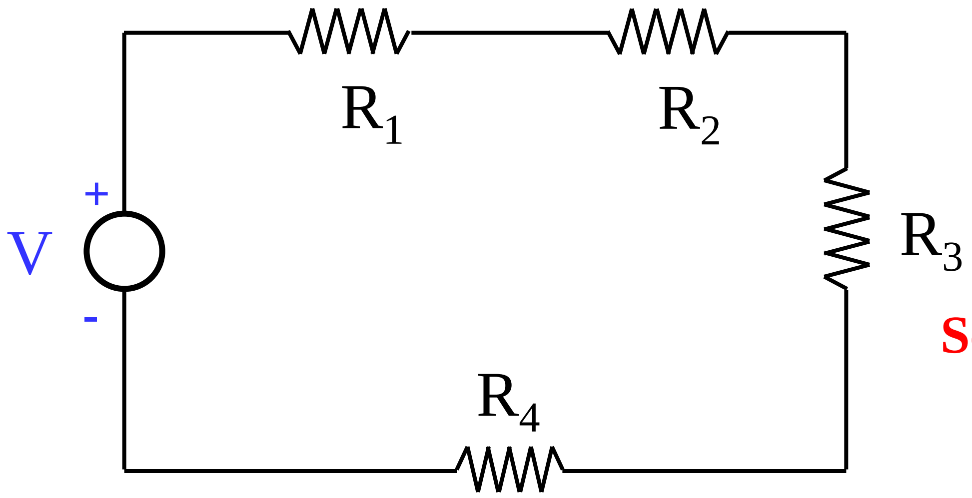
Chapter-2

Methods of Circuit Analysis and Circuit Theorems (3/3)

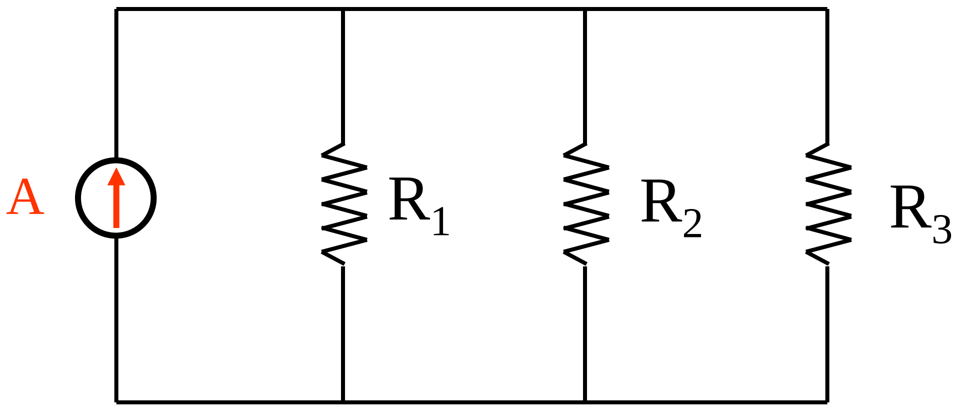
Circuit Reduction

Before we start…



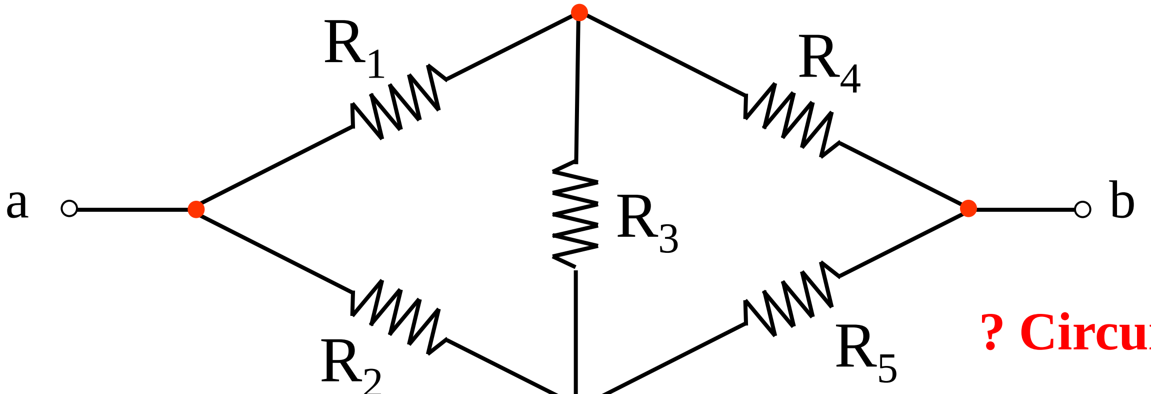
R_{eq}=?

Series Circuit



R_{eq}=?

Parallel Circuit



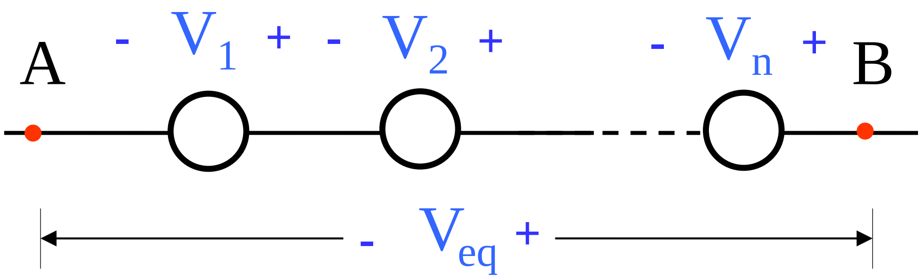
R_{eq}=?

? Circuit

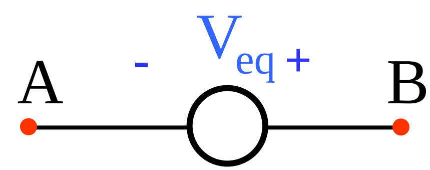
Circuit Reduction-Series Circuits

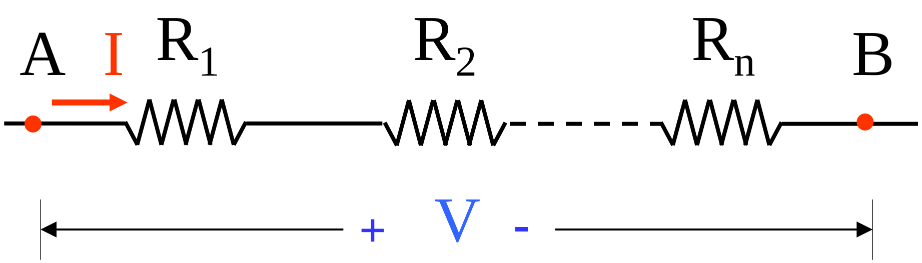
One of the methods used to reduce circuit complexity is circuit reduction. The circuit to be reduced may contain sources or circuit elements.

1- **<u>In a series circuits</u>** <u>common quantity is</u> **<u>current</u>**.



$$V_{equivalent} = V_1 + V_2 + V_3 + \ldots\ldots + V_n$$

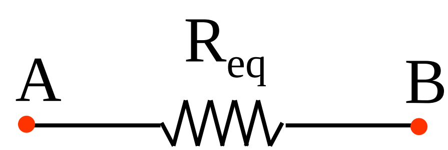




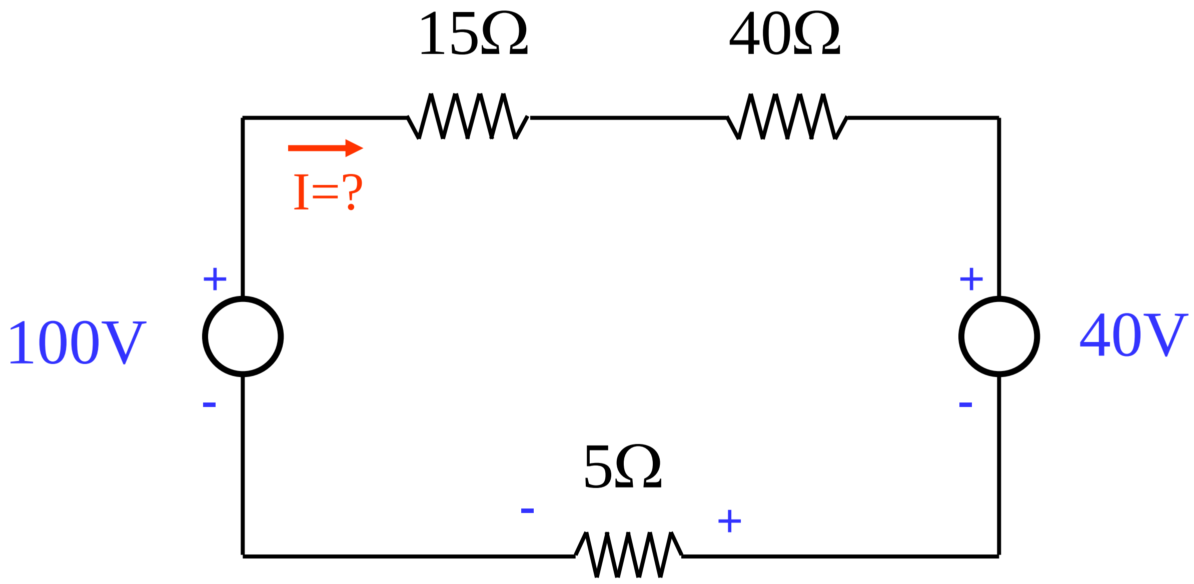
$$V = IR_1 + IR_2 + IR_3 + \ldots\ldots + IR_n$$

$$V = I(R_1 + R_2 + R_3 + \ldots\ldots + R_n) = IR_{eq}$$

$$R_{equivalent} \equiv R_1 + R_2 + R_3 + \ldots\ldots + R_n$$



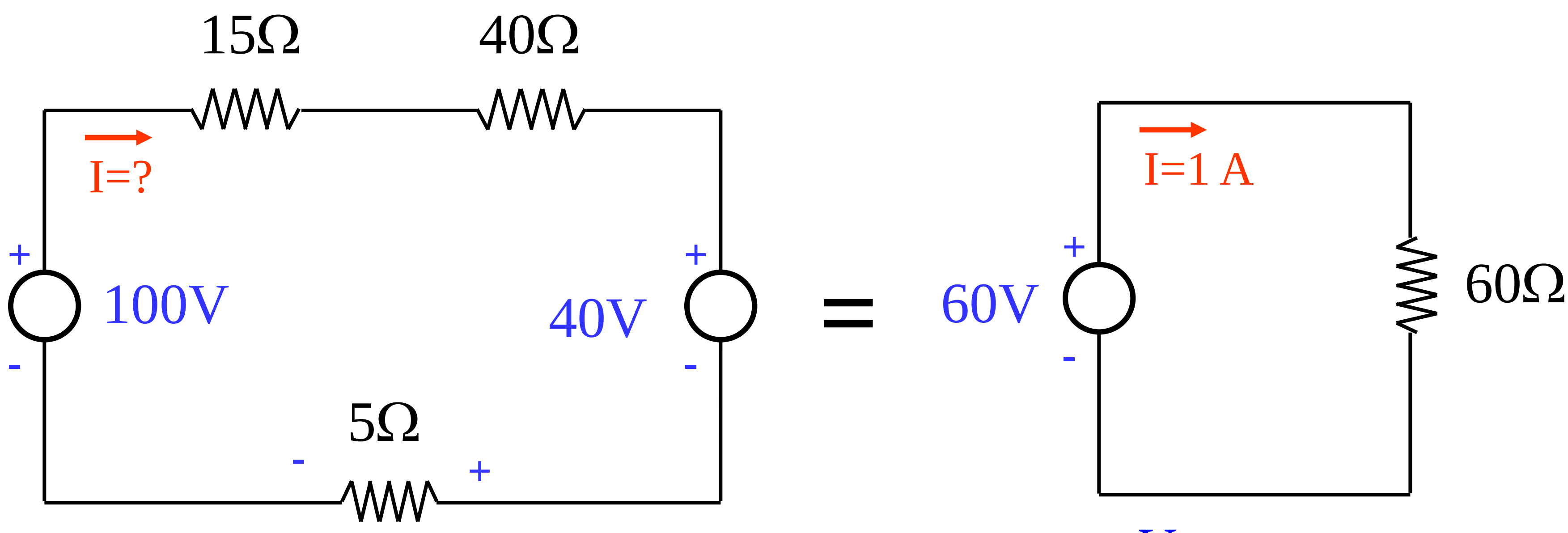
Example-2.10: Find the current I in the following circuit.



Solution:

Equivalent voltage source: $E_{eq} = 100V - 40V = 60V$

Equivalent Resistor: $R_{eq} = 15\Omega + 40\Omega + 5\Omega = 60\,\Omega$ (in series)

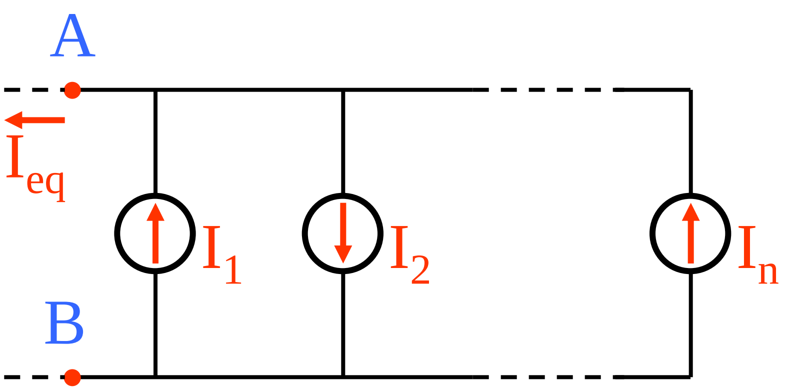


$$I = \frac{V_{eq}}{R_{eq}} = \frac{60V}{60\Omega} = 1A$$

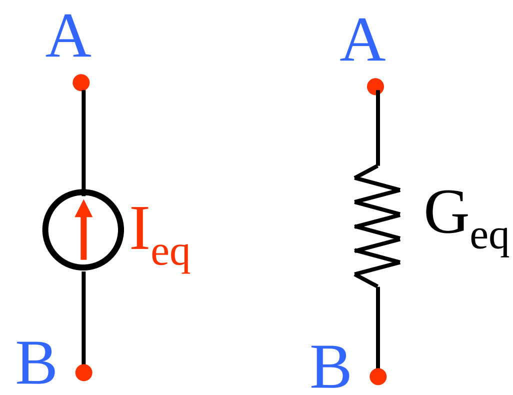
Circuit Reduction-Parallel Circuits

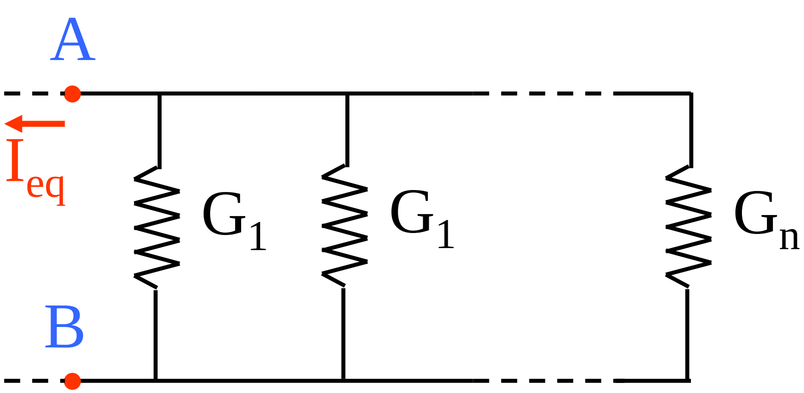
Another connection is a parallel connection. Parallel connections of circuit elements and sources are shown below.

2- <u>Common quantity in **parallel circuit** is</u> **voltage**.



$$I_{equivalent} = I_1 + I_2 + I_3 + ... + I_n$$





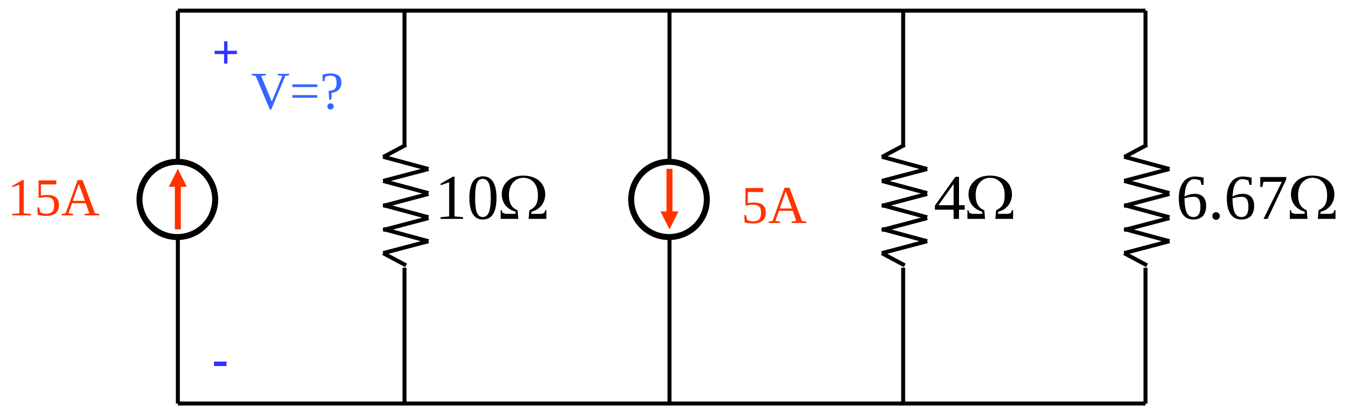
$$I = VG_1 + VG_2 + VG_3 + ... + VG_n$$

$$I = V(G_1 + G_2 + G_3 + ... + G_n)$$

$$G_{eqiavalent} \equiv G_1 + G_2 + G_3 + ... + G_n$$

$$G_{equivalent} = \frac{1}{R_{eş}} \equiv \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$

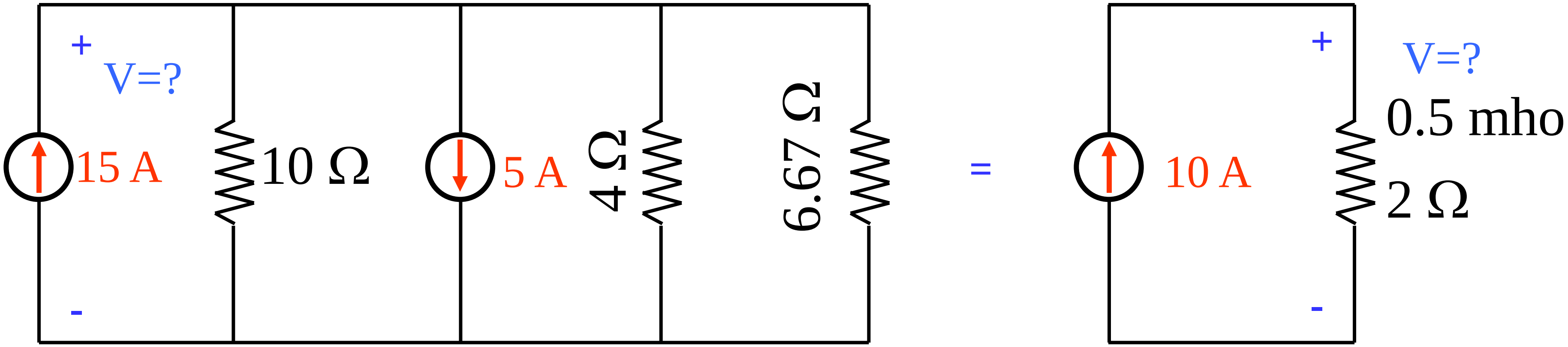
Example-2.11: Find the voltage between the ends of the parallel circuit below.



