Ankara University
Engineering Faculty
Department of Engineering Physics

PEN207

Circuit Design and Analysis

Prof. Dr. Hüseyin Sarı

Chapter-4

Exponential Input and Transformed Circuits
(1/2)

Circuit Responses
Content

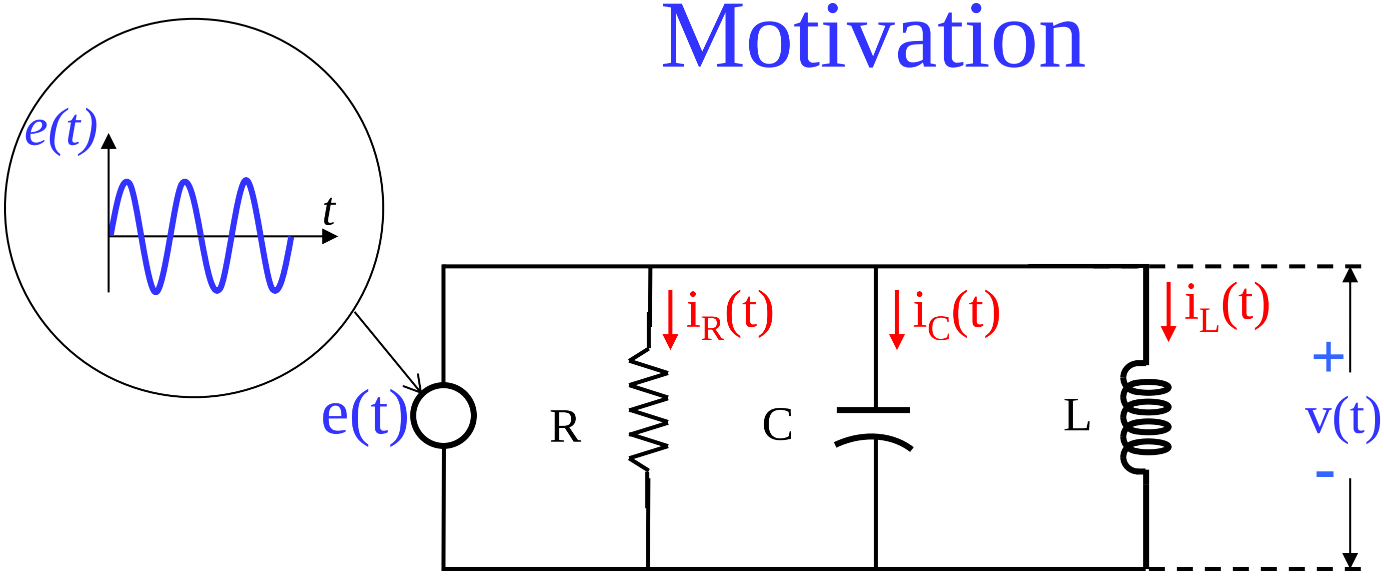
- **Representation of Circuit Input by Exponential Function**
- **Response of Indivudual Circuit Elements**
- **Exponential Input**
- **Sinusoidal Input**
- **Transformed Circuits**
- **Impedance and Admittance**
- **Circuit Analalysis with Transformed Circuits**

In this chapter,

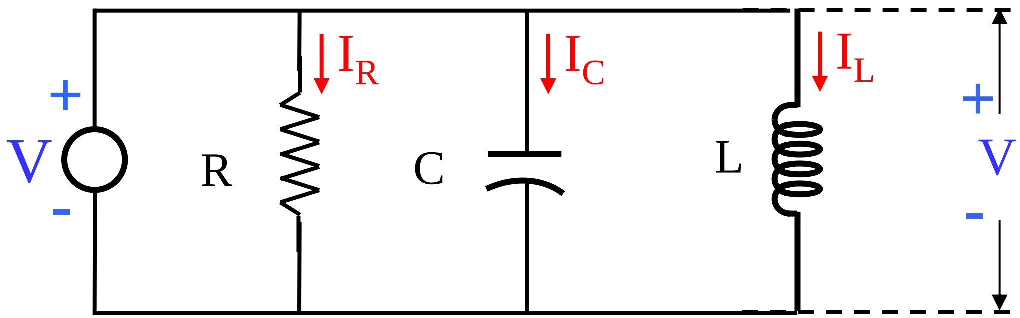
- The response of a circuit to the exponentially varying input,
- Transformation of a circuit containing capacitor and inductor to a simple resistive circuit,
- Impedance and admittance

will be learned.