Solution:

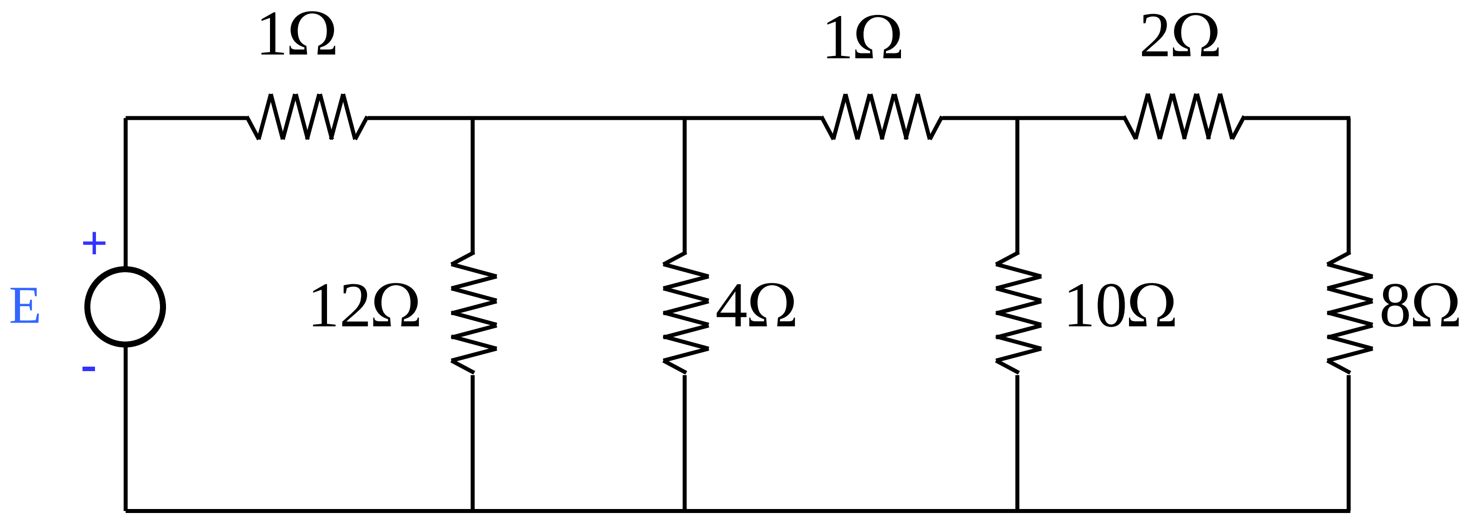
Equivalent current: $I_{eq} = 15A - 5A = 10A$

Equivalent resistor $G_{eq} = \frac{1}{10} + \frac{1}{4} + \frac{1}{6.67} = 0.5\,mho \Rightarrow R_{eq} = 2\,\Omega$

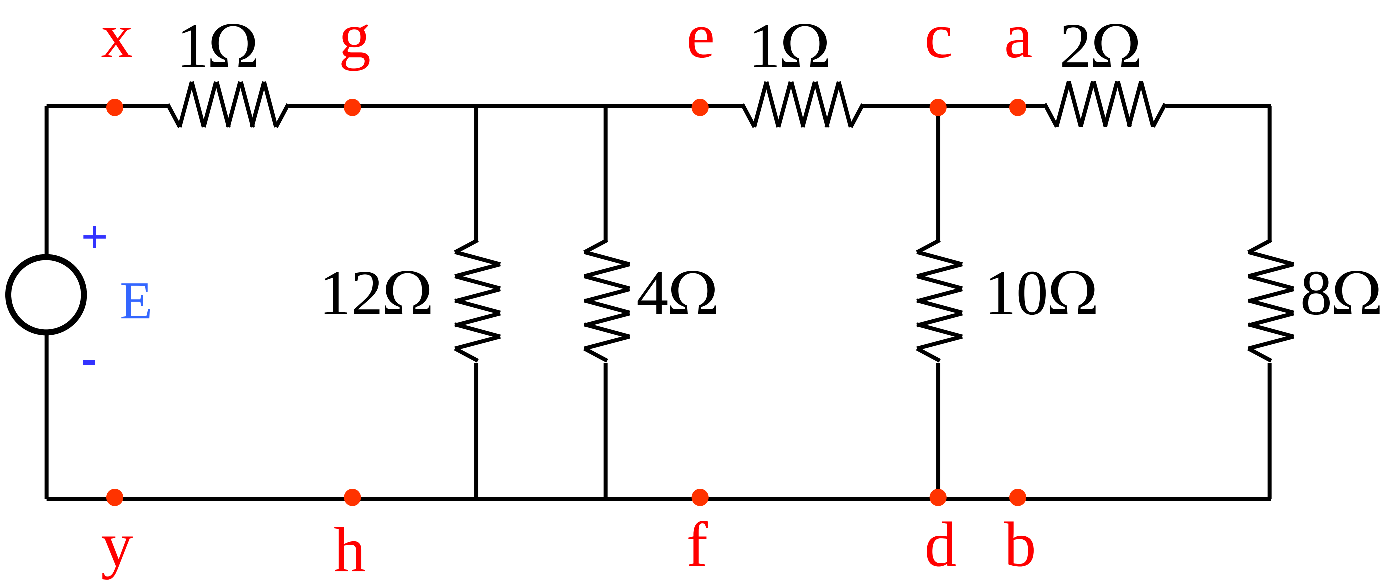


Voltage: $V = I_{eq} R_{eq} = (10A)(2\,\Omega) = 20V$

Example-2.12: In the following circuit find the equivalent resistor (R).

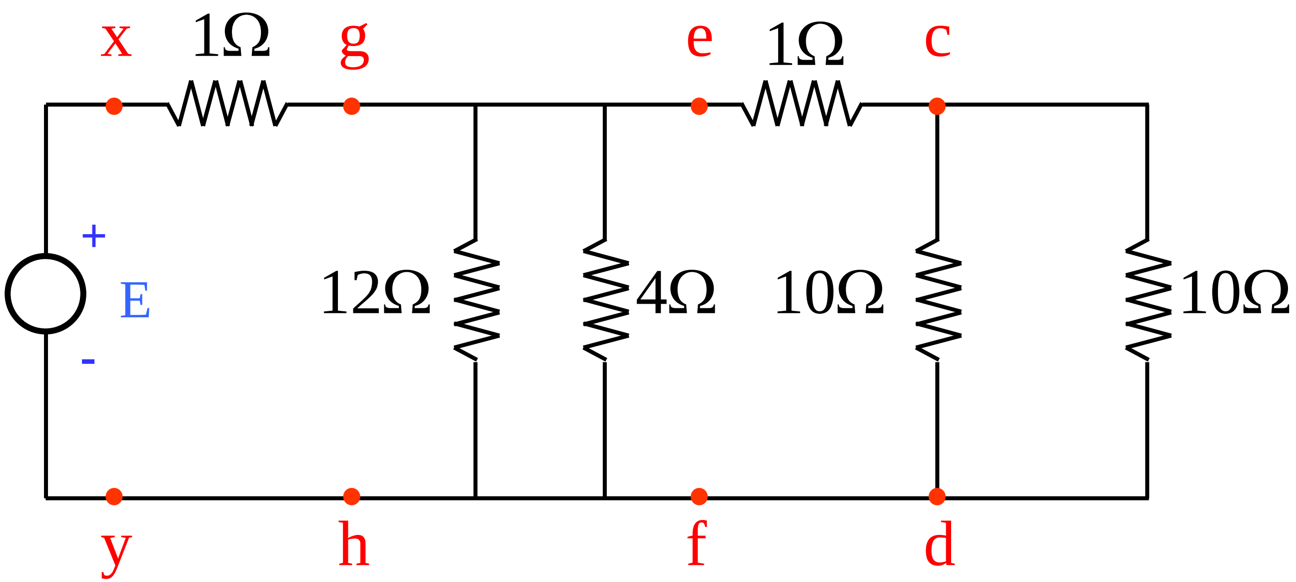


Solution: When applying the circuit reduction method, starting from the point of the the resistors are connected farthest away from the source and going to the source.

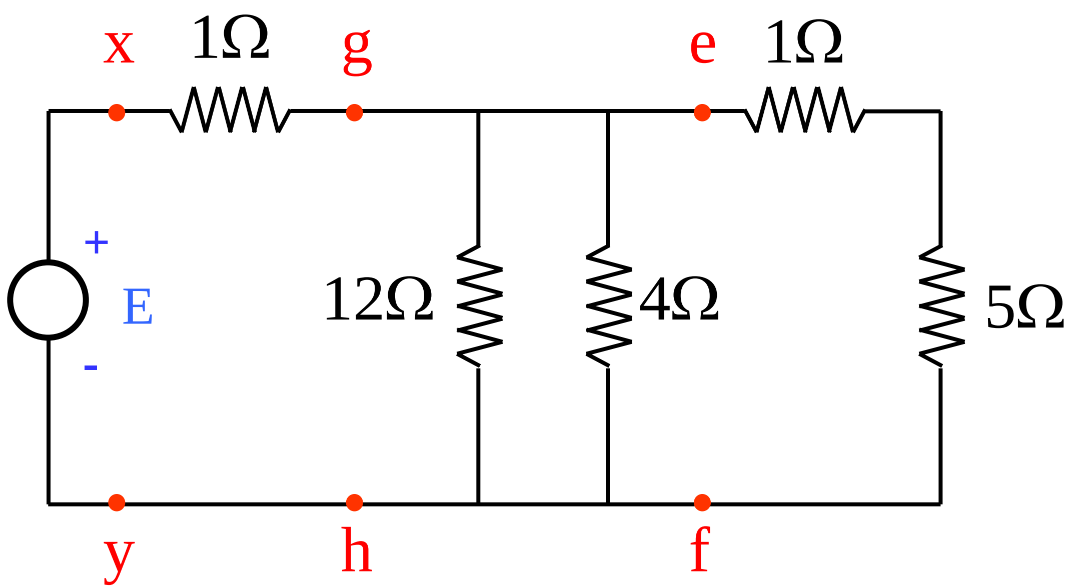


Between a-b (2Ω and 8Ω resistors are in series).

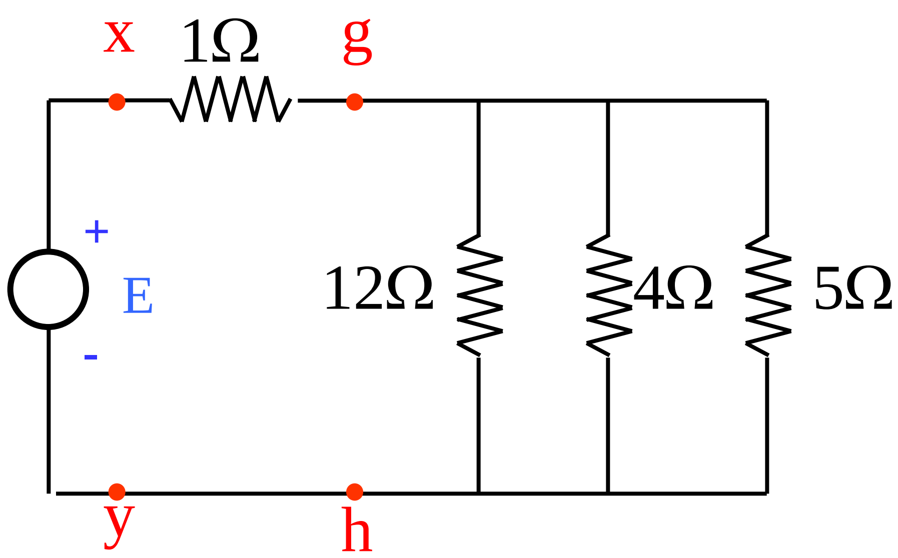
$$R_{ab} = 2\,\Omega + 8\,\Omega = 10\,\Omega$$



Between c-d (10Ω and 10Ω resistors are in parallel) : $R_{cd} = \dfrac{(10\Omega)(10\Omega)}{10\Omega + 10\Omega} = 5\,\Omega$

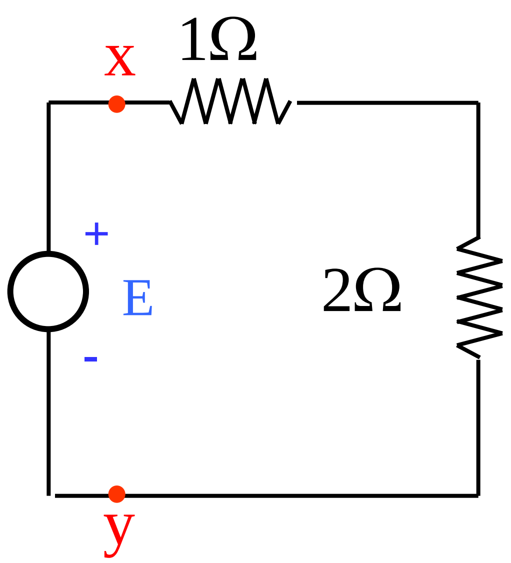


Between e-f (5Ω and 1Ω resistors are in series): $R_{ef} = 5\,\Omega + 1\Omega = 6\,\Omega$

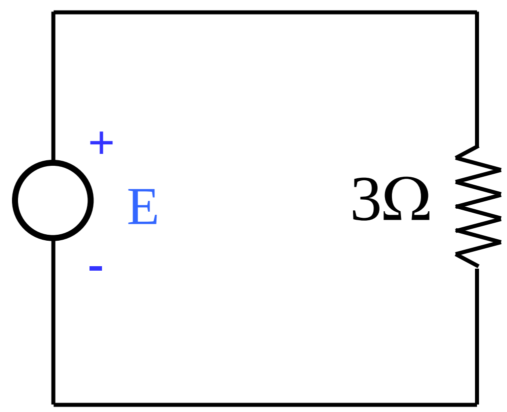


Between g-h (12Ω, 4Ω and 5Ω resistors are in parallel).

$$\frac{1}{R_{gh}} = \frac{1}{6\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{12\,\Omega} = \frac{1}{2\,\Omega} \Rightarrow R_{gh} = 2\,\Omega$$

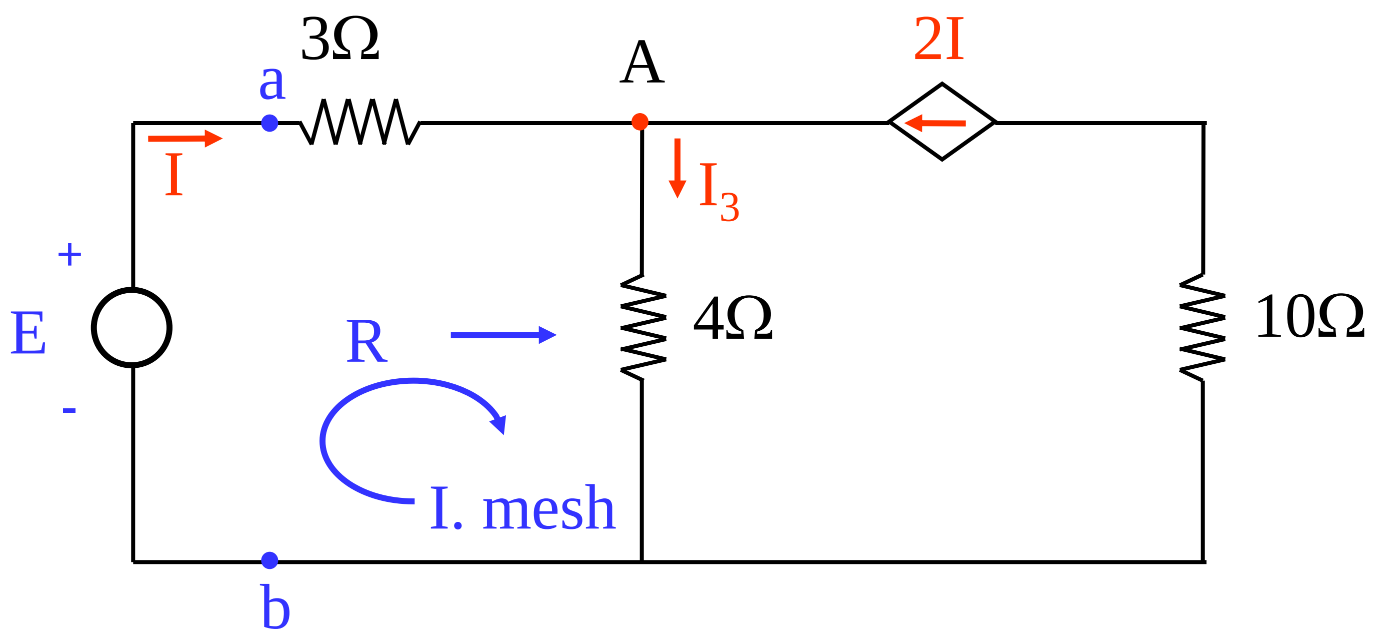


Between x-y (2Ω and 1Ω resistors are in series).



$$R_{xy} = 2\,\Omega + 1\,\Omega = 3\,\Omega$$

Example-2.13: Find the resistor (R_{ab}).

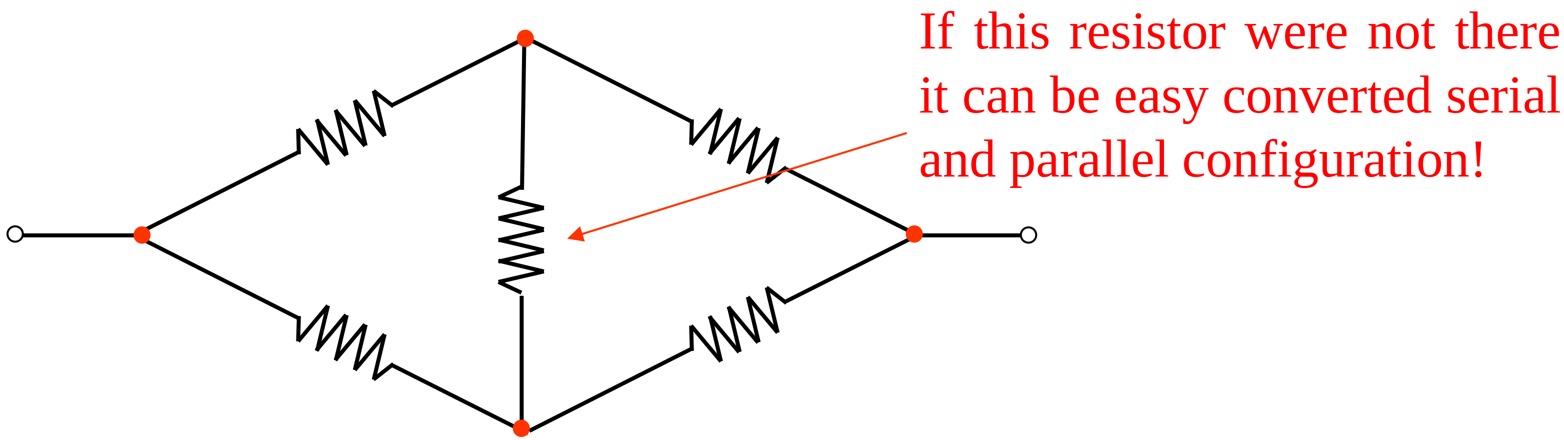


Solution: KCL at point A we can find current I_3 $$I_3 = I + 2I = 3I$$

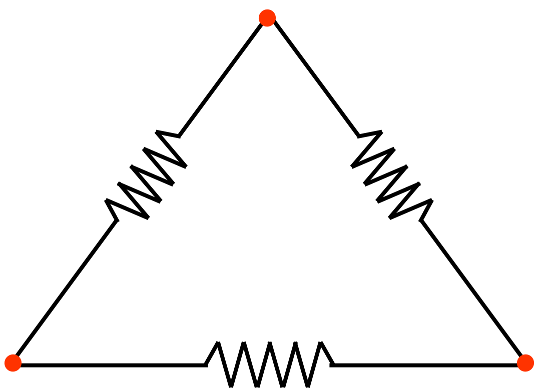
KVL for the mesh at left: $$E = 3I + 4(3I) = 15I$$

Resistor: $$R = \frac{E}{I} = 15\ \Omega$$

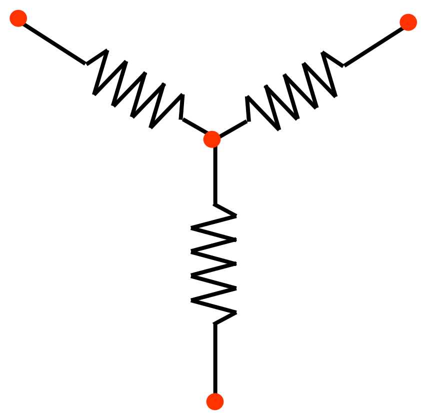
3- There are **certain circuit configuration** that cannot be solved only by serial and parallel connections. These transformations can often be resolved by using Y–Δ conversion. For example, the circuit below is neither fully serial nor fully parallel.



This conversion allows the three Y-connected resistors to be connected to the Δ-shaped and vice versa.



Δ-Configuration
(Π–Configuration)

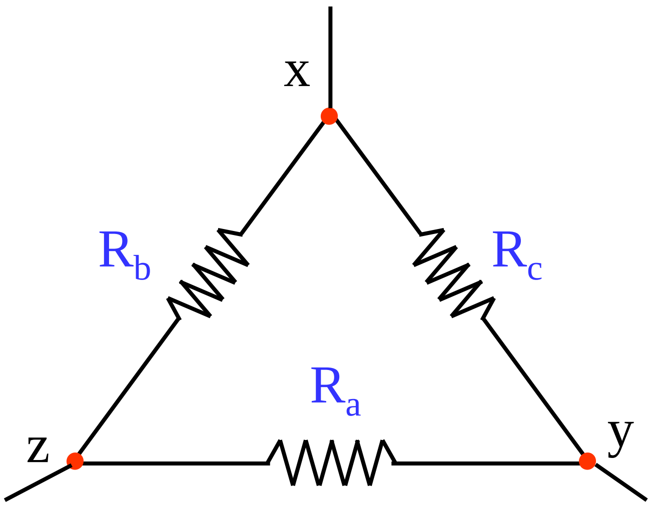


Y-Configuration
(T-Configuration)

Y-Δ (T-Π) Conversions

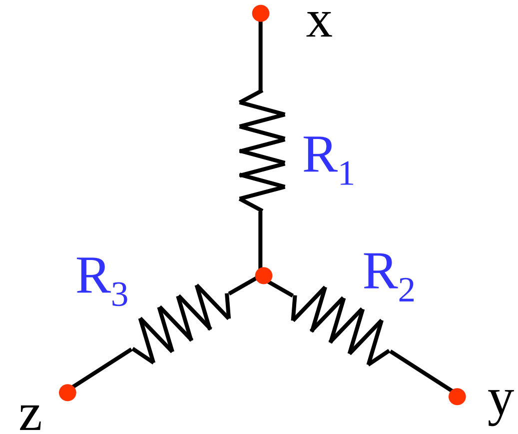
This conversion allows Y-connected three resistors to be converted to the Δ-connected three resistors.