Motivation



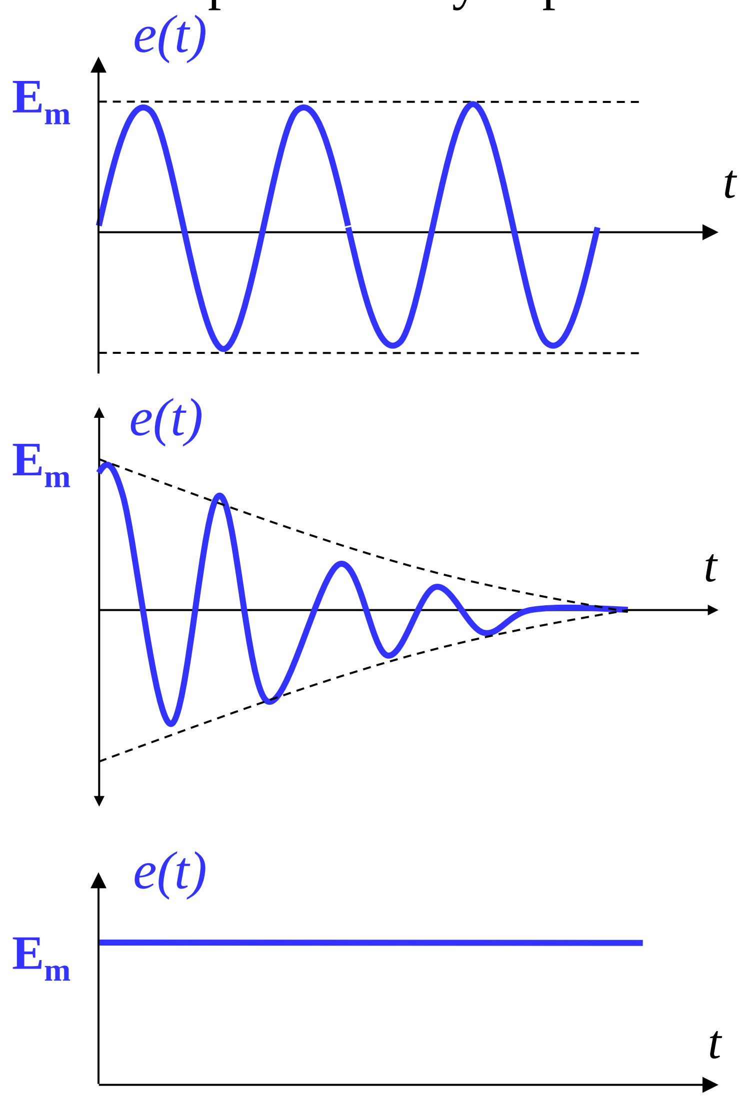
How can above circuit (ac) be analysed as a direct current (dc) circuit?

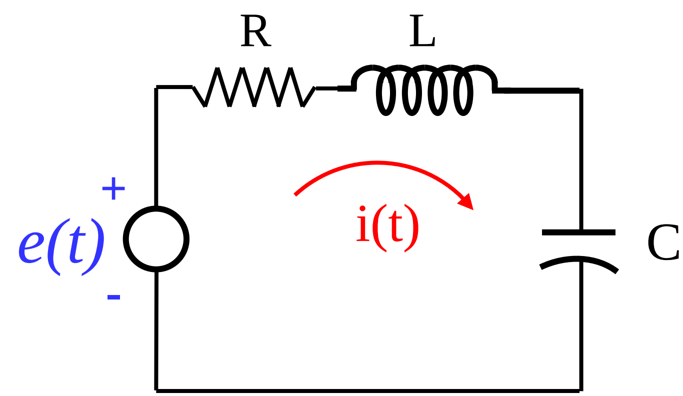


How is the equivalent resistance (impedance) of the circuit including resistance, inductor and capacitor calculated? Is the resistance dependent on frequency in such circuits?

Representation of Input by Exponential Functions

In applications, many circuit excitation can be represented with exponential functions. Thus, a periodically changing signal (ac) and direct signal (dc) can also be represented by exponential functions.





$$e(t) = E_m e^{st}$$

s=a+jb complex number

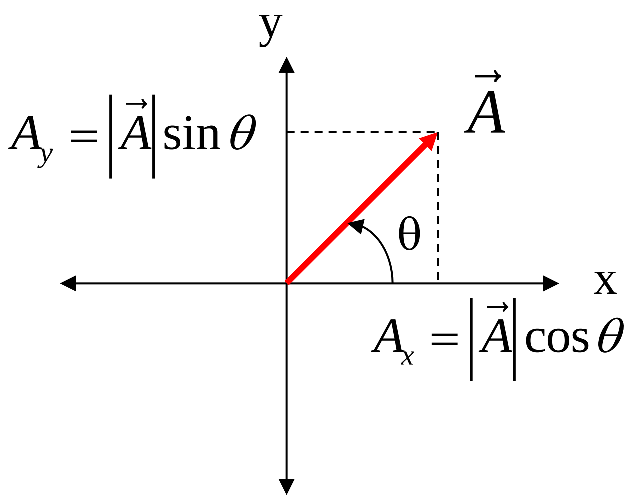
s=α+jω complex number

If α is zero: $i(t) = Ie^{i\omega t}$

$$v(t) = Ve^{i\omega t}$$

Complex Numbers-Reminder

A vector (A) on a plane can be expressed in terms of its components.