Δ- or Π- Configuration



$\leftrightarrow$

Y- or T- Configuration



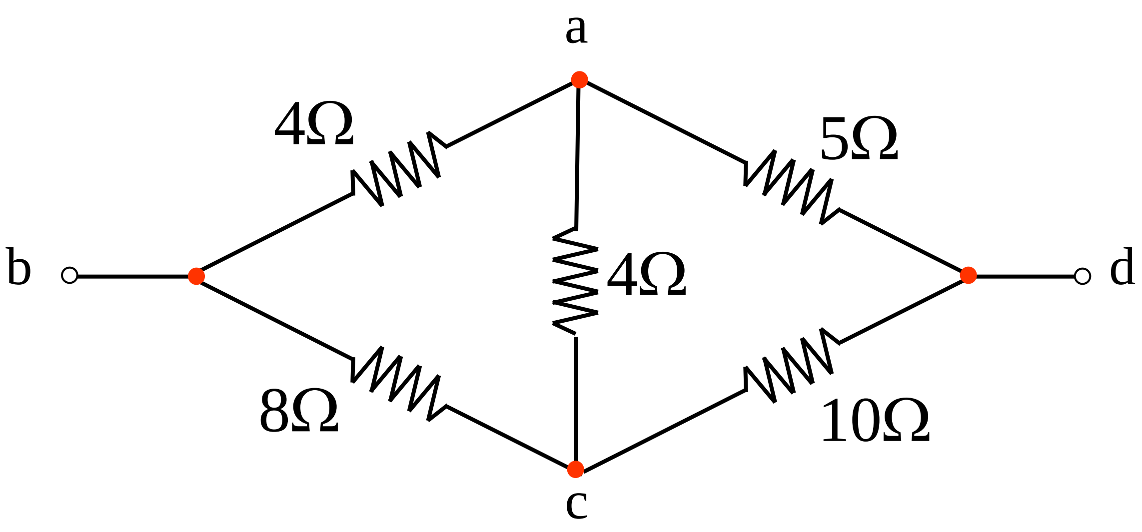
$$R_{xy} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{(R_a + R_b) + R_c}$$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

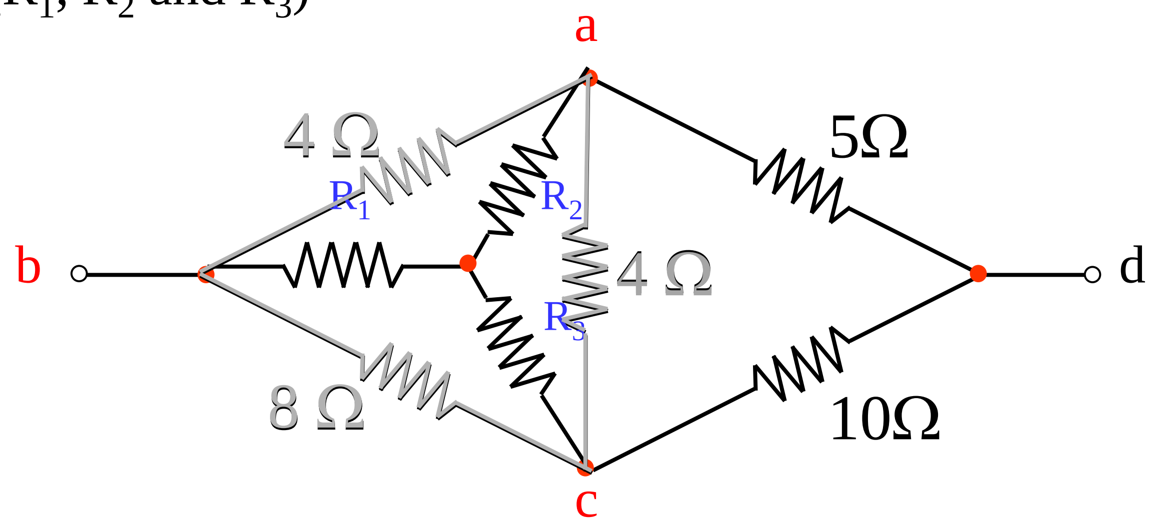
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \quad \leftrightarrow \quad R_2 = \frac{R_aR_c}{R_a + R_b + R_c}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \qquad R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$

Example-2.14: Find the equivalent resistor between the b-d terminals



Solutions: If the Δ-resistors connected to the point a-b-c in the circuit is converted to Y-resistors by the Δ -> Y conversion, the new resistors (R_1, R_2 and R_3)



R_a=4 Ω, R_b =8 Ω, R_c=4 Ω

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

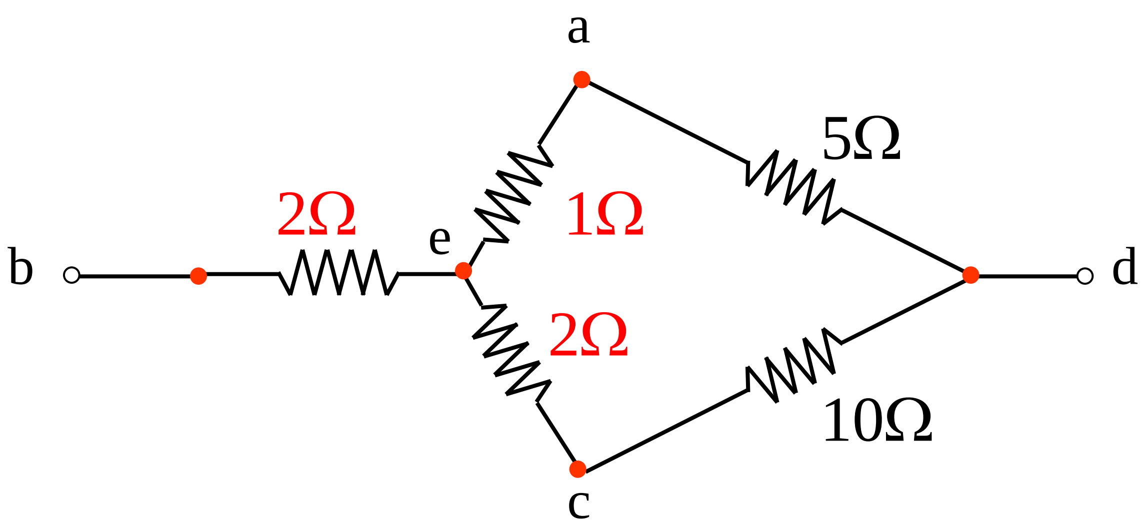
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_1 = \frac{(8\,\Omega)(4\,\Omega)}{4\,\Omega + 4\,\Omega + 8\,\Omega} = 2\,\Omega$$

$$R_2 = \frac{(4\,\Omega)(4\,\Omega)}{4\,\Omega + 4\,\Omega + 8\,\Omega} = 1\,\Omega$$

$$R_3 = \frac{(4\,\Omega)(8\,\Omega)}{4\,\Omega + 4\,\Omega + 8\,\Omega} = 2\,\Omega$$

After the conversion, the new circuit can be simplified with serial and parallel connections.



$$R_{ead} = 1\,\Omega + 5\,\Omega = 6\,\Omega$$

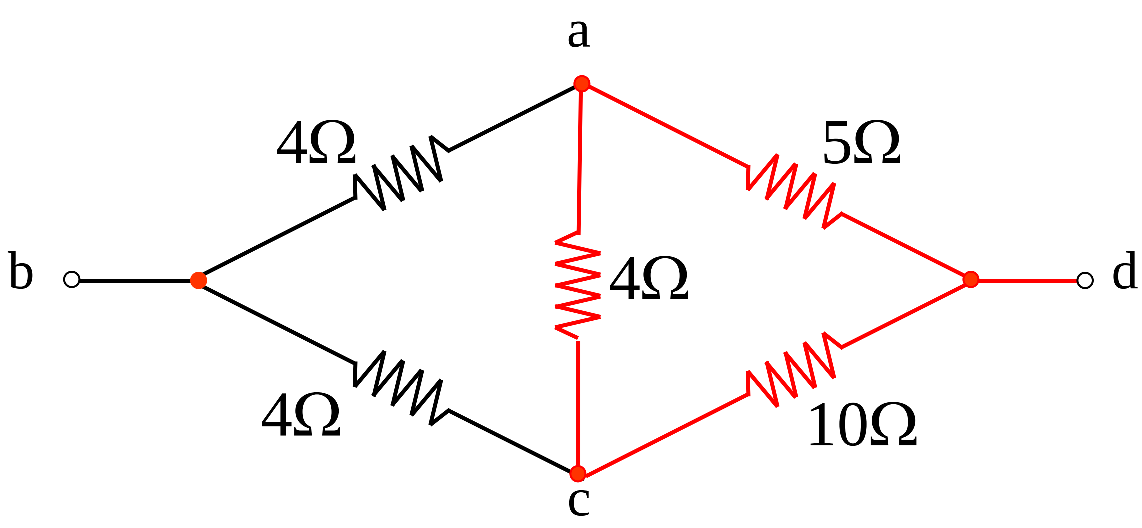
$$R_{ecd} = 2\,\Omega + 10\,\Omega = 12\,\Omega$$

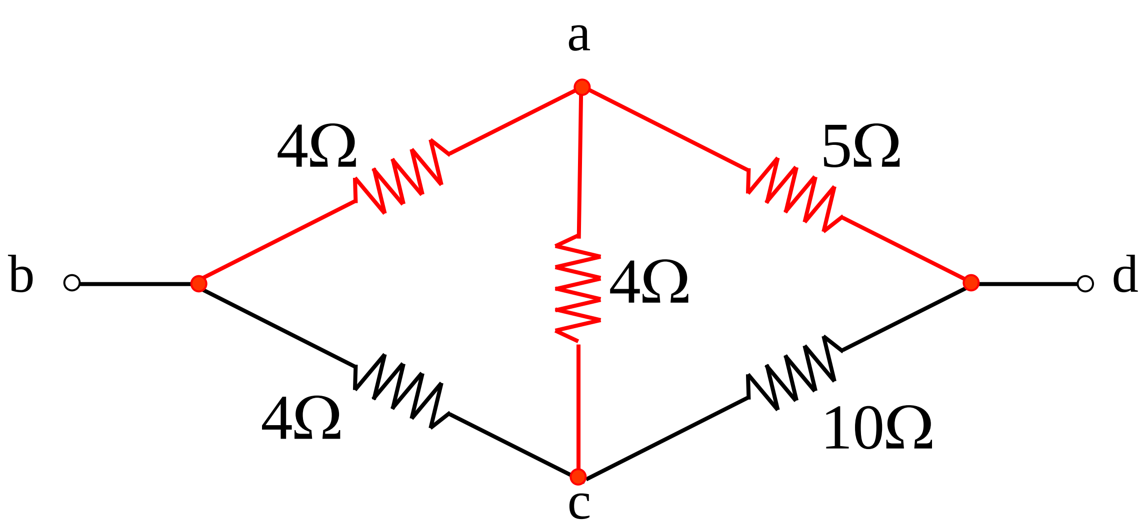
$$R_{ed} = \frac{(6\,\Omega)(12\,\Omega)}{6\,\Omega + 12\,\Omega} = 4\,\Omega$$

$$R_{bd} = 2\,\Omega + 4\,\Omega = 6\,\Omega$$

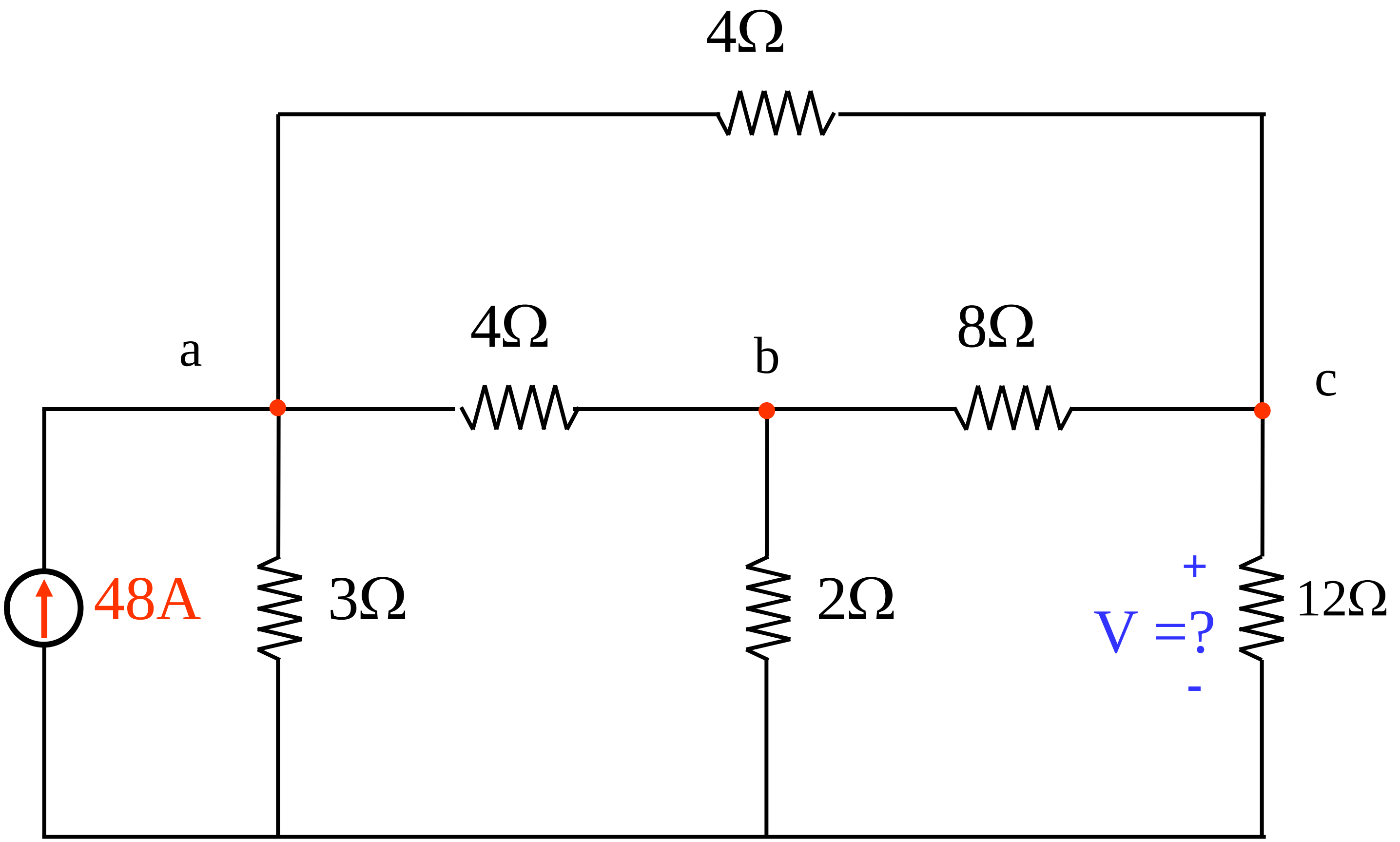
Homevork : (a) Find the equivalent resistor between the b-d terminals by convertint the Δ -shape the **acd** resistors to Y-converion.

(b) Convert the Y-resistor connected to the points **bcd** to Δ-shape.

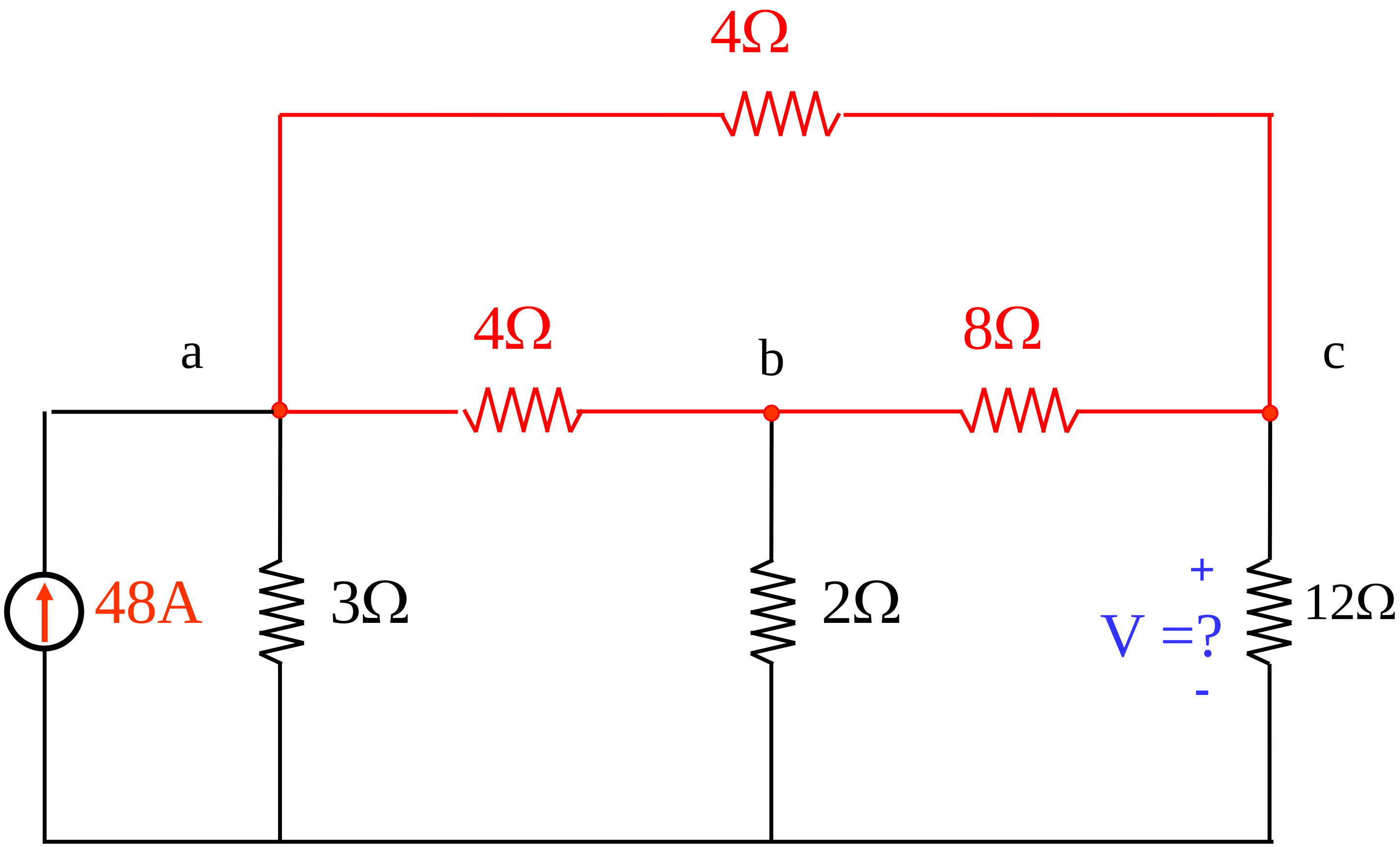


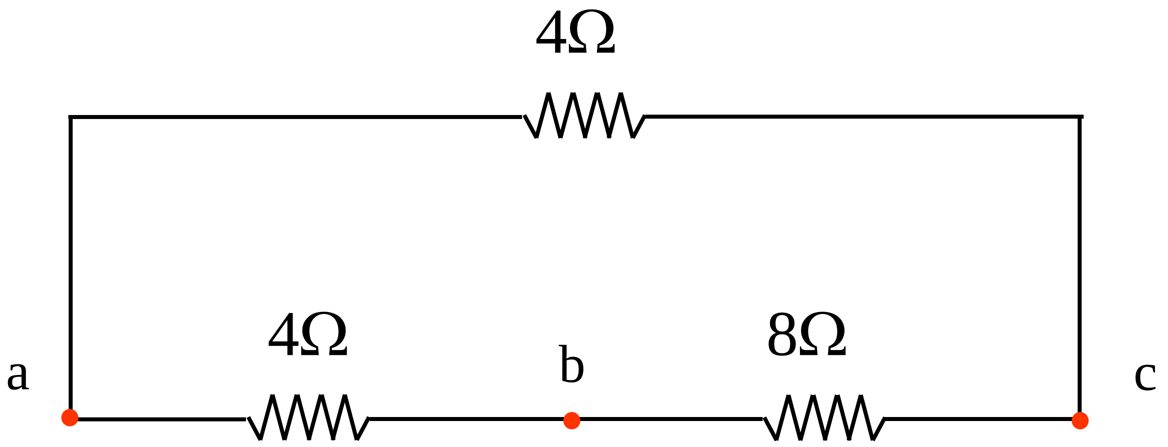


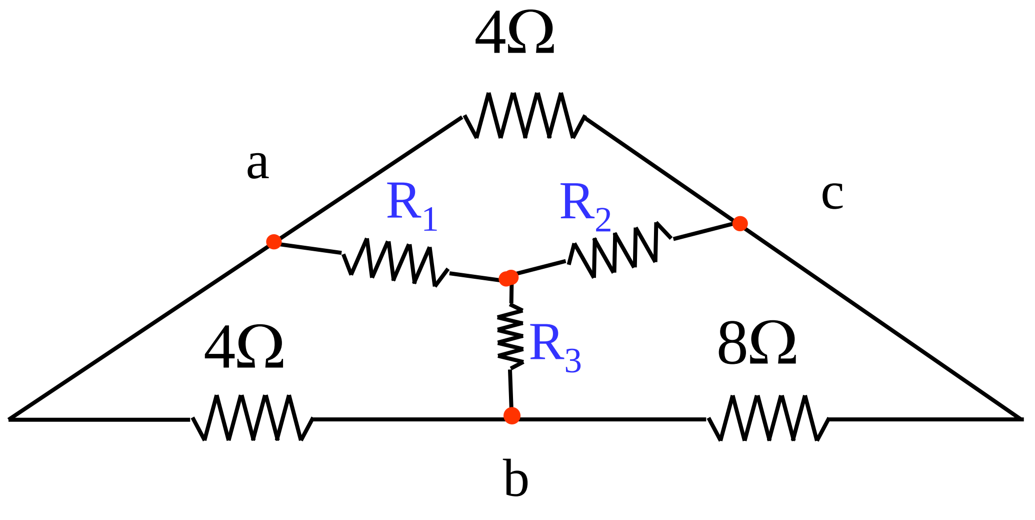
Example-2.15: Using the circuit reduction method, find the voltage V in the following circuit.

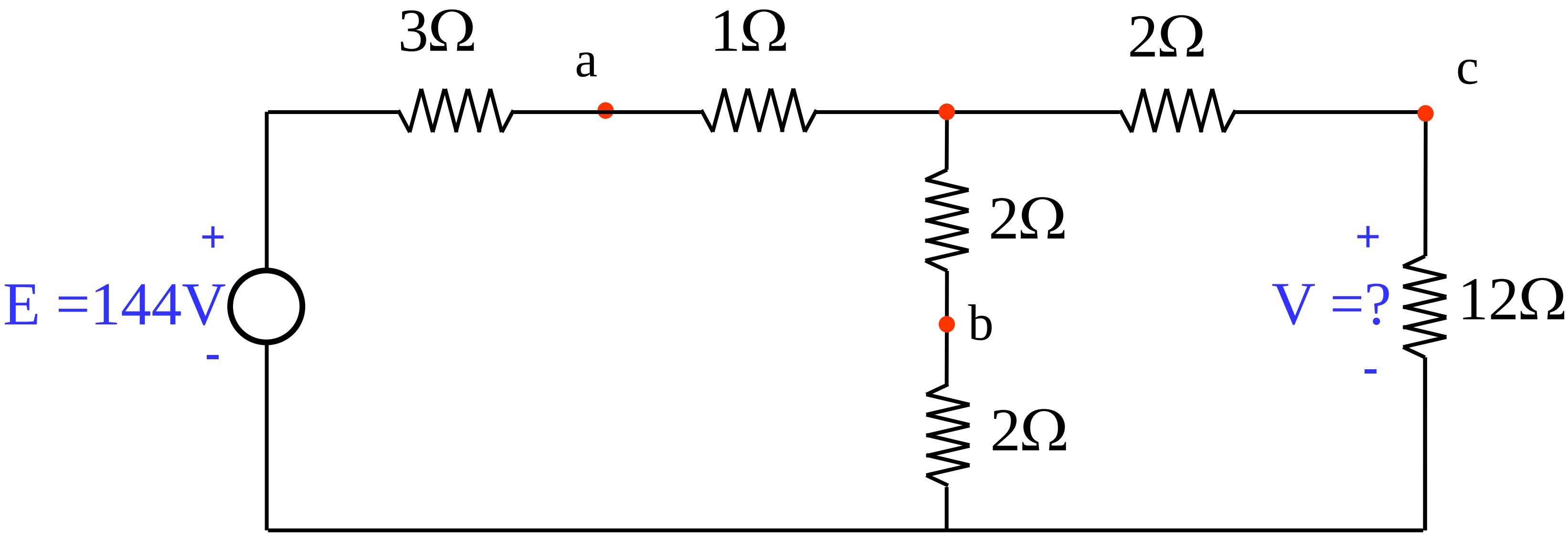


Solution: Converting resistors Δ-connected to a, b and c to Y-shape. Then converting current sourse 48A and 3Ω to voltage source

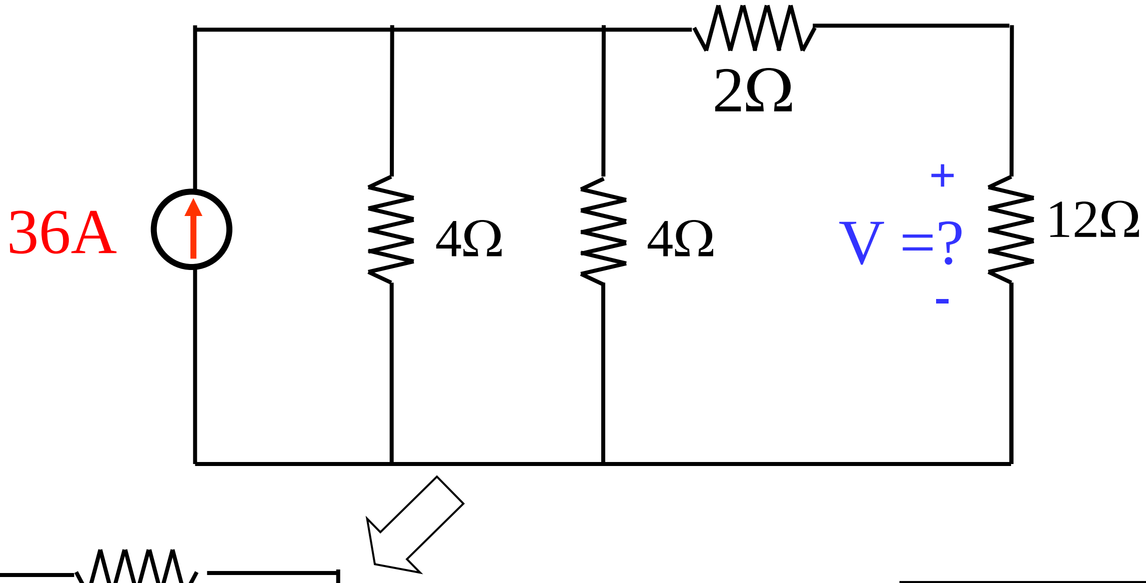



4Ω
4Ω
8Ω
a
b
c

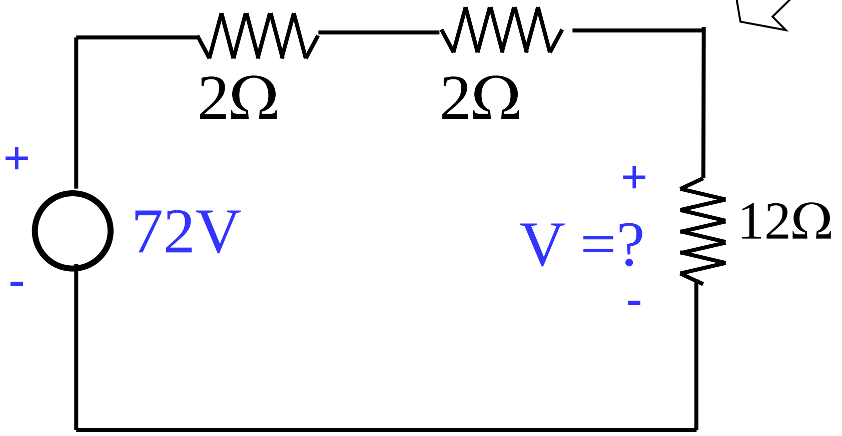

4Ω
a
R_1
R_2
c
4Ω
R_3
8Ω
b



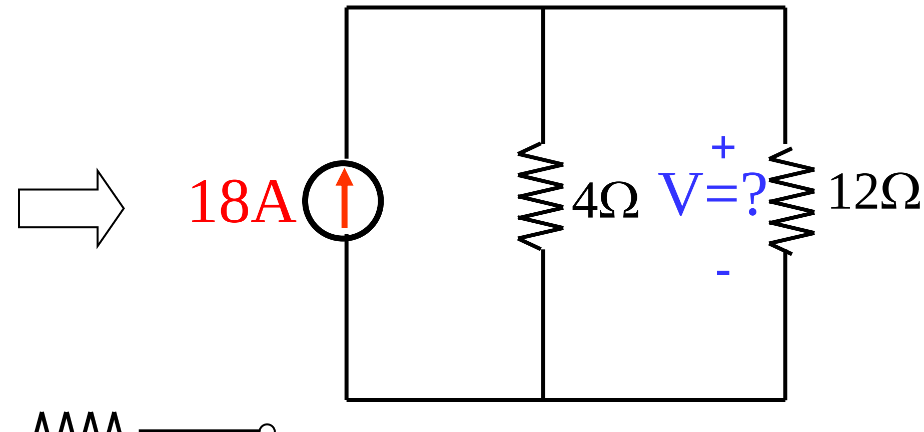
3Ω
a
1Ω
2Ω
c
+
E =144V
-
2Ω
b
2Ω
+
V =?
-
12Ω



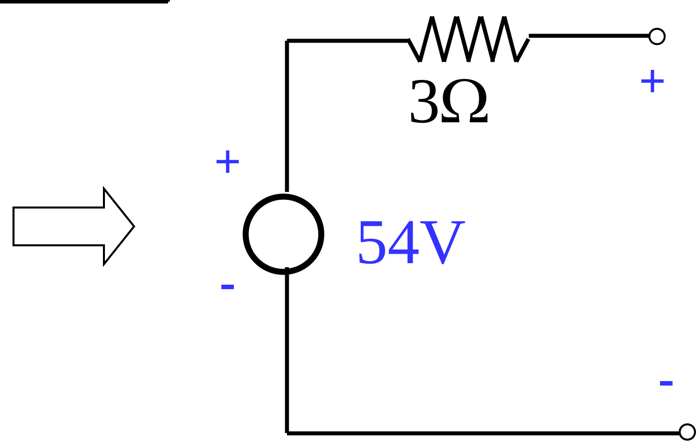
2Ω
36A
4Ω
4Ω
+
V =?
-
12Ω



2Ω
2Ω
+
72V
-
+
V =?
-
12Ω



18A
4Ω
+
V=?
-
12Ω



3Ω
+
54V
-
+
-

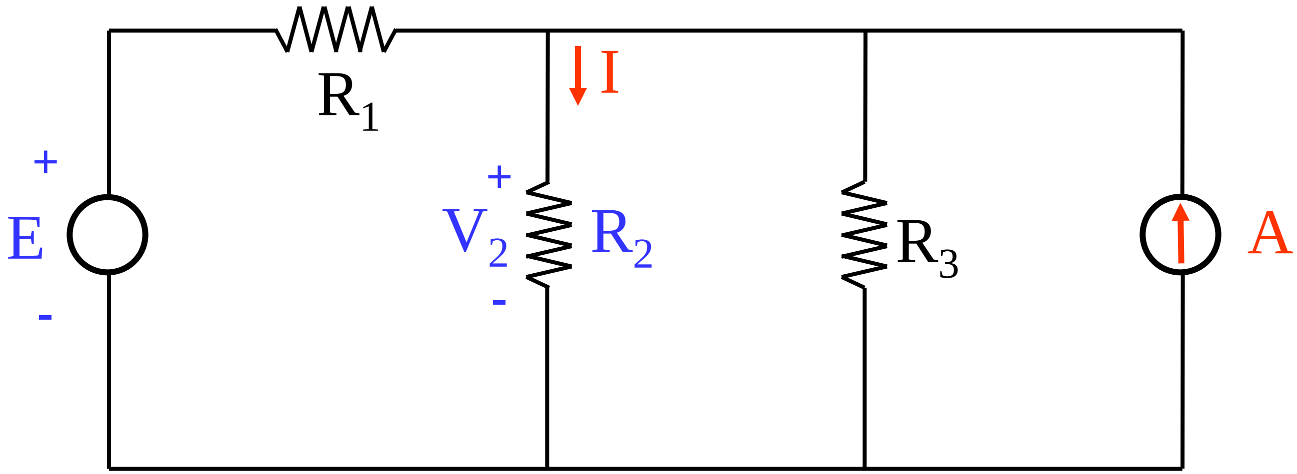
Superposition Principle

If there are **more than one sources** in a circuit, the voltage and current can be considered as a sum of contribution of each sources.

This principle arises from the fact that the current through any resistor is directly proportional to the voltage.

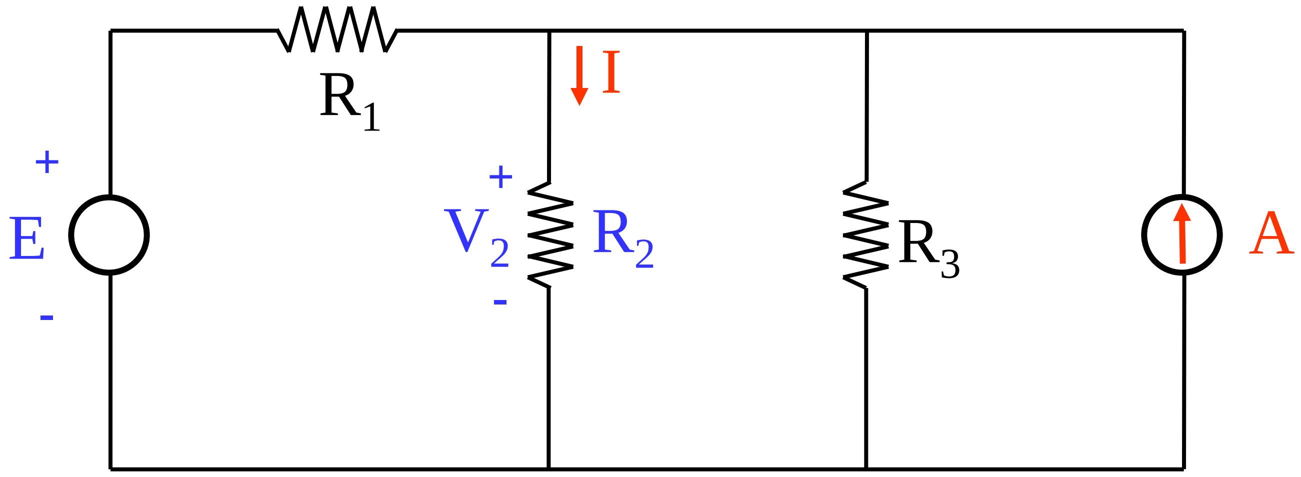
$$f(x_1) + f(x_1) = f(x_1 + x_2)$$

If there are several sources in the circuit, the each source is disabled while taking into account the effect of other source.



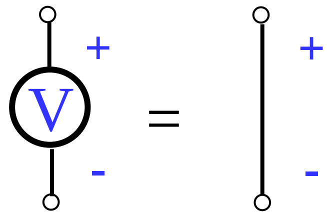
$$I_E + I_A = I$$

Superposition Principle

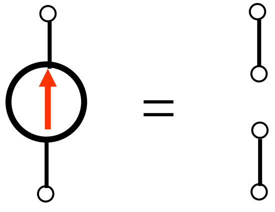


$$I_E + I_A = I$$

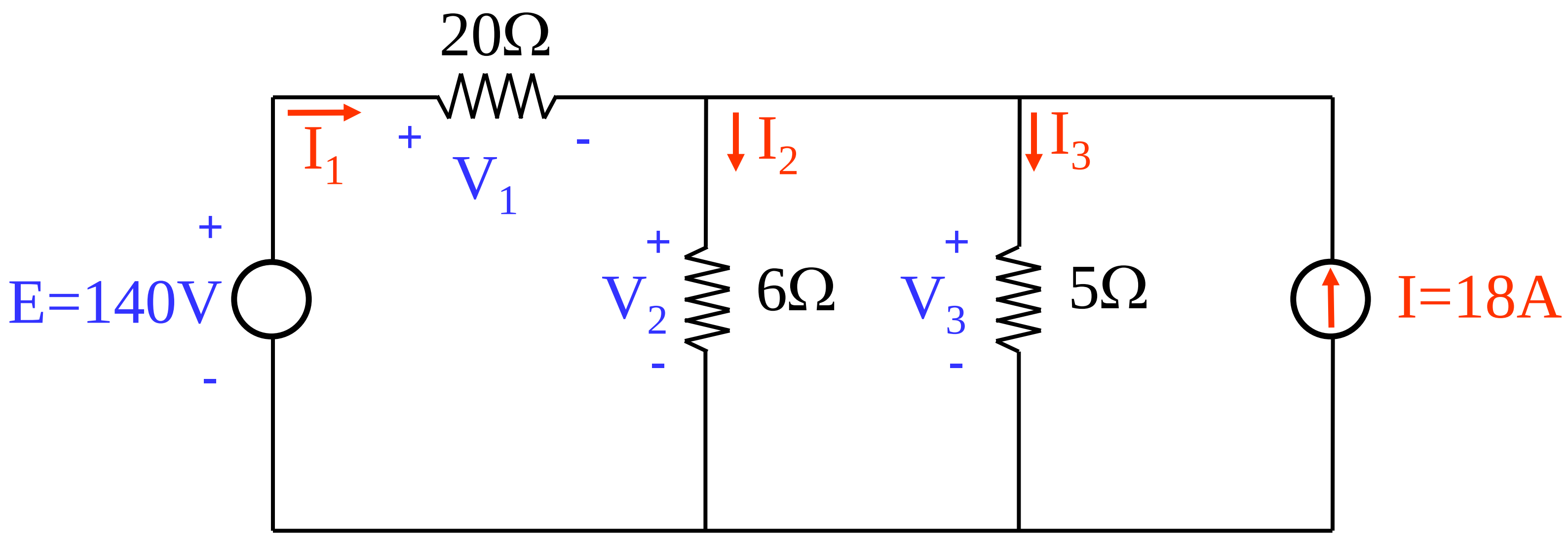
Voltage Source → Short Circuit



Current Source → Open Circuit

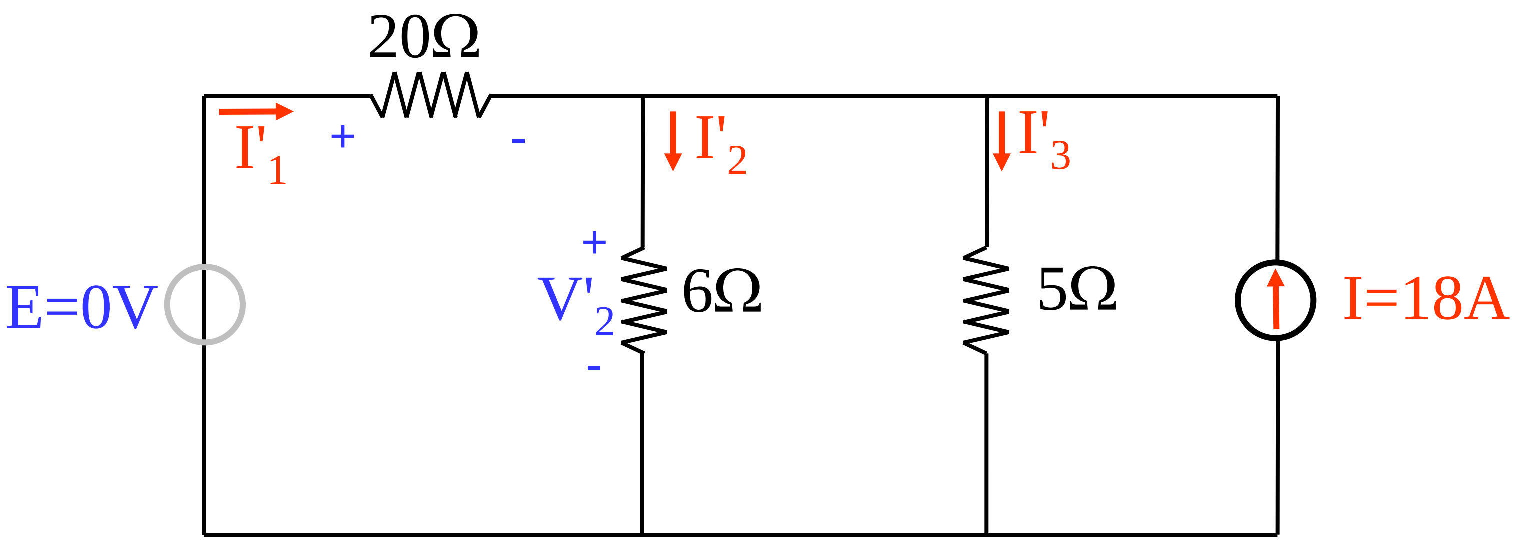


Example-2.16: Find the currents I_1, I_2 and I_3 using the superposition principle in the following circuit (This problem was solved by direct implementation of the basic laws in Example-2.1).