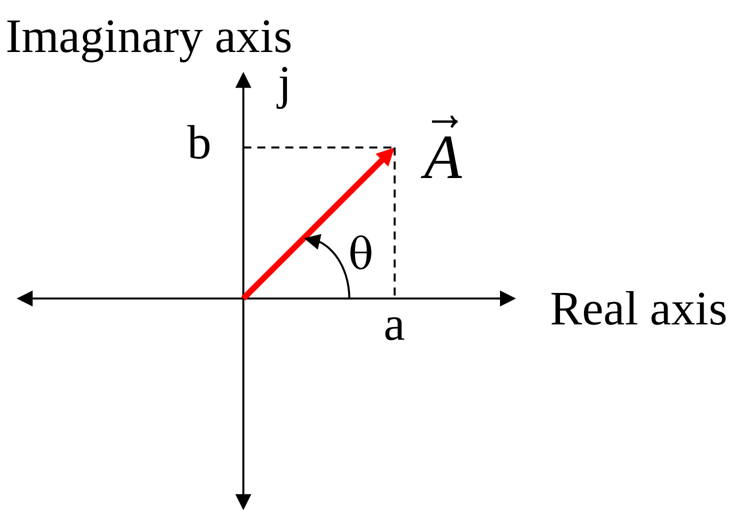
$$\vec{A} = \left(|\vec{A}|\cos\theta\right)\hat{i} + \left(|\vec{A}|\sin\theta\right)\hat{j}$$

$$\vec{A} = \left(A_x\right)\hat{i} + \left(A_y\right)\hat{j}$$

$$|A| = \sqrt{A_x^2 + A_y^2}$$

$$\theta = -\tan^{-1}\left(\frac{A_y}{A_x}\right)$$

The vector A can be represented by a complex number in the complex plane.



$$\vec{A} = a + jb$$

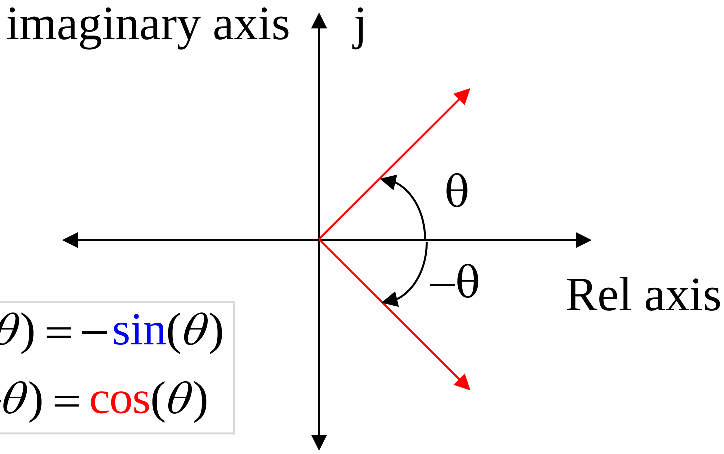
$$|A| = (a + jb).(a - jb)$$

$$= \sqrt{a^2 + b^2}$$

$$\theta = -\tan^{-1}\left(\frac{b}{a}\right)$$

Complex Numbers-Reminder

Relationship between complex number, trigonometric and exponential functions:



$$\sin(-\theta) = -\sin(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\theta \rightarrow -\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

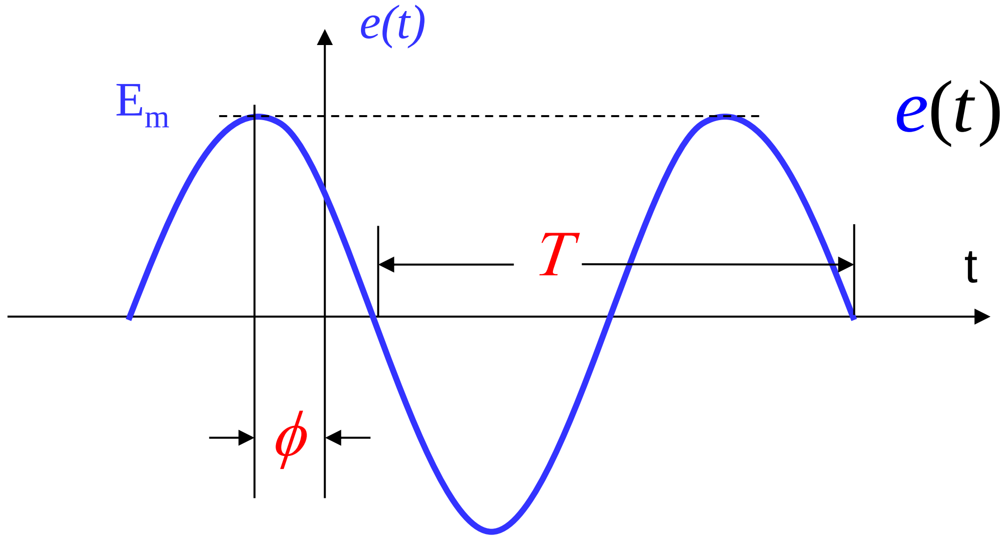
Trigonometric functions:

$$\sin(\omega t + \phi) = \cos(\omega t + \phi - \pi/2)$$

$$A\cos(\omega t) + B\sin(\omega t) = \sqrt{A^2 + B^2}\cos(\omega t - \tan^{-1}(B/A))$$

Note: $\tan^{-1}(B/A)$ must be represented as *rad* unit

A periodic function can be expressed by exponential functions.