Solution: At first, the currents will be found by assuming that the 140V source has no effect (assuming zero); secondly, the currents as if the current source of 18A has no effect (assuming open circuit). Then we fill find the algebraic sum of the currents

1- Suppose the 140V voltage source has no effect (assuming zero-short circuit). Find the desired currents using the node voltage method.



$$V_2' \cdot \left(\frac{1}{20} + \frac{1}{6} + \frac{1}{5} \right) = 18A \quad \Rightarrow$$

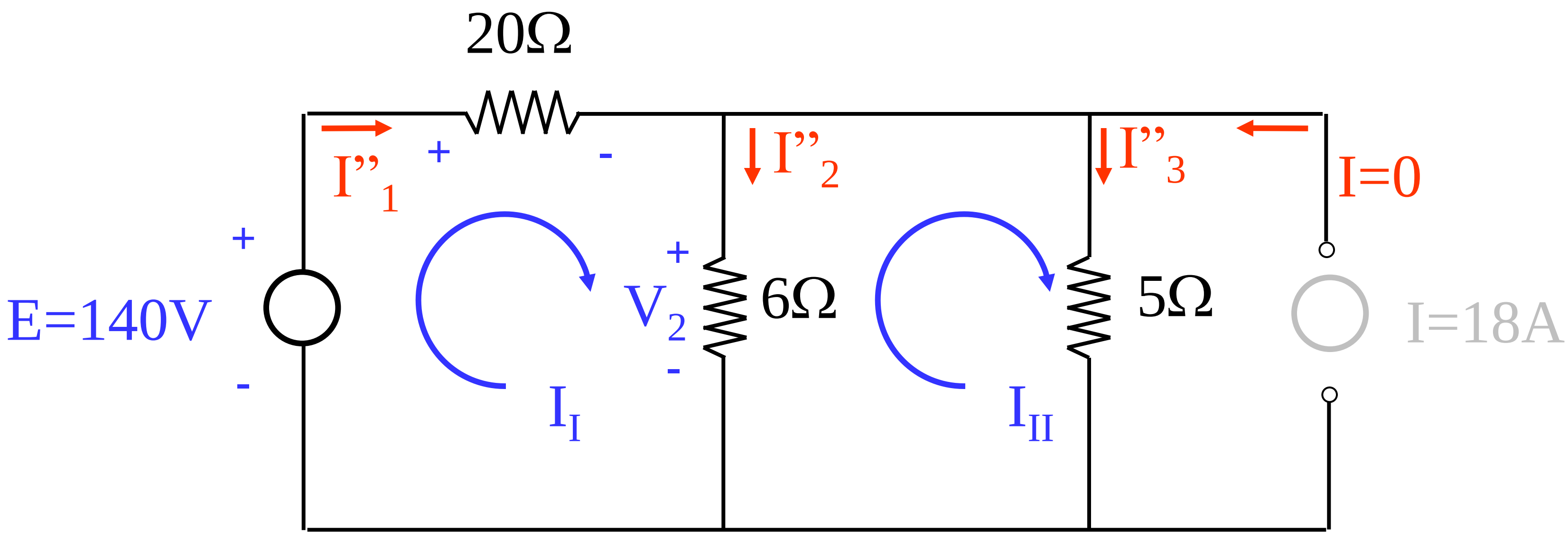
$$V_2' = 43.2\,V$$

$$I_1' = -43.2\,V / 20\,\Omega = -2.16A$$

$$I_2' = 43.2\,V / 6\,\Omega = 7.20A$$

$$I_3' = 43.2\,V / 5\,\Omega = 8.64A$$

2- Let us find the currents as if the current source has no effect (open circuit). To find the currents, mesh currents method can be applied I and II.



$$26I_I - 6I_{II} = 140$$
$$-6I_I + 11I_{II} = 0$$

⇨

$$I_I = 6.16A \qquad I_{II} = 3.36A$$
$$I_1'' = I_I = 6.16A$$
$$I_2'' = I_I - I_{II} = 2.80A$$
$$I_3'' = I_{II} = 3.36A$$

The net currents will be the sum of the currents found above.

$$I_1 = I_1' + I_1'' = -2.16 + 6.16 = 4.00A$$

$$I_2 = I_2' + I_2'' = 7.20 + 2.80 = 10.00A$$

$$I_3 = I_3' + I_3'' = 8.64 + 3.36 = 12.00A$$

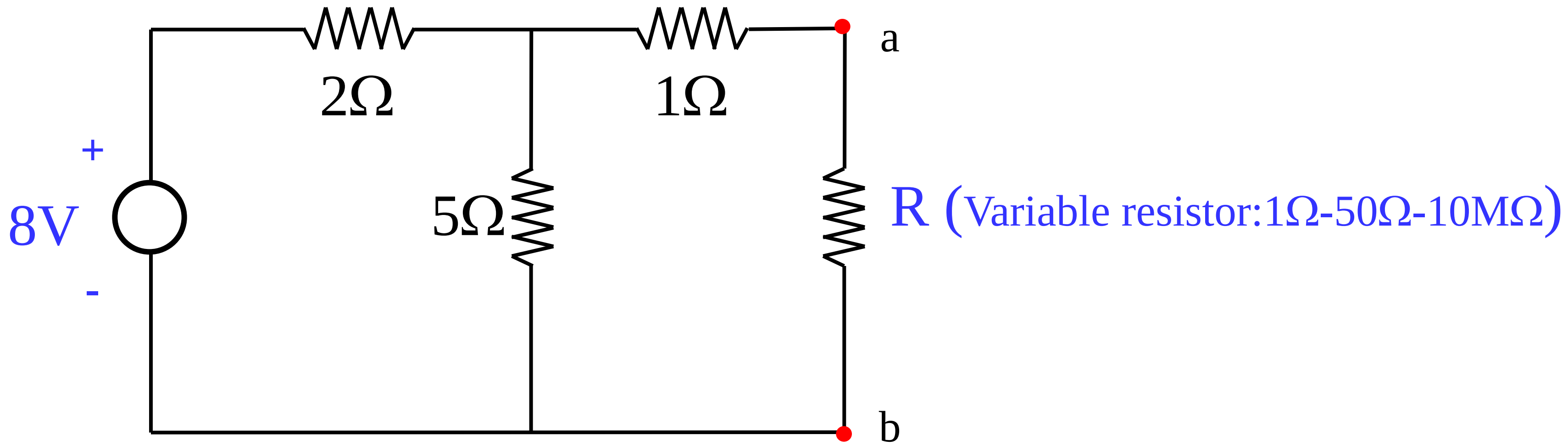
It is the same as the currents previously found in Example-2.1 obtained by the application of the basic laws.

Thevenin’s Theorem

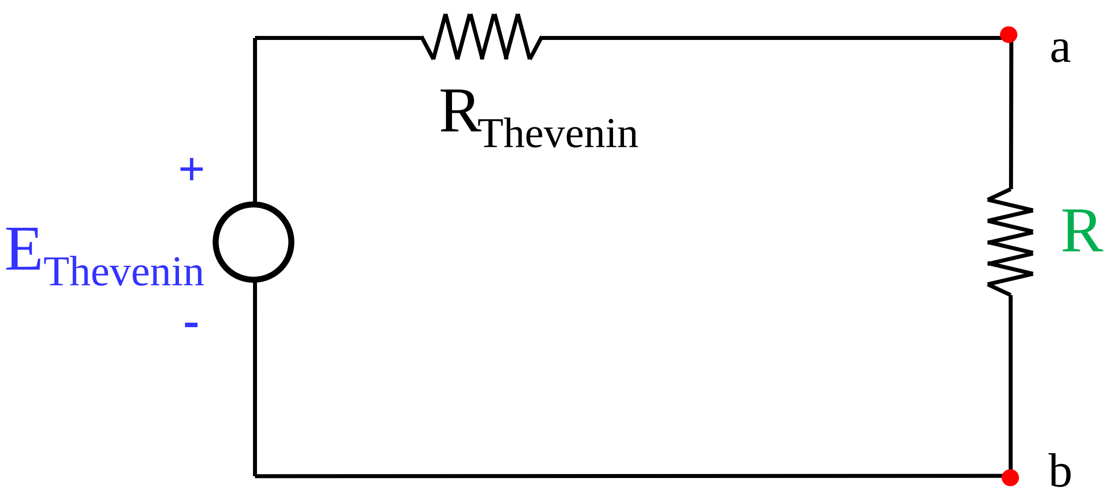
Thevenin's Theorem is basically permits a simple representation of the circuit when viewed from any pair of output terminals of a complex circuit. As a result of this a load connected to the output of the circuit it allows to determine the effect of the circuit on the load or effect of the load on the circuit itself.

Thevenin’s Theorem: Any linear two-terminal circuit consisting of resistors and sources can be represented by a voltage source and a series resistor.

Thevenin's Theorem

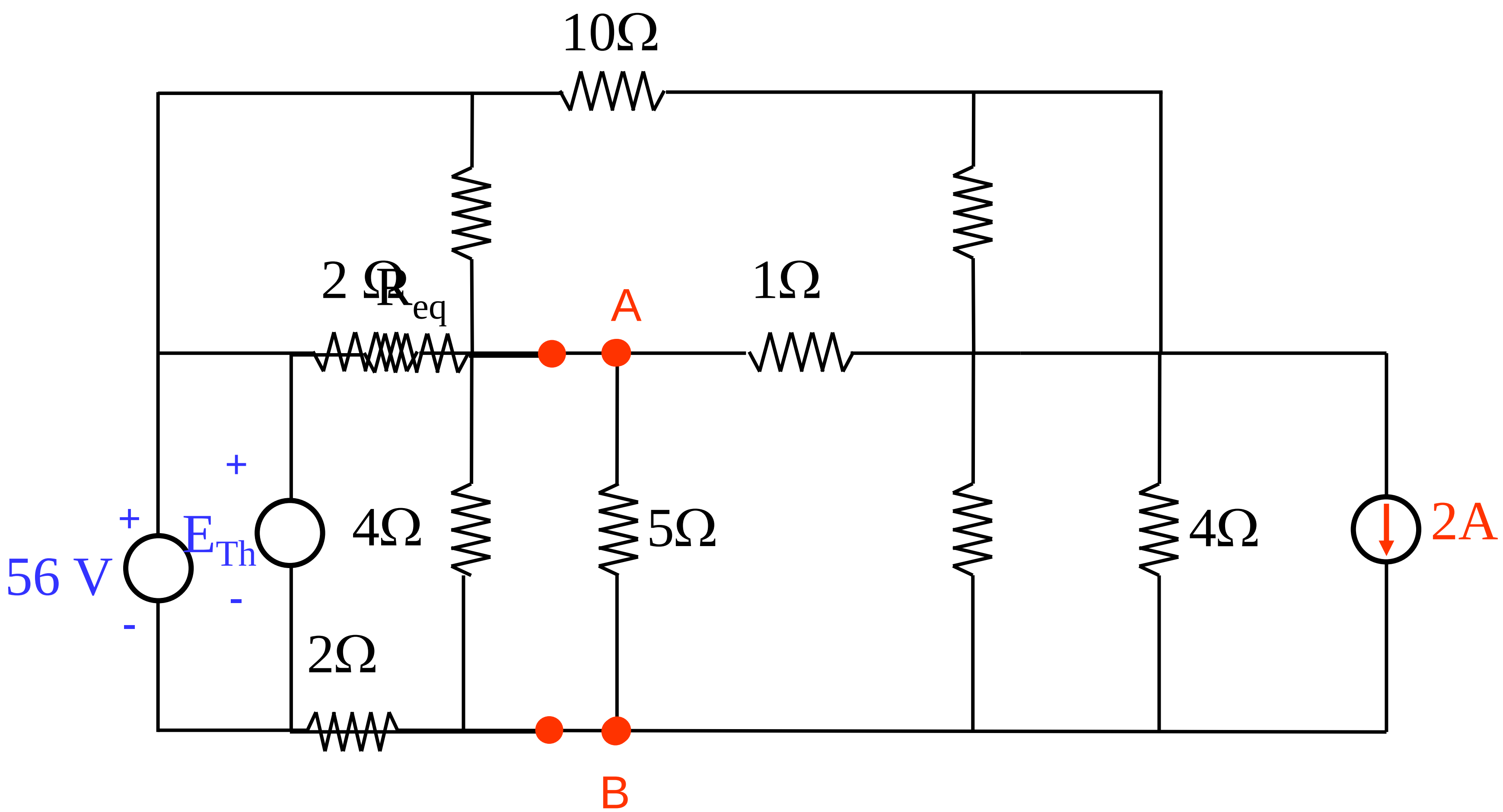


Let us find the current that will pass through the **ab** branch for different values of R resistor. For this we will have to re-analyze the entire circuit for each resistor. Instead, we simply avoid the analysis of the circuit to the left of point ab (if we can express it with a voltage source and a resistor) each time.



$$I_R = \frac{E_{Thevenin}}{R_{Thevenin} + R}$$

Consider the resistance between points A and B in the circuit below:



Thevenin Theory

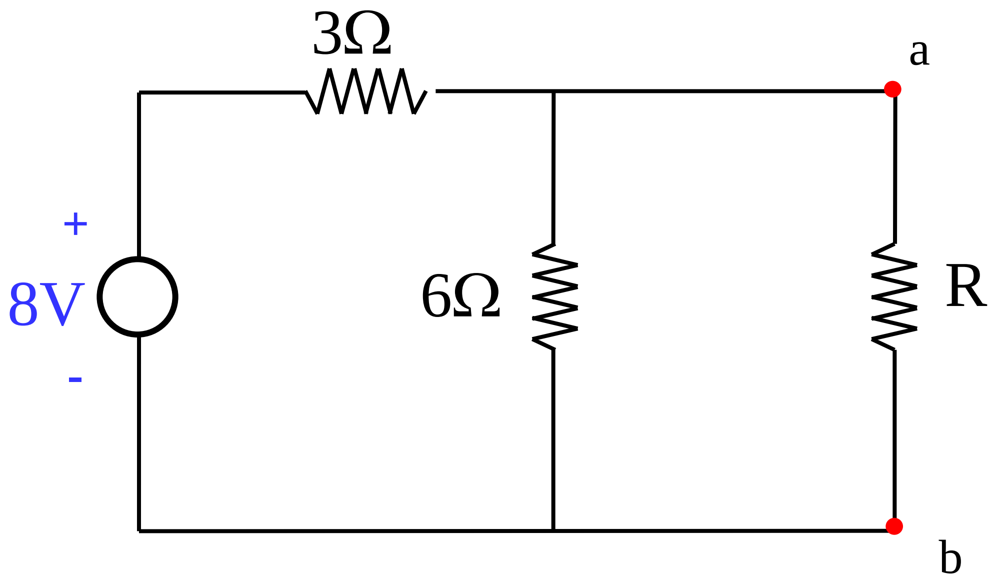
How to find Thevenin's Circuit

1- The circuit element between the selected two terminals is **removed**,

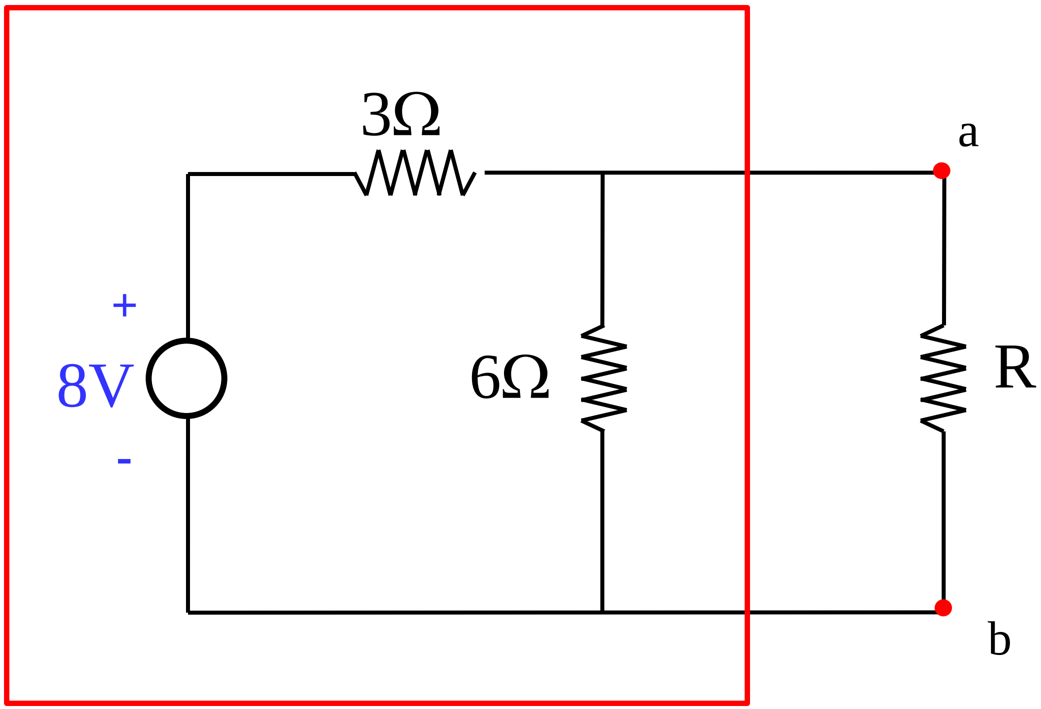
2- **Voltage** between these two terminals is found,

3- **Equivalent resistance** of the circuit is calculated (setting Current sources open circuit, voltage sources short circuit).

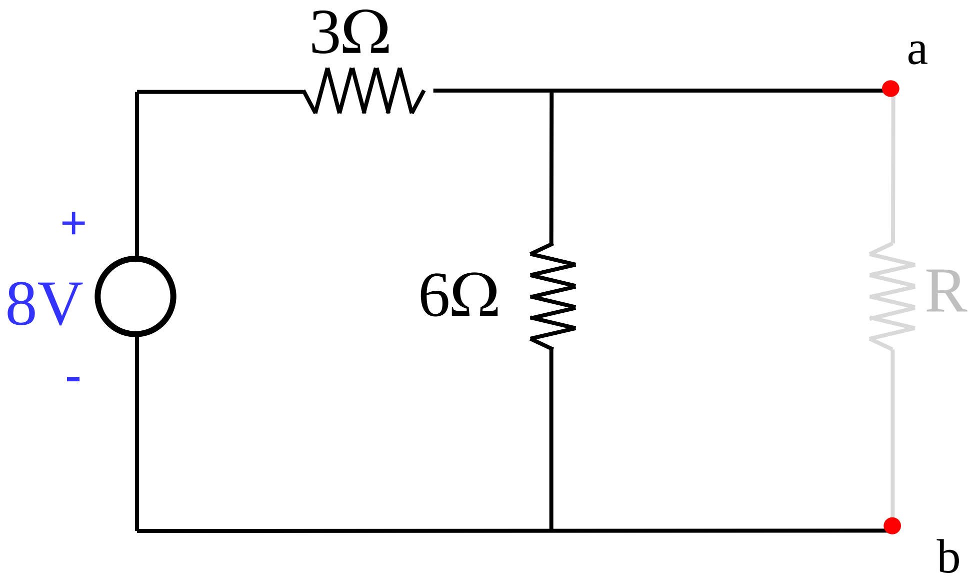
Example: Find the current through the branch ab for the 2Ω, 20Ω and 500Ω of the R resistor in the circuit below. Find the equivalent circuit of Thevenin

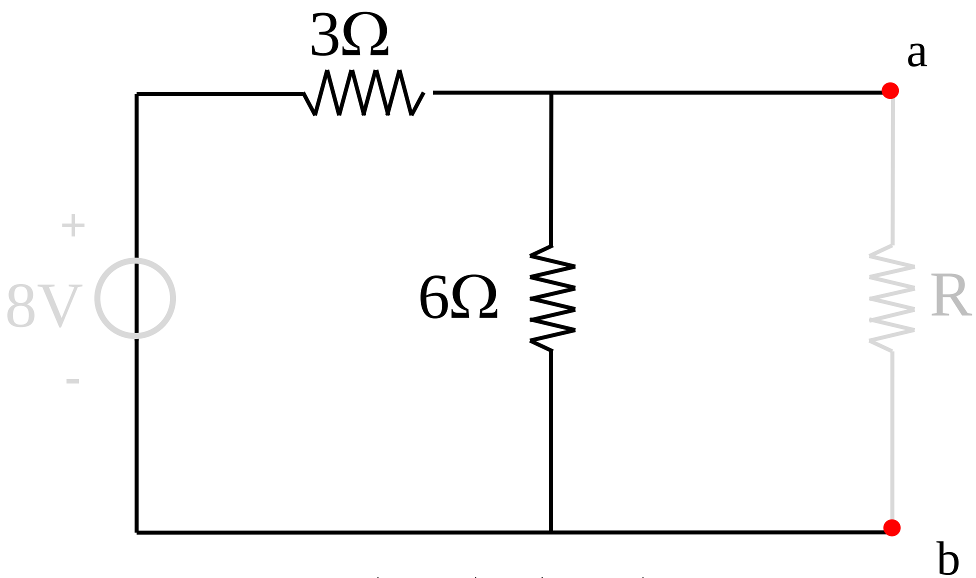


Solution: The remainder of the circuit when looked from the points ab can be represented by a voltage source and a resistor, ie an equivalent Thevenin circuit.



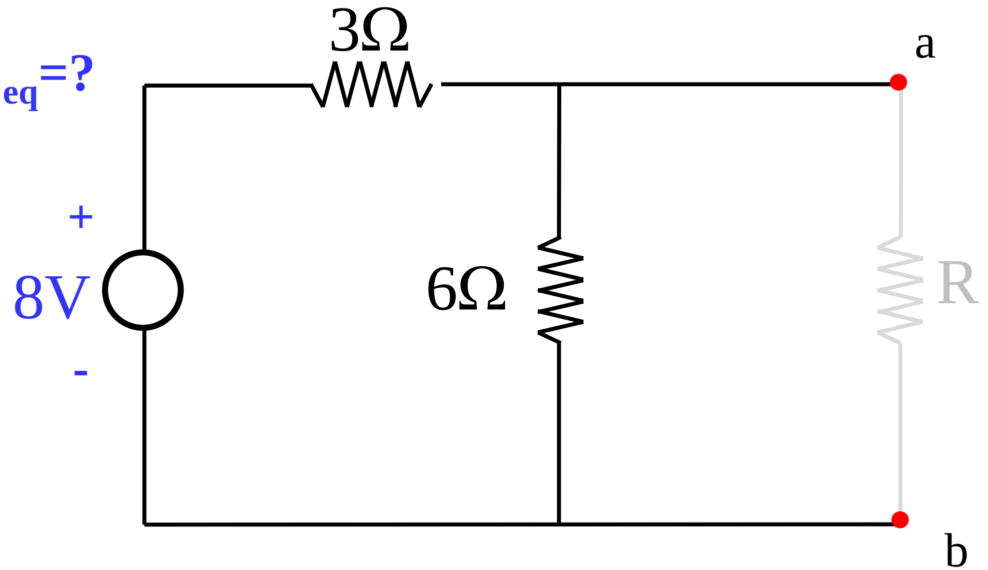
R_{eq}=?