$$e(t) = E_m \cos(\omega t + \phi)$$

$$\omega = \frac{2\pi}{T} = 2\pi\upsilon$$

ϕ = phase angle

ω = angular frequency

T = periyod

ν = frequency

$$e(t) = E_m \cos(\omega t + \phi) \quad \Rightarrow \quad e(t) = \frac{E_m}{2}\left[e^{j(\omega t+\phi)} + e^{-j(\omega t+\phi)}\right]$$

$$e(t) = \frac{E_m}{2} e^{j\phi} e^{j(\omega t)} + \frac{E_m}{2} e^{-j\phi} e^{-j(\omega t)} = \left(\frac{E_m}{2} e^{j\phi}\right) e^{j(\omega t)} + \left(\frac{E_m}{2} e^{-j\phi}\right) e^{-j(\omega t)}$$

$$e(t) = \mathbf{E_1} e^{j(\omega t)} + \mathbf{E_2} e^{-j(\omega t)}$$

Amplitudes:

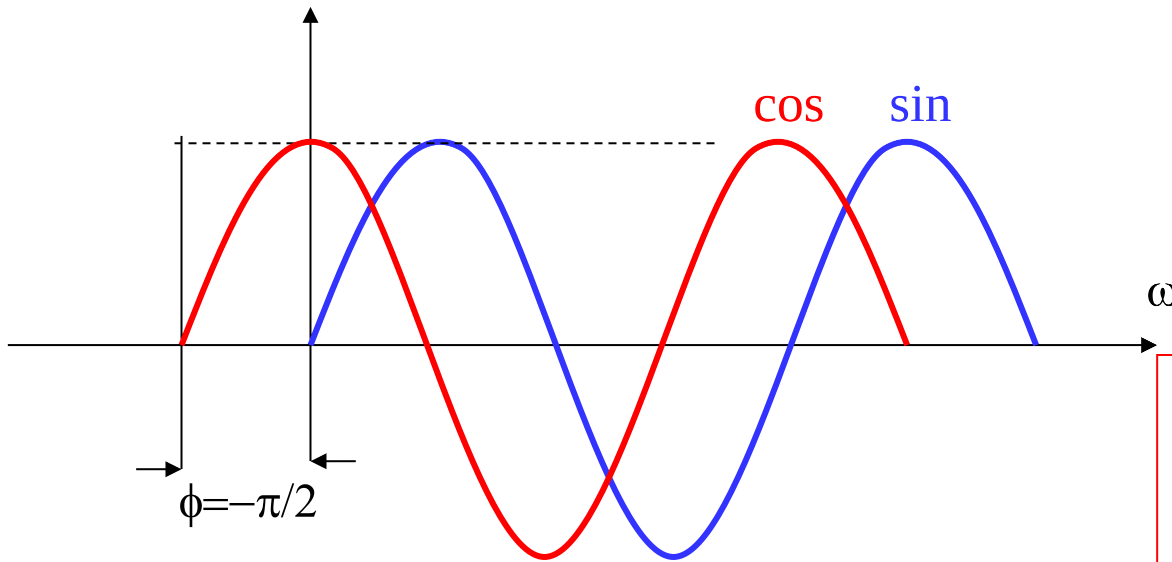
$$\mathbf{E_1} \equiv \frac{E_m}{2} e^{j\phi} \qquad \mathbf{E_2} \equiv \frac{E_m}{2} e^{-j\phi}$$

E_1 and E_2 are amplitıdes represented by complex numbers

Expressing voltages, currents, or current-voltage, voltage-current ratios in a circuit with complex numbers makes calculations quite simple.

Other functions can also be represendet by periodic function.

$$\sin(\omega t+\phi)=\cos(\omega t+\phi-\pi/2)$$



Note:
tan^{-1}(B/A) must be represented as *rad* unit

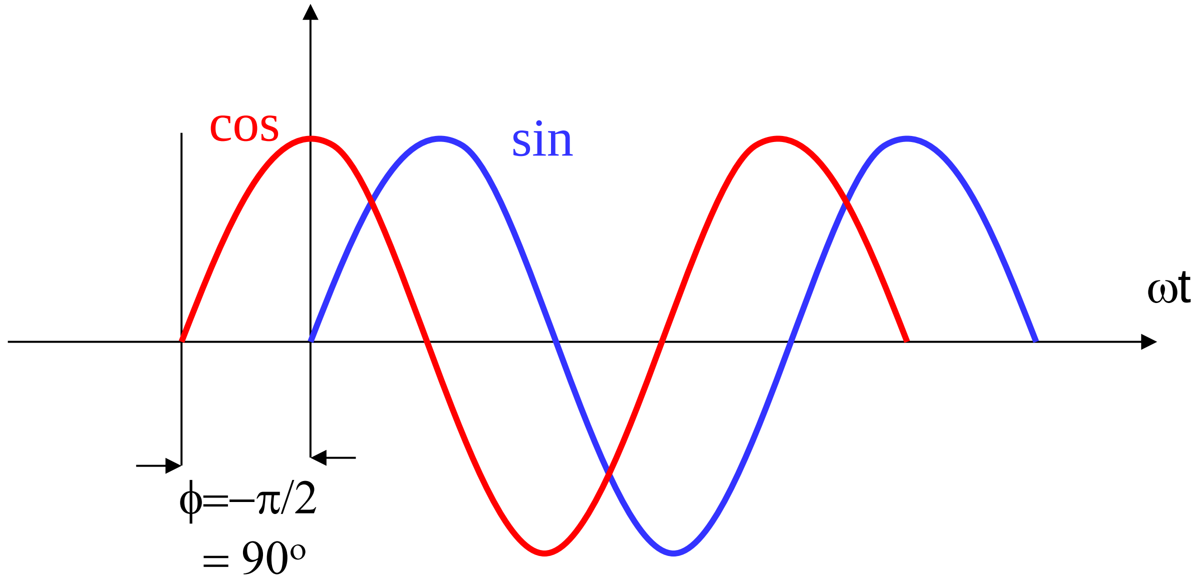
$$A\cos(\omega t)+B\sin(\omega t)=\sqrt{A^2+B^2}\cos(\omega t-\tan^{-1}(B/A))$$