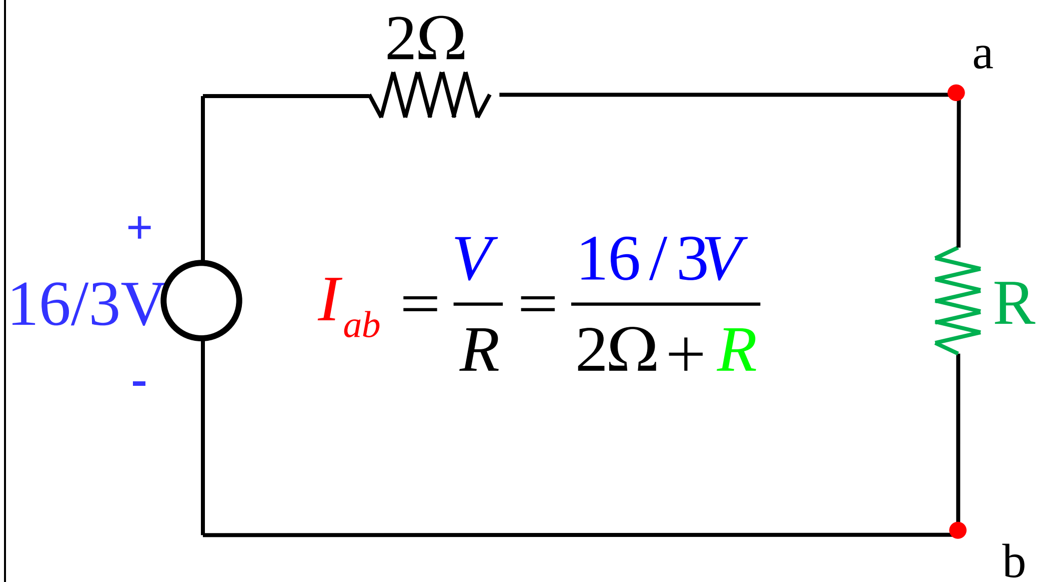


$$R_{eq} = \frac{(3\Omega).(6\Omega)}{3\Omega + 6\Omega} = 2\,\Omega$$

$E_{eq}=?$



$$E_{eş}=\left(\frac{8V}{3\Omega+6\Omega}\right)(6\Omega)=\frac{16}{3}V$$



$$I_{ab}=\frac{V}{R}=\frac{16/3V}{2\Omega+R}$$

$$I_{ab}=\frac{16/3V}{2\Omega+R}$$

R=2Ω $\quad I_{ab}=\frac{16/3V}{2\Omega+2\Omega}=1.33A$

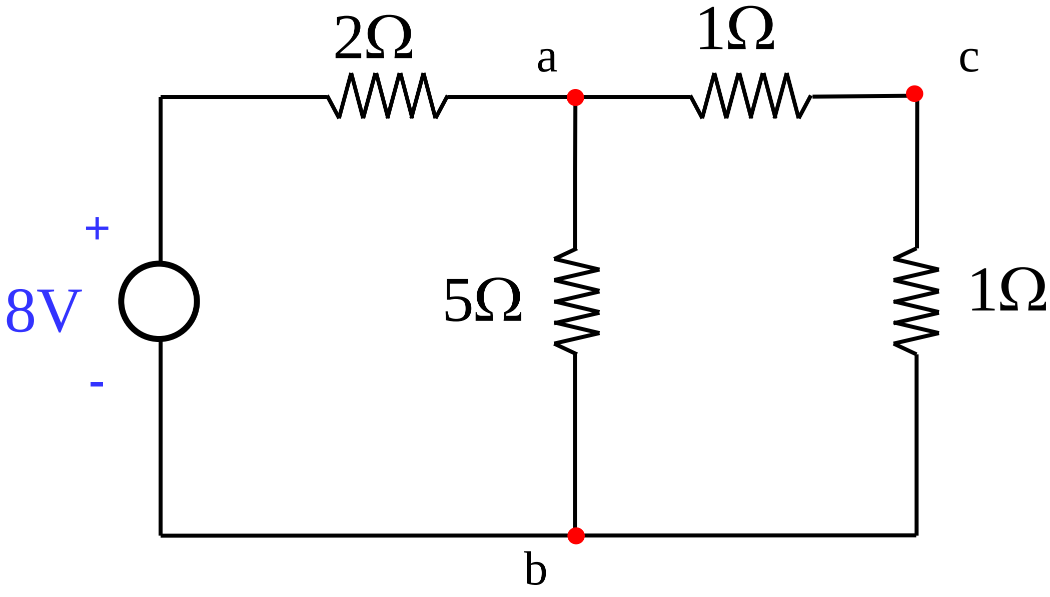
R=20Ω $\quad I_{ab}=\frac{16/3V}{2\Omega+20\Omega}=0.24A$

R=500Ω $\quad I_{ab}=\frac{16/3V}{2\Omega+500\Omega}=0.01A$

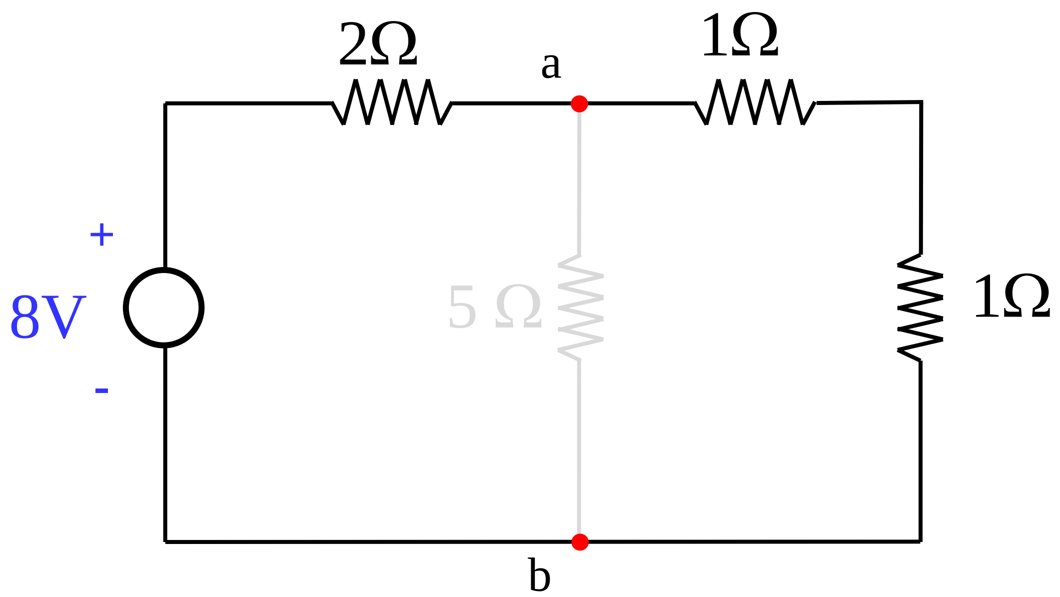
Example: Find the Thevenin equivalent circuit when viewed from

a) ab terminals

b) ac terminals.



Solution: a) Thevenin equivalent circuit viewed from **ab** terminals

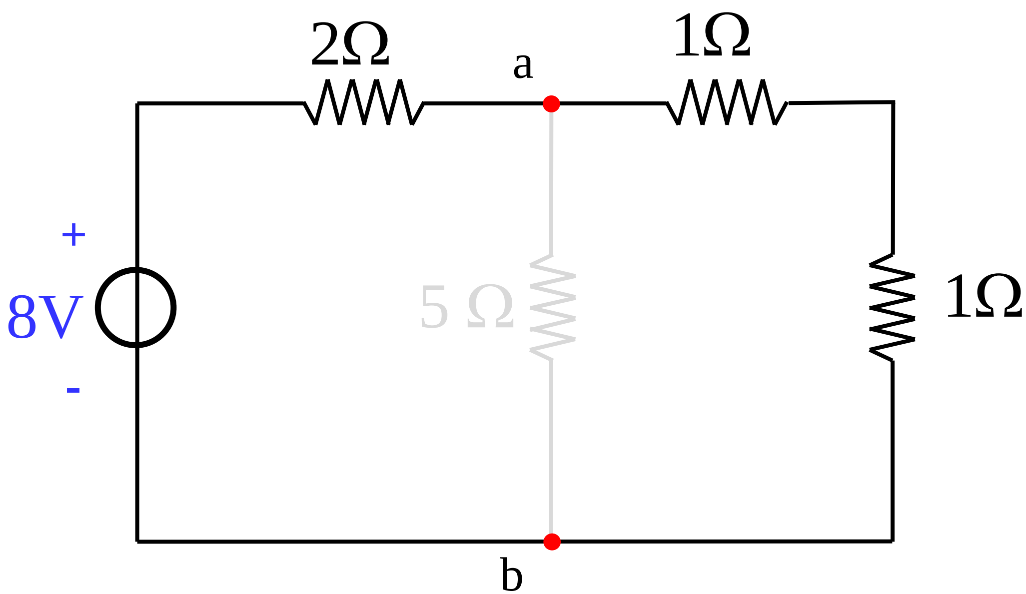


$R_{eq} = ?$

$E_{Th} = ?$

Equivalent Resistance

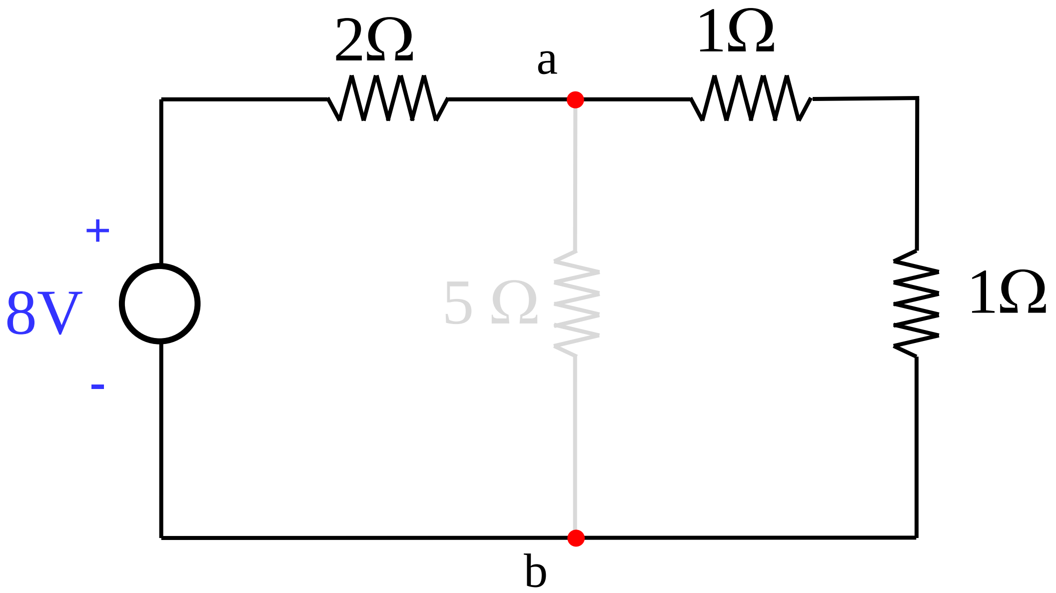
$$R_{eq} = ?$$



$$R_{eq} = \frac{(1\Omega + 1\Omega).(2\Omega)}{(1\Omega + 1\Omega) + 2\Omega} = 1\Omega$$

Equivalent Voltage Source

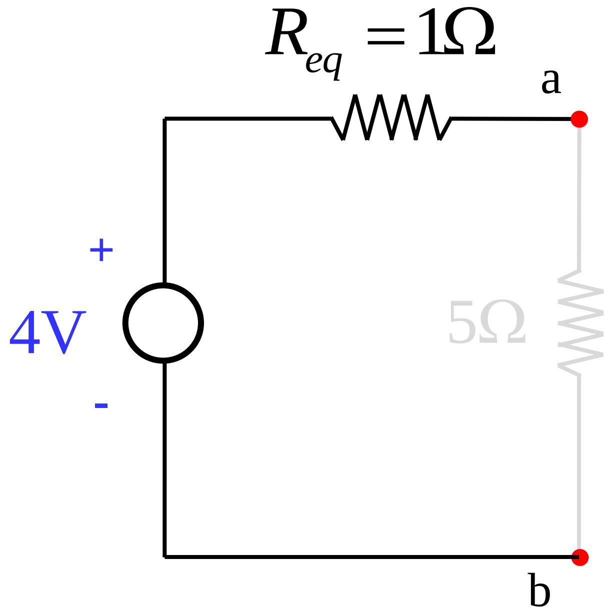
$$E_{Th} = ?$$



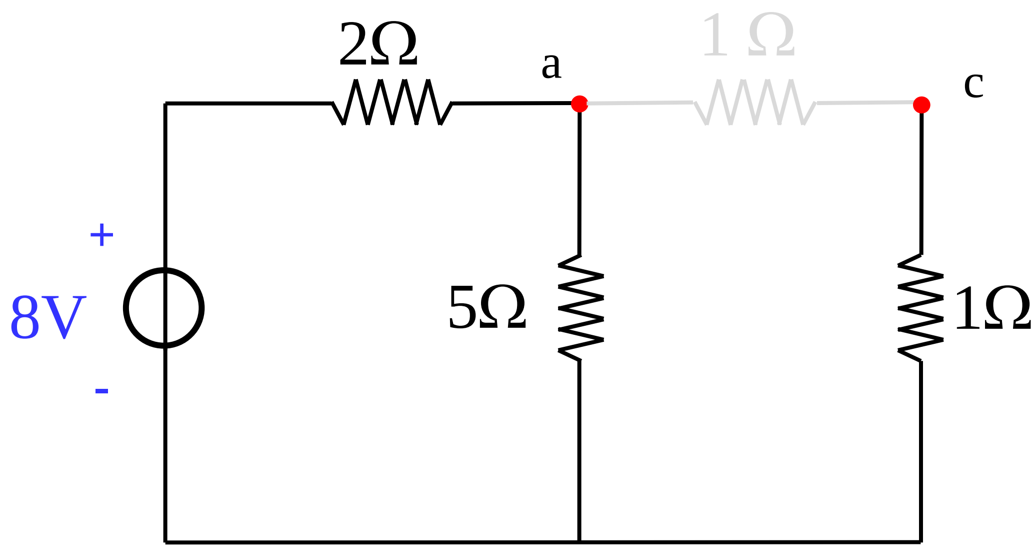
$$I = \frac{8V}{4\Omega} = 2A$$

$$E_{ab} = (2\Omega)(\ 2A) = 4V \Rightarrow \boxed{E_{Th} = 4V}$$

Thevenin equivalent circuit when viewed from **ab terminals** :



b) Thevenin equivalent circuit viewed from **ac** terminals



$$R_{eq} = ?$$

$$E_{Th} = ?$$

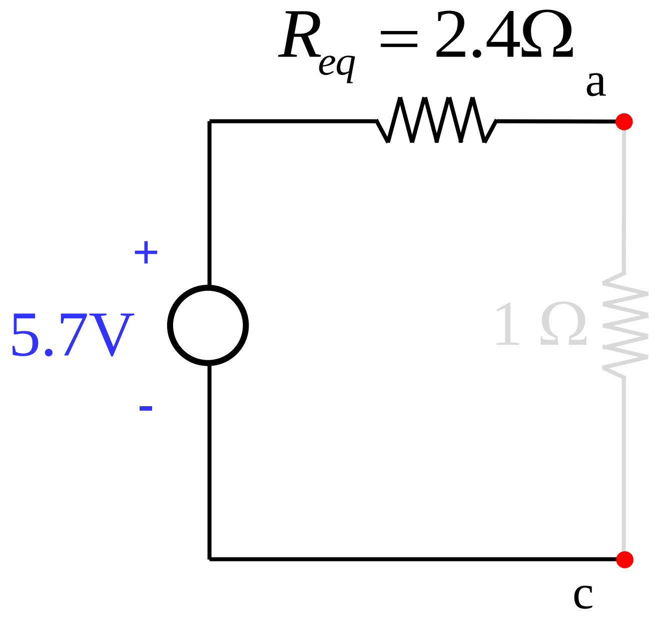
$$R_{eş} = \frac{(5\Omega).(2\Omega)}{5\Omega + 2\Omega} + 1\Omega = 2.4\Omega$$

$$I = \frac{8V}{7\Omega} = 1.14A$$

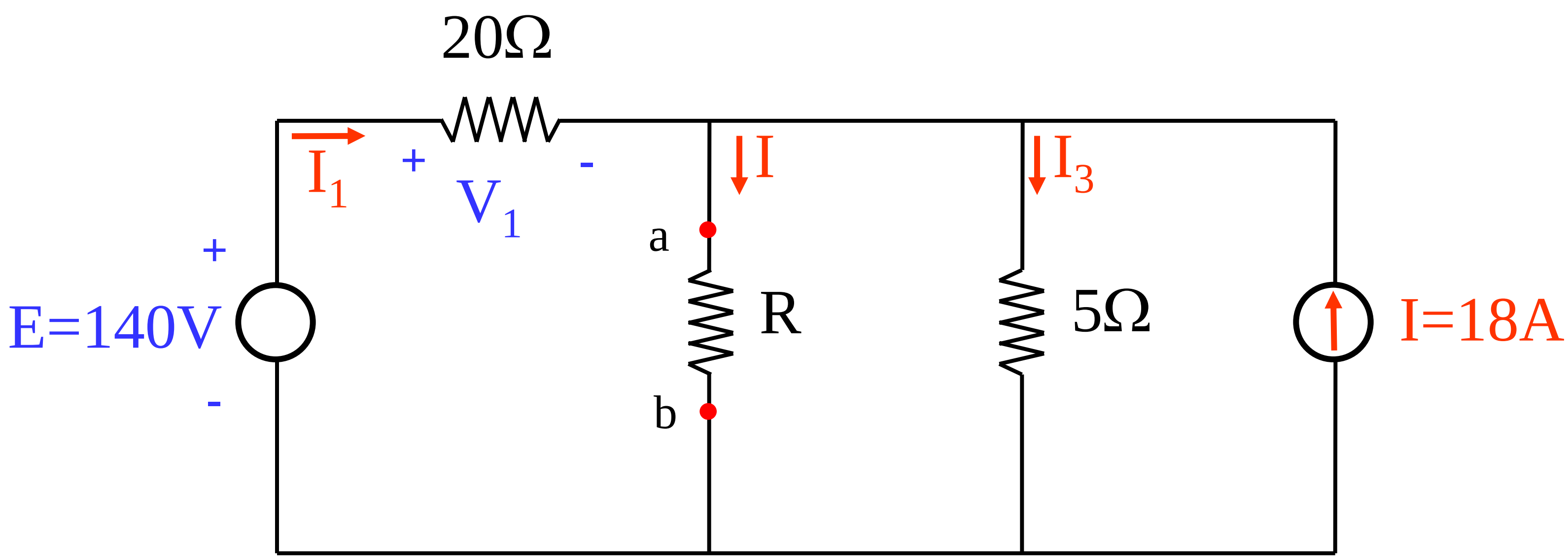
$$E_{ac} = (5\Omega)(1.14A) = 5.7V \Rightarrow$$

$$E_{Th} = 5.7\,V$$

Thevenin equivalent circuit when viewed from **ac terminals**:

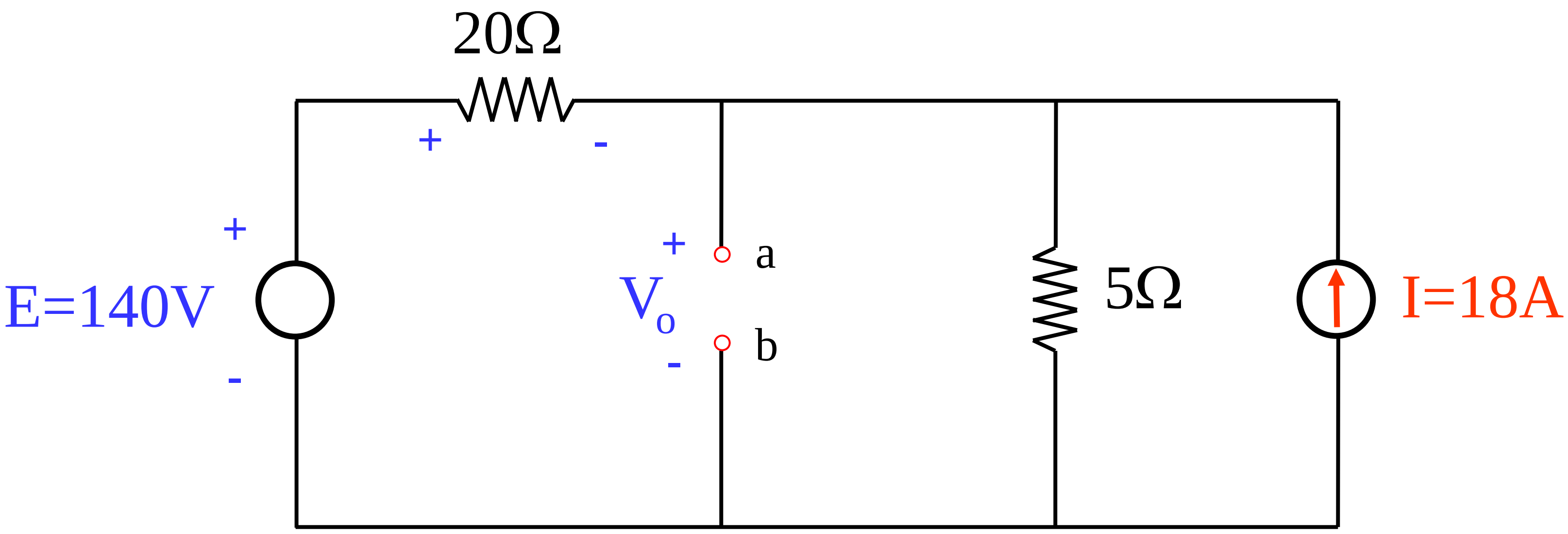


Example-2.17: Find the resistance R that can absorb the highest power from the circuit below and calculate the power.



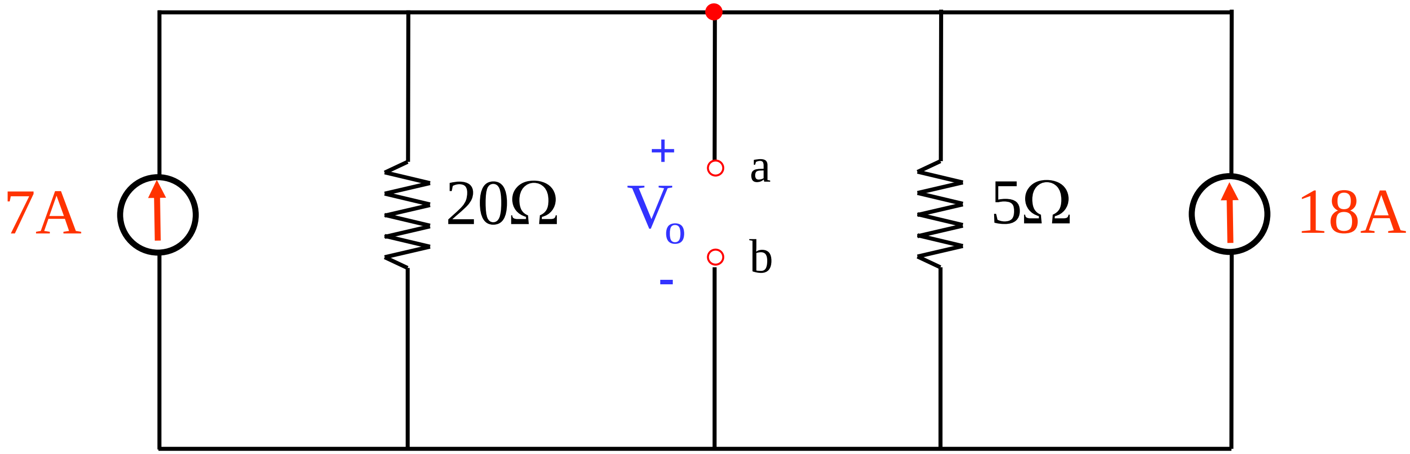
Solution: To find power, current and resistance R must be known. Since power is the product of current and voltage (P=I.V), multiplication of current and voltage (or resistance) is more important than their individual values. Therefore, there must be an I(R) relation that gives I as a function of R.

Since it is desired to find the resistor R, the resistor R is taken out and the Thevenin equivalent circuit of the rest of the circuit is formed.



To find V_o voltage, the 140V voltage source can be converted into a current source with a 20Ω resistor (Node Voltage Method).

$$V_o = I.R_o \Rightarrow I = \frac{V_o}{R_o} = \frac{140V}{20\Omega} = 7A$$ Source conversion

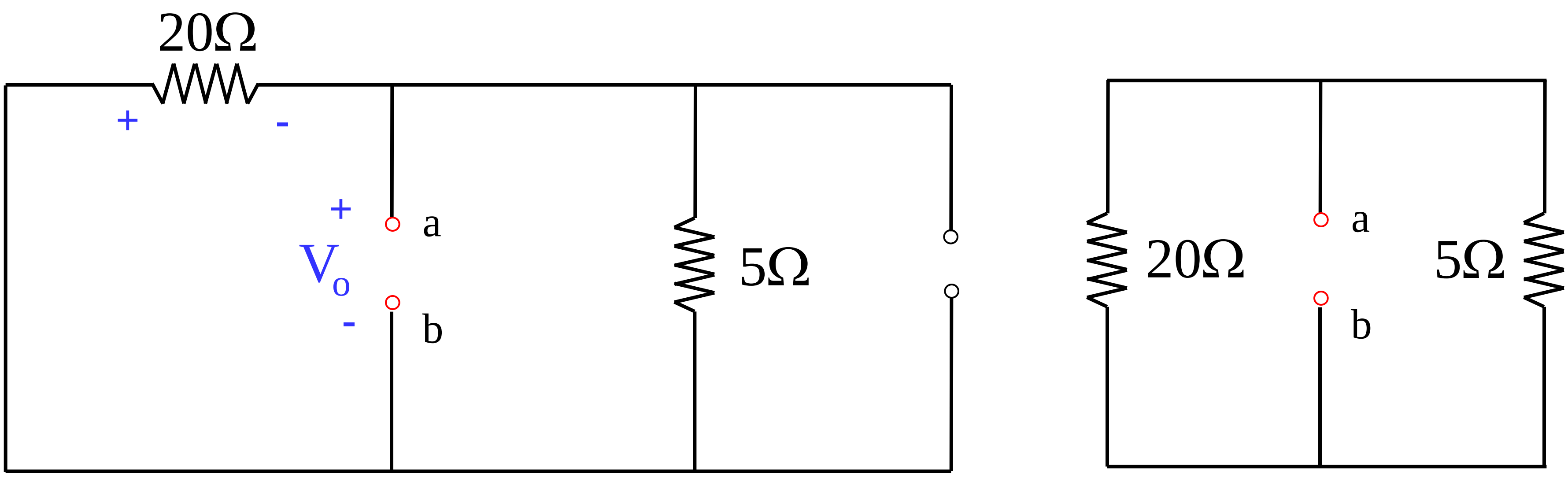


If the Kirchhoff's Current Law (KCL) equation is used for the above circuit, the V_o voltage can be found:

$$V_o.\left(\frac{1}{20\Omega} + \frac{1}{5\Omega}\right) = 7A + 18A \Rightarrow \qquad V_o = 100V$$

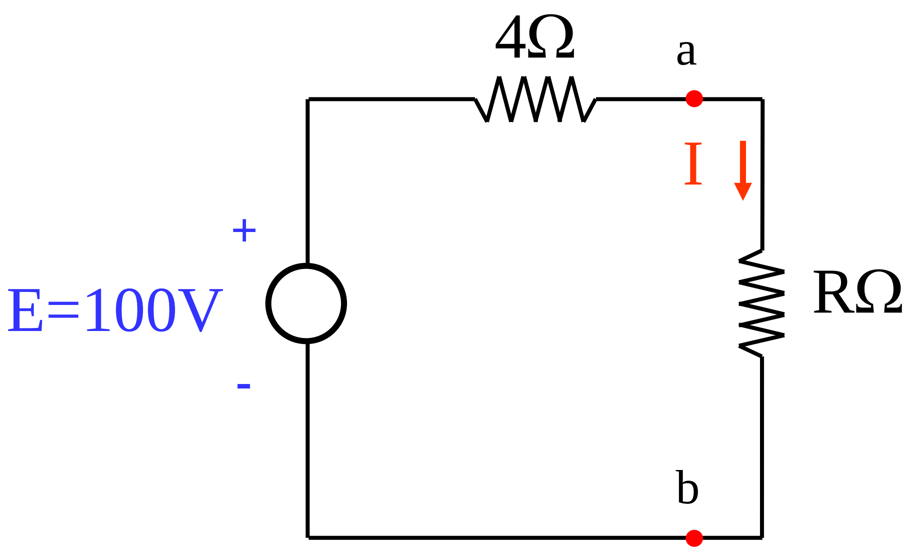
To find the equivalent resistance R_o, the sources are eliminated (voltage supply short circuit; current supply open circuit).

(The same result is obtained when both circuits are made for a voltage or current source). In case of voltage supply



$$R_o = \frac{(20\Omega).(5\Omega)}{20\Omega + 5\Omega} = 4\Omega$$

The Thevenin equivalent circuit viewed from the terminals a-b can be represented by a voltage source of a100V and a resistance of 4Ω.

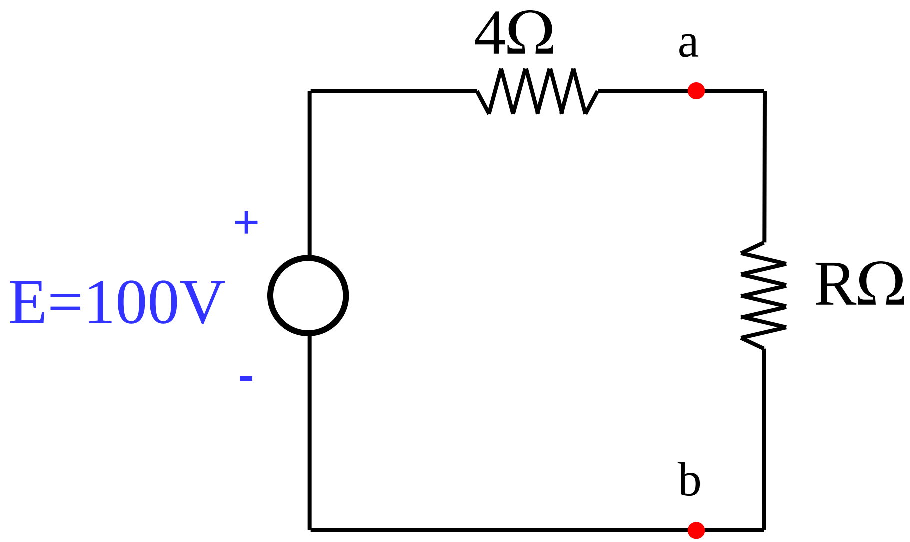


Using the KVL current passing through resistor R:

$$100V - 4I - RI = 0 \Rightarrow I = \frac{100V}{4+R}$$

Power (P) on the resistor R:

$$P = I^2 R = \frac{10000R}{(4+R)^2}$$

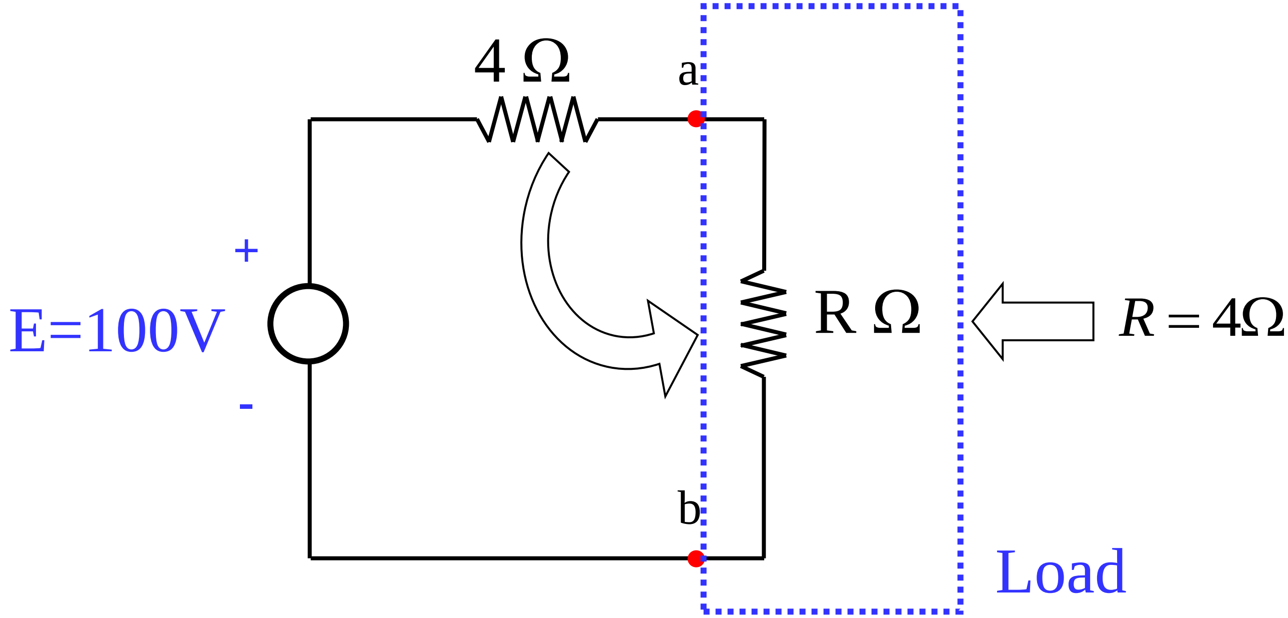


If the maximum resistance value is found from the expression giving the change of power according to the resistance:

$$\frac{dP}{dR} = \frac{10000(4+R)^2 - 20000(4+R)R}{(4+R)^4} = 0 \quad \Rightarrow \quad R = 4\Omega$$

$$I = \frac{100}{4+4} = 12.5A \quad \Rightarrow \quad P_{max} = (12.5A)^2 (4\Omega) = 625W$$

Resistance Match (Impedance Matching)

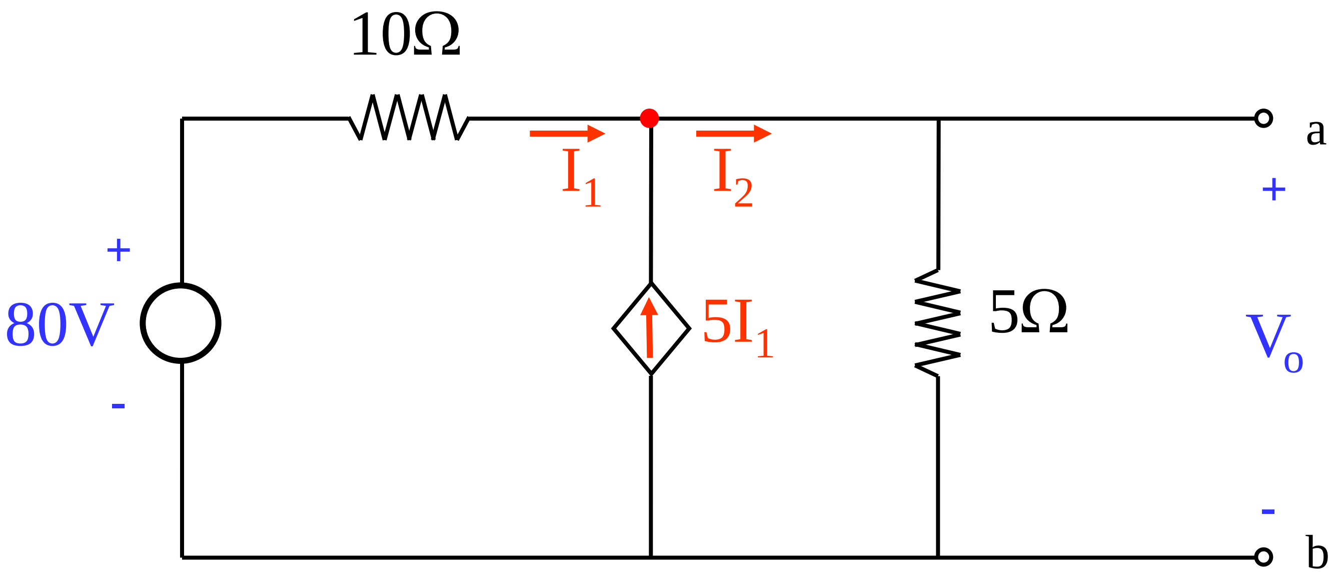


This example is equivalent to Example-2.1. In example-2.1, instead of a 6Ω resistor, this example has a R resistor.

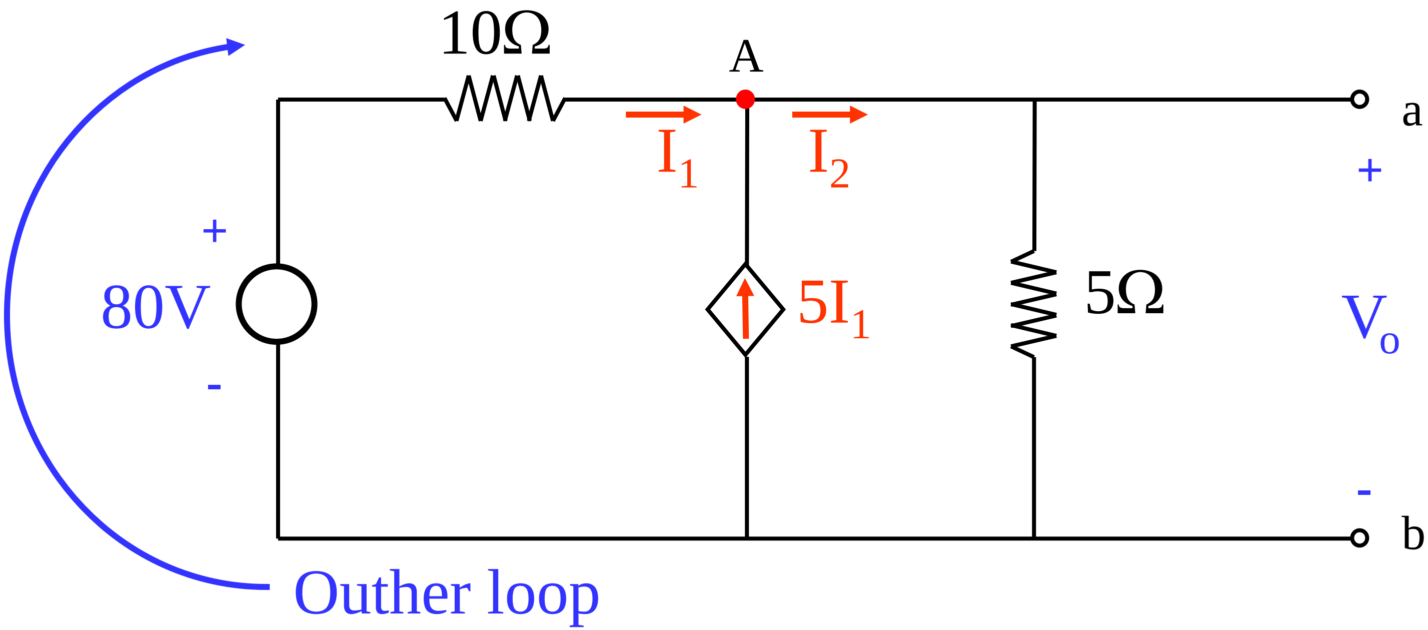
This example shows **how to find the current and voltage of a circuit according to a single circuit element with the help of Thevenin's theorem.** **Also note that for maximum power transmission the load resistance (R) is equal to the equivalent resistance of the source when viewed from the terminals of R.**

*Equivalent source resistor is called **output resistor**, and the method of equalizing the output resistor to the load resistor is known as resistance matching (most commonly **impedance matching**).*

Example-2.18: Find the Thevenin equivalent circuit as seen from the a-b terminals in the circuit below.



Solution: There is a dependent source in the circuit. Therefore, it should be examined separately from the circuit where only independent resources exist. For this reason first, the short-circuit voltage V_o and the open-circuit current I_o; then there will be equivalent resistance.



KCL at the junction **A**: $I_2 = I_1 + 5I_1 = 6I_1$

KVL through outher loop: $+80V - 10I_1 - 5I_2 = 0$

$\Rightarrow 10I_1 + 5I_2 = 10I_1 + 5(6I_1) = 80V \Rightarrow I_1 = 2A;\ I_2 = 12A$

$\Rightarrow V_o = 5\Omega.I_2 = 5\Omega.(12A) = 60V$

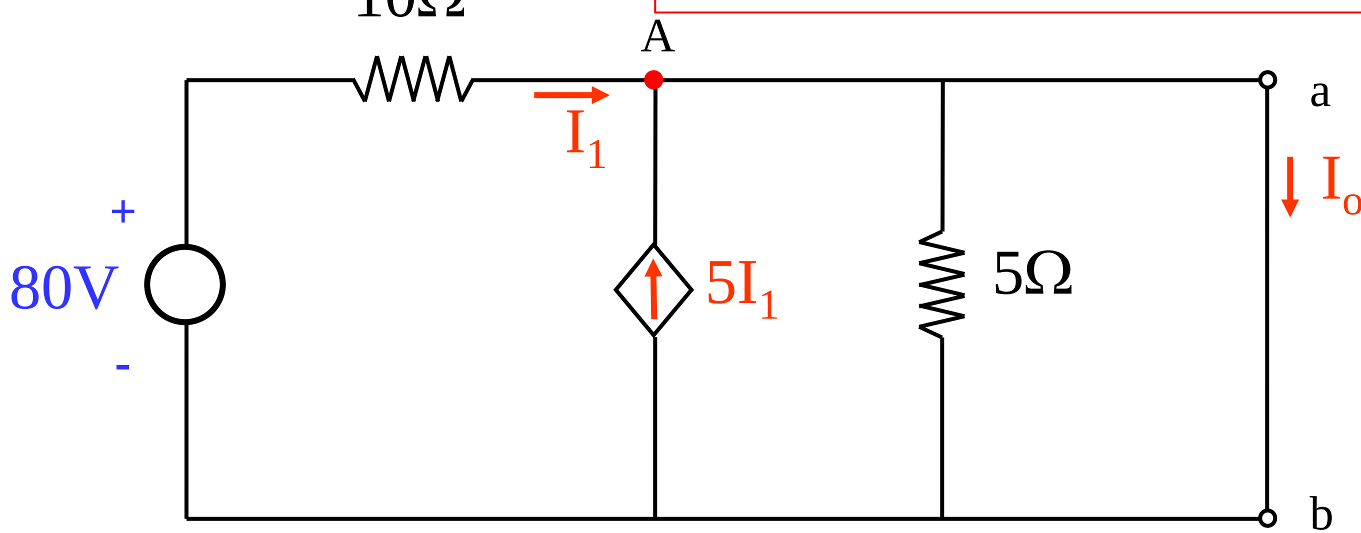
Finding Thevenin resistor R_o:

The circuit is redrawn as shown below. As a result of the short circuit, the output voltage and consequently the current at the 5Ω resistor are zero. KVL and KCL equations :

Since there is a dependent current source in the circuit, I_o ve E_o are found between ab to find the equivalent resistance: so their ratio (E_o / I_o) is R_o. If there were not the dependent source, the normal way to find R_o would be to eliminate the effect of power supplies (current source is open circuit, voltage source is short circuit).