$$\left[A\cos(\omega t)+B\sin(\omega t)\right]^2=A^2\cos^2(\omega t)+B^2\sin^2(\omega t)+2AB\cos(\omega t)\sin(\omega t)$$

$$\sin(x)\cos(y)=\frac{1}{2}\left[\sin(x+y)+\cos(x-y)\right]$$

$$\sin^2(x)=1-\cos^2(y)$$

$$\left[A\cos(\omega t)+B\sin(\omega t)\right]^2=(A^2-B^2)\cos^2(\omega t)+B^2+2AB\cos(\omega t)\sin(\omega t)$$



Sine function is 90º (or π/2) behind of cosine function.

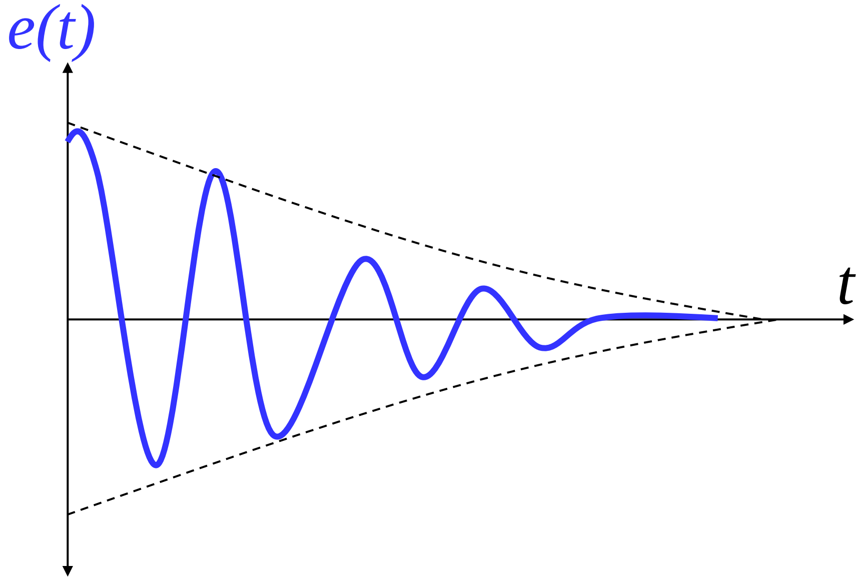
$$\sin(\omega t) = \cos(\omega t - \pi/2)$$

$$\frac{d\sin(t)}{dt} = \cos(t) \qquad \frac{d\cos(t)}{dt} = -\sin(t)$$

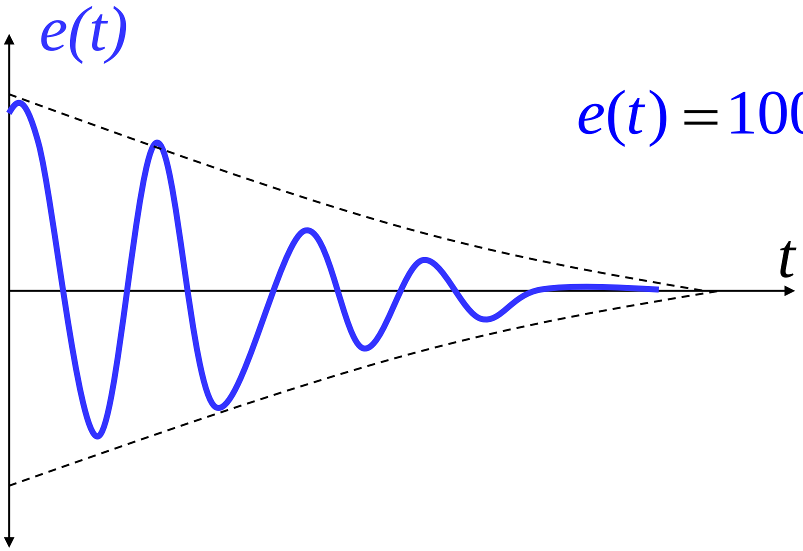
Cosine function is 90º (or π/2) ahead of sine function

Example-4.1: Express the following waveform as an exponential stimulation (function).

$$e(t) = 100e^{-2t}\sin(2t + \frac{\pi}{4})$$



Solution-4.1:



$$e(t) = 100e^{-2t}\sin(2t + \frac{\pi}{4})$$

First, the quantity sin(2t+π/4) is converted to cosine form.

$$\sin(x) = \cos(x - \pi/2)$$

$$\sin(2t + \frac{\pi}{4}) = \cos(2t + \frac{\pi}{4} - \frac{\pi}{2})$$

$$e(t) = 100e^{-2t}\cos(2t + \frac{\pi}{4} - \frac{\pi}{2}) = 100e^{-2t}\cos(2t - \frac{\pi}{4})$$

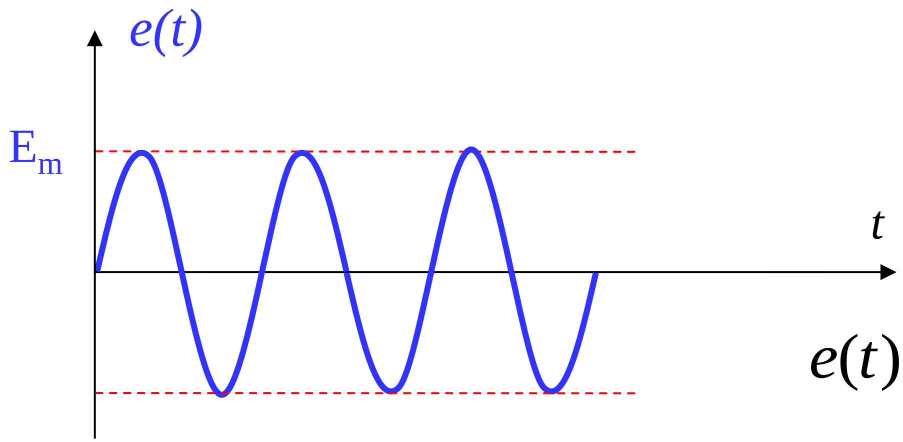
$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$e(t) = 100e^{-2t}\left[\frac{e^{j(2t-\pi/4)} + e^{-j(2t-\pi/4)}}{2}\right]$$

$$e(t) = 50e^{-j\pi/4}e^{-t(2-j2)} + 50e^{j\pi/4}e^{-t(2+j2)}$$

$s = -(2 \pm j2)$

Expression of a Periodic Function with Exponential Functions

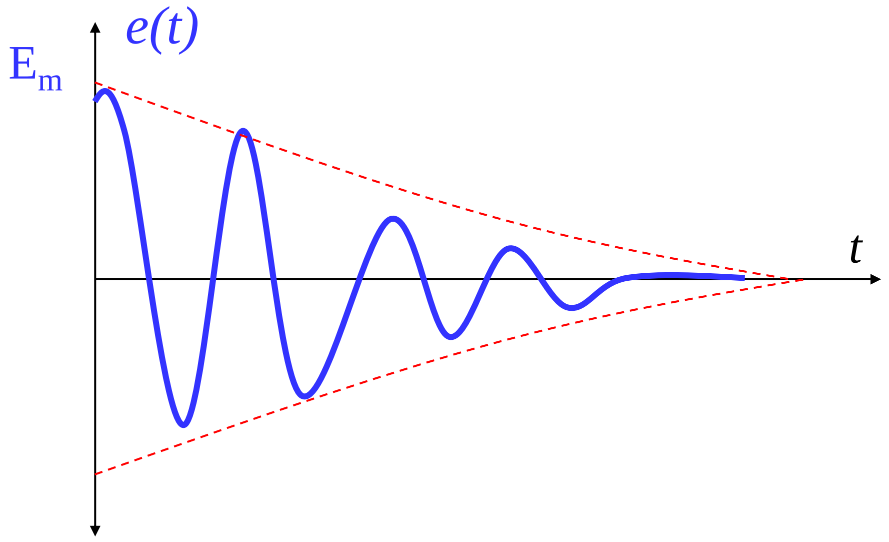


$$e(t)=E_m e^{st}$$

s=0+jb (pure) complex number:

$$e(t)=E_m e^{-(0-jb)t}=E_m e^{jbt}$$

$$e(t)=E_m e^{jbt}=E_m e^{j\omega t}=E_m Sin(\omega t+\phi)$$

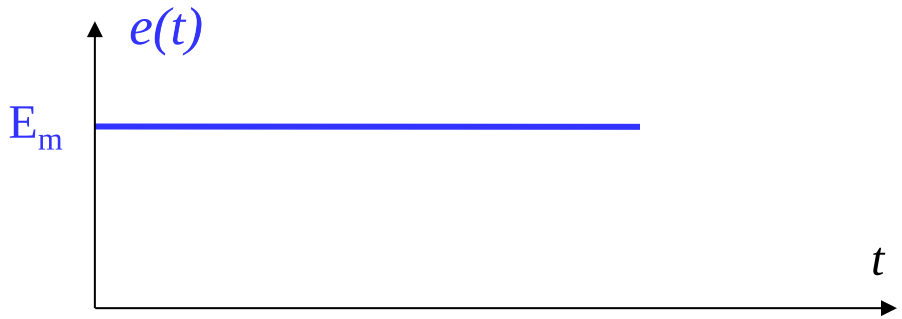


$$e(t)=E_m e^{st}$$

s=a+jb complex number:

$$e(t)=E_m e^{-(a-jb)t}=(E_m e^{-at})\sin(bt)$$

$$e(t)=(E_m e^{-at})\sin(\omega t)$$



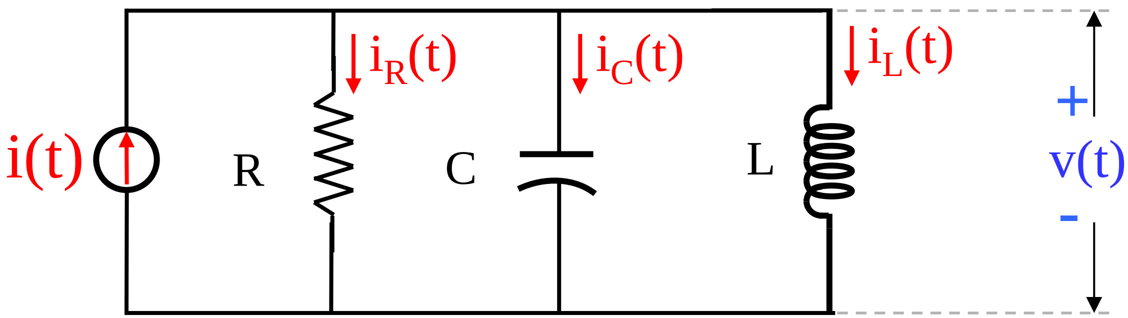
$$e(t)=E_m e^{st}$$

s=0 (ω=0)

$$e(t)=E_m e^{-(0)t}=E_m$$

Response of Individual Circuit Elements

Let's look at the most general situation where stimulation is $\mathbf{e^{st}}$. Suppose a **current** $i(t)=Ie^{st}$ is applied to the following circuit:



Each of the circuit elements will see a common voltage of $v(t)=Ve^{st}$ (V is the amplitude of the voltage).

Amplitudes of current and voltage:

$$i_R(t)=\frac{v(t)}{R}=\frac{V}{R}e^{st}=I_Re^{st}$$

$$i_C(t)=C\frac{dv(t)}{dt}=CsVe^{st}=I_Ce^{st}$$

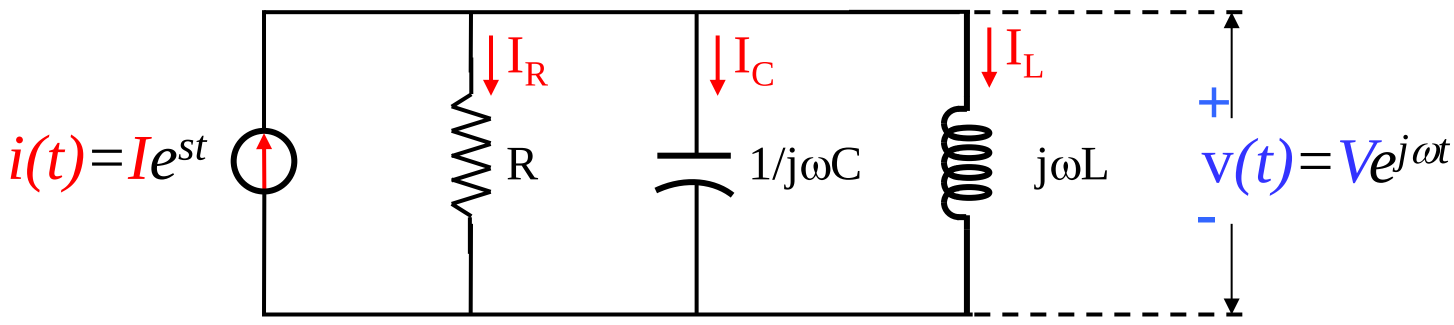
$$i_L(t)=\frac{1}{L}\int v(t)dt=\frac{1}{sL}Ve^{st}=I_Le^{st}$$

$$I_R=\frac{V}{R}\Rightarrow\frac{V}{I_R}=R=R_R=\frac{1}{G}$$

$$I_C=CsV\Rightarrow\frac{V}{I_C}=\frac{1}{sC}\equiv R_C$$

$$I_L=\frac{1}{sL}V\Rightarrow\frac{V}{I_L}=sL\equiv R_L$$

Response of Individual Circuit Elements-2



If a circuit has an exponential stimulator (source), the resistance of the circuit is not constant! It is a function of frequency s of the source.

$$i(t) = I_R e^{j\omega t}$$

$$v(t) = V e^{j\omega t}$$

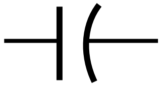
| s=frequency (ω) |

$$i(t) = I e^{j\omega t} \qquad v(t) = V e^{j\omega t}$$

R

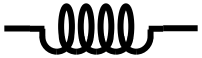
Resistor: $\dfrac{V}{I_R} = R_R$ Independent of frequency! (Real)

C



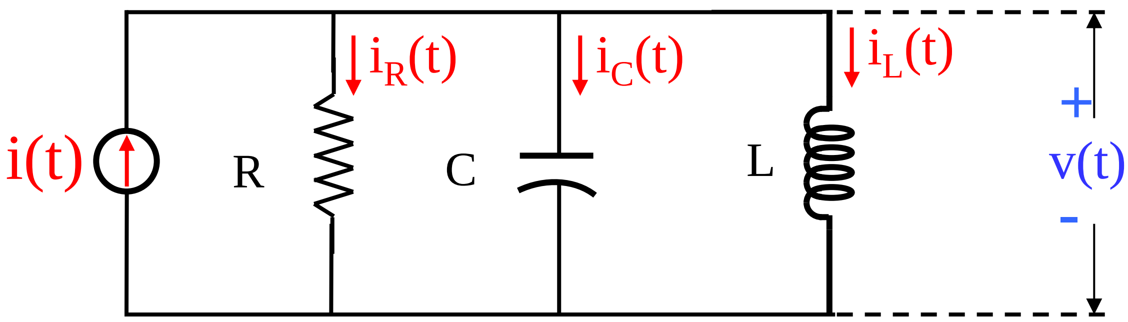
Resistor of capacitor: $\dfrac{V}{I_C} = \dfrac{1}{j\omega C} \equiv R_C$ Decreases with frequency! (Complex)