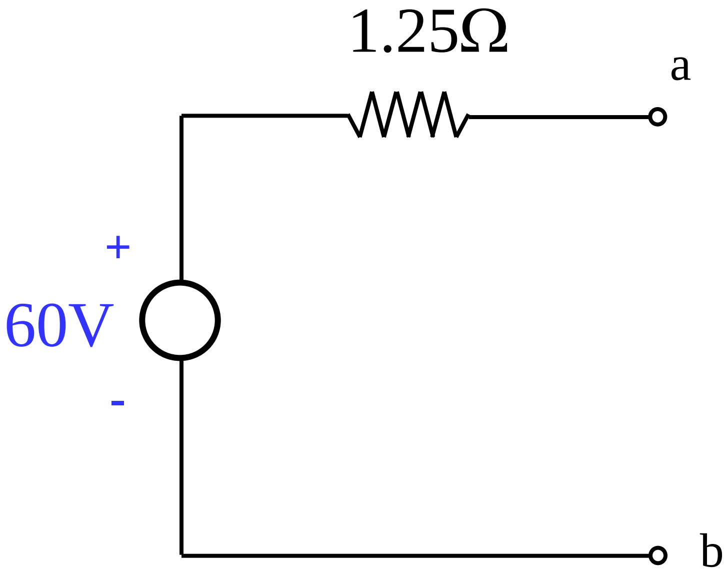
$$10I_1 = 80V \Rightarrow I_1 = 8A$$

$$I_o = I_1 + 5I_1 = 48A$$

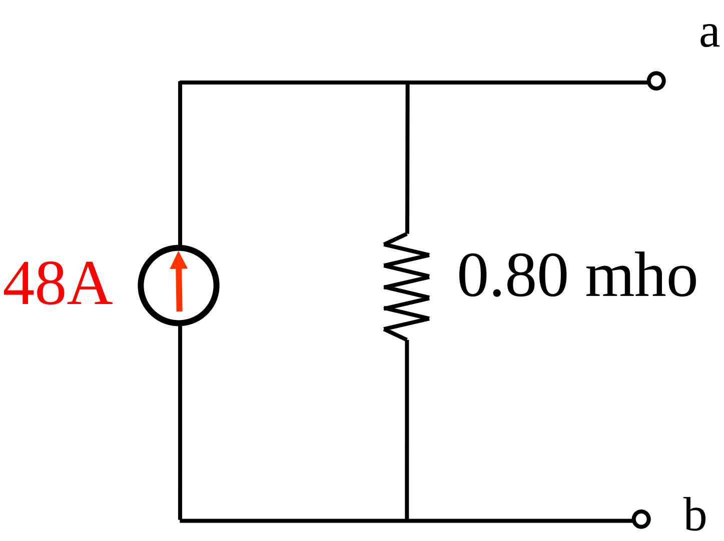


R_o resistor:

$$R_o = \frac{V_o}{I_o} = \frac{60V}{48A} = 1.25\Omega$$

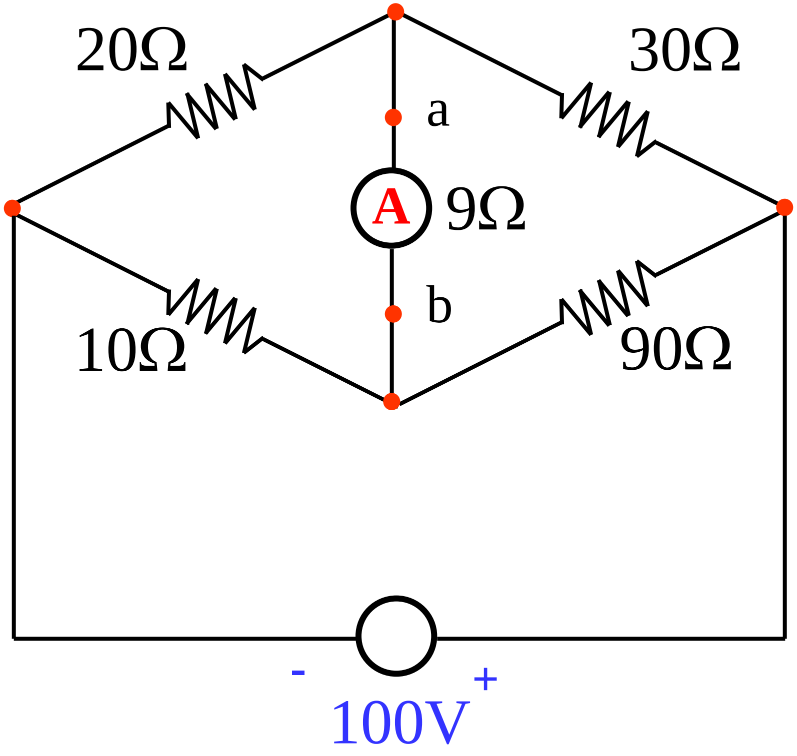


Thevenin equivalent

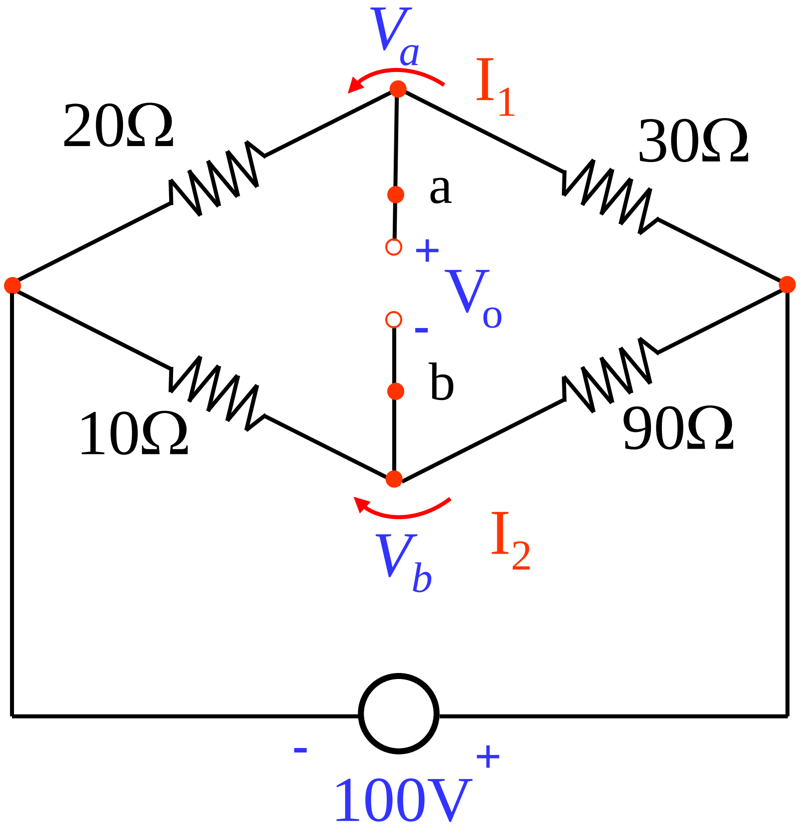


Norton equivalent

Example-2.19: The following circuit is the circuit of an unbalanced bridge used to measure resistance. Find the current through the ammeter A. The internal resistance of the ammeter is 9Ω.



Solution: The solution of this problem can be greatly simplified by using the Thevenin equivalent circuit of the circuit. The first step is to take out the ammeter and find the open circuit voltage V_o.

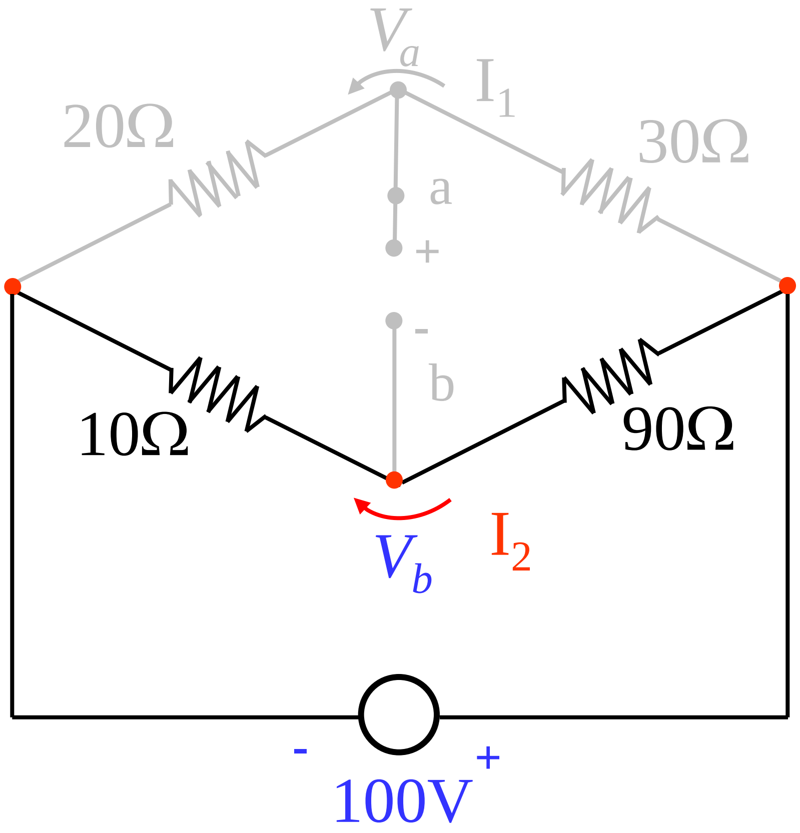


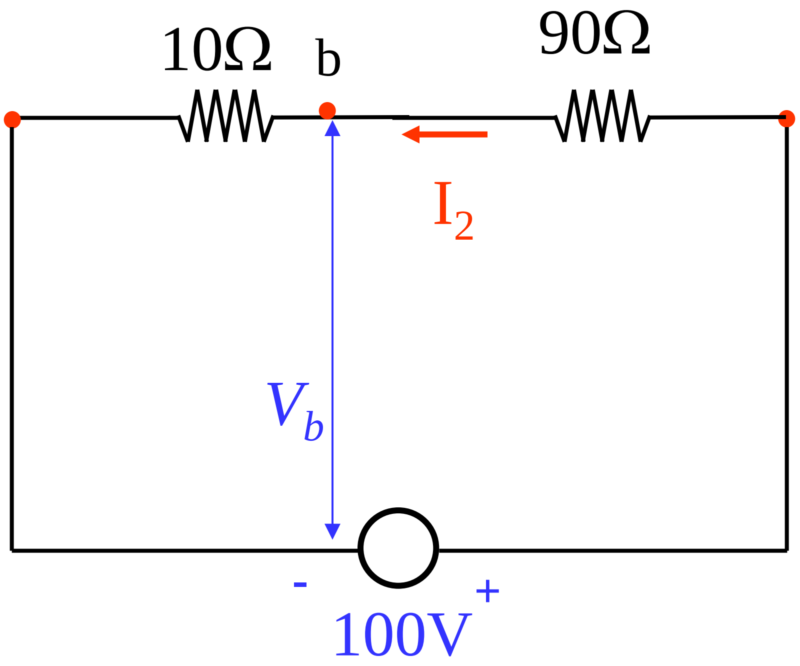
$$V_o = V_a - V_b$$

$$V_o = (20\Omega) I_1 - (10\Omega) I_2$$

$$I_1 = \frac{100V}{20\Omega + 30\Omega} = 2A \qquad I_2 = \frac{100V}{10\Omega + 90\Omega} = 1A$$

$$V_o = 20I_1 - 10I_2 = (20\Omega)(2A) - (10\Omega)(1A) = 30V$$



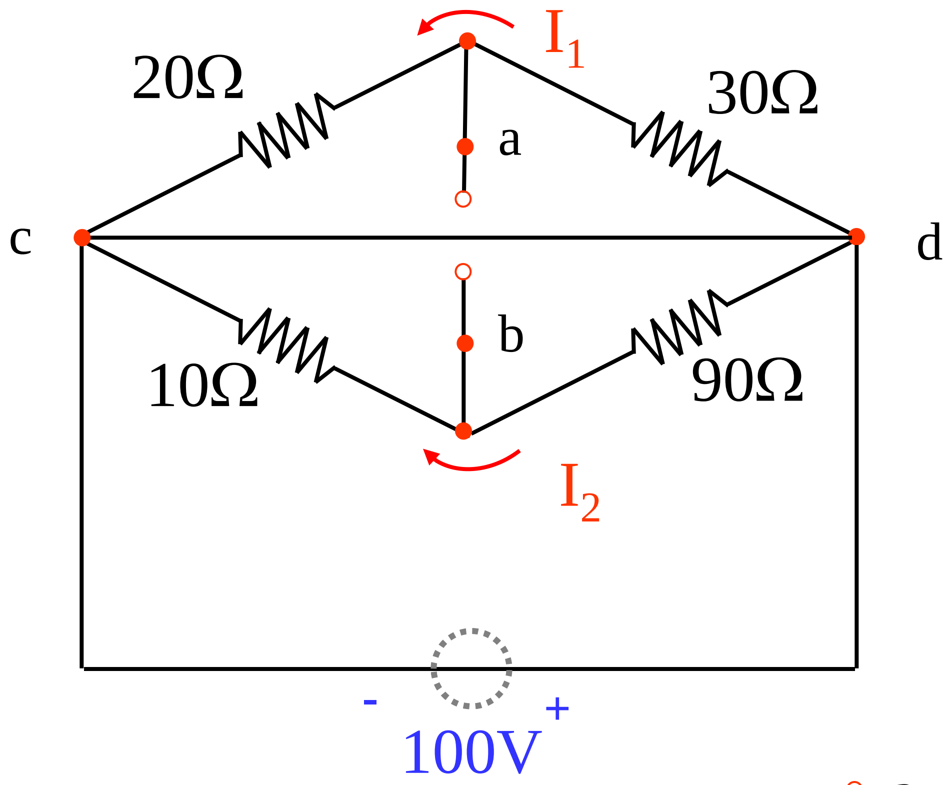


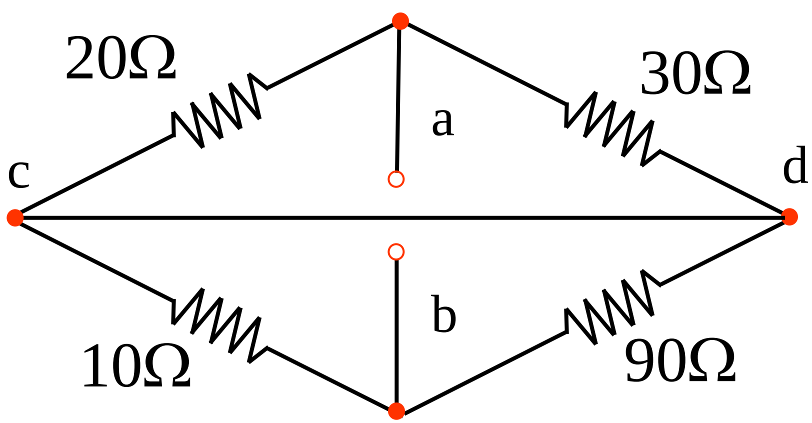
$$I_1 = \frac{100V}{20\Omega + 30\Omega} = 2A$$

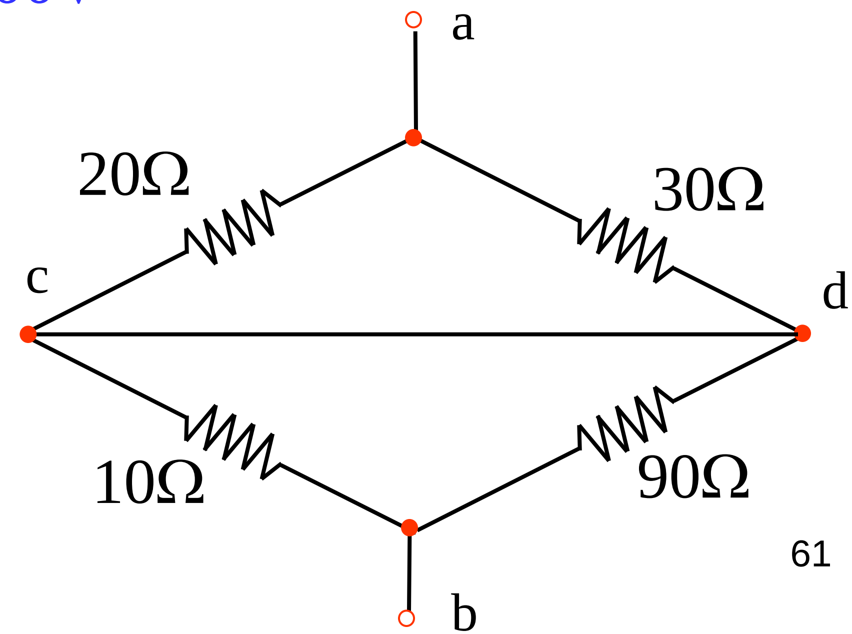
$$I_2 = \frac{100V}{10\Omega + 90\Omega} = 1A$$

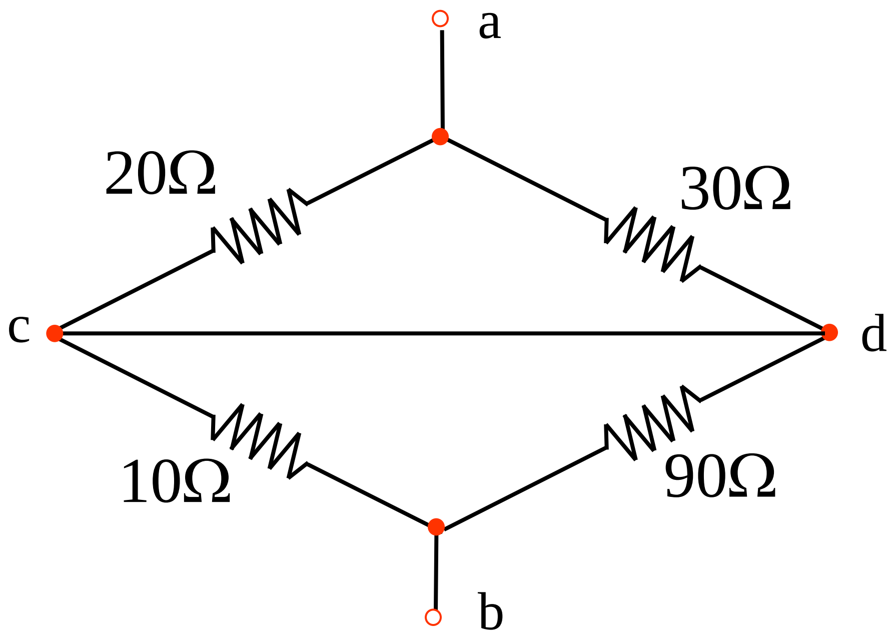
$$V_o = 20I_1 - 10I_2 = (20\Omega)(2A) - (10\Omega)(1A) = 30V$$

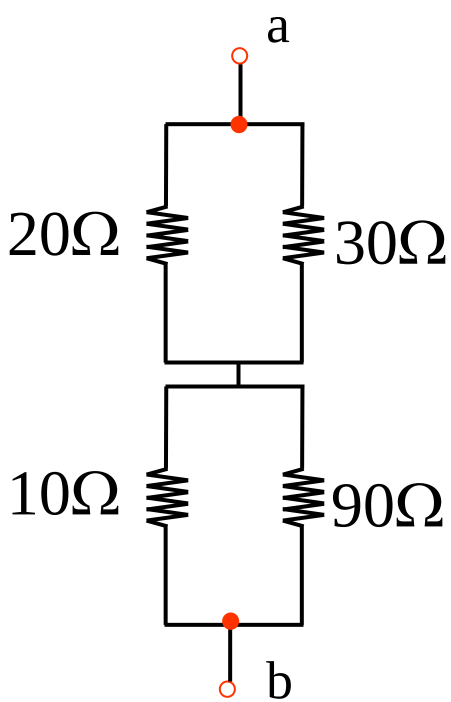
Equivalent R_o resistor:







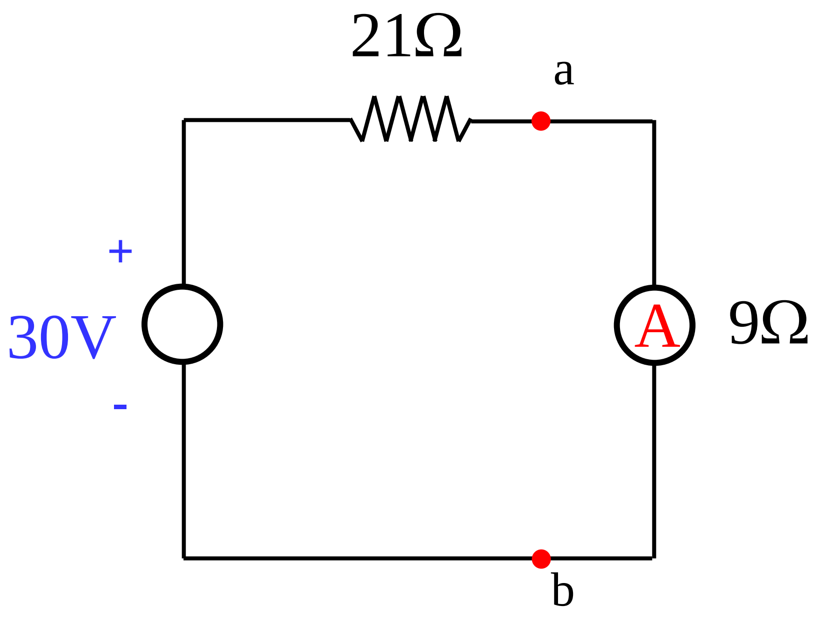




$$R_{ab} = \frac{(20\Omega)(30\Omega)}{20\Omega + 30\Omega} + \frac{(10\Omega)(90\Omega)}{10\Omega + 90\Omega} = 21\Omega$$

The equivalent circuit of the Thevenin will consist of a 30V voltage source and a 21Ω series connected resistor. If the ammeter is placed, current will flow through it (considering the internal resistance of the 9Ω ammeter)

$$I = \frac{30V}{21\Omega + 9\Omega} = 1A$$



Thevenin equivalent circuit