L



Resistor of inductor: $\dfrac{V}{I_L} = j\omega L \equiv R_L$ Increases with frequency! (Complex)

Response of Individual Circuit Elements-3



In general the resistance of the elements in the circuit is the function of s (frequency) and also the relations between the amplitudes of the voltages and currents the voltage-current relations is represented in *frequency domain*.

$$I_R = \frac{V}{R} \Rightarrow \quad \frac{V}{I_R} = R \qquad V_R = R_C I_C$$

$$I_C = CsV \Rightarrow \frac{V}{I_C} = \frac{1}{sC} \equiv R_C \qquad V_C = \left(\frac{1}{j\omega C}\right) I_C = \left(-j\frac{1}{\omega C}\right) I_C \equiv -jR_C I_C$$

$$I_L = \frac{1}{sL} V \Rightarrow \frac{V}{I_L} = sL \equiv R_L \qquad V_L = j\omega L I_L \equiv jR_L I_L$$

R_L and R_C complex number; a vector!

Impedance and Admittance

$$I_R = \frac{V}{R} \Rightarrow \frac{V}{I_R} = R = \frac{1}{G}$$

For many applications, the ratio of the voltage between the two ends to the current is important. This ratio is called ***impedance*** and is indicated by the Z(ω) symbol.

$$I = \frac{V}{Z(\omega)} \Rightarrow \frac{V}{I} = Z(\omega) = \frac{1}{Y(\omega)}$$

$$\frac{V}{I} \equiv Z(\omega) \qquad \text{Impedance}$$

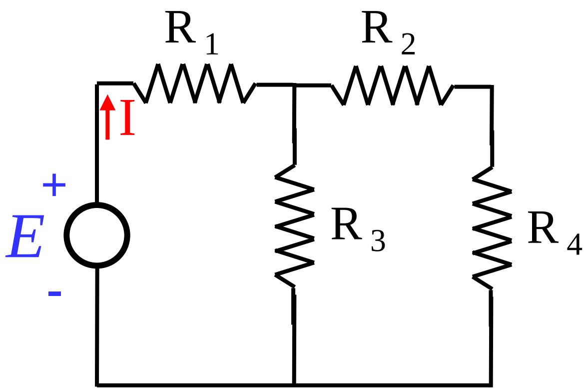
The unit of impedance is **ohms**

Admittance is the incerse of Impedence.

$$Y(\omega) \equiv \frac{1}{Z(\omega)} \qquad \text{Admittance}$$

Impedance and Admittance

DC Current and Voltage

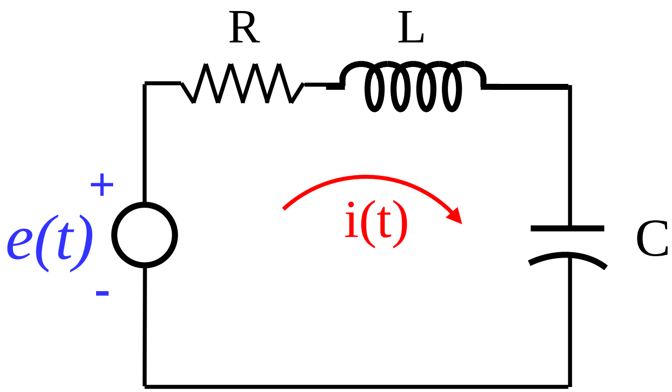


Equivalent Resistor $\frac{V}{I} = R_{eq}$

Equivalent Conductor $G_{eq} = \frac{1}{R_{eq}}$

$$R_{eq} = constant$$

AC Current and Voltage



Empedance $\frac{V}{I} = Z(\omega)$

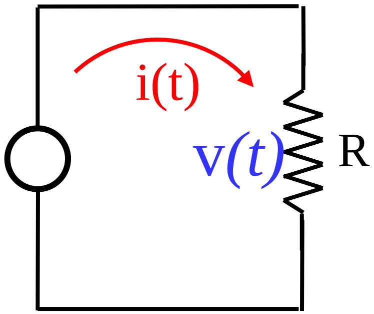
Admittance $Y(\omega) = \frac{1}{Z(\omega)}$

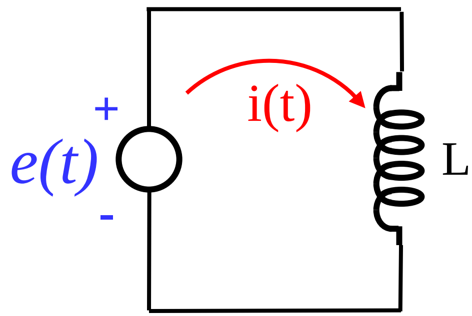
$$Z(\omega) = function\ of\ \ frequency(\omega)!$$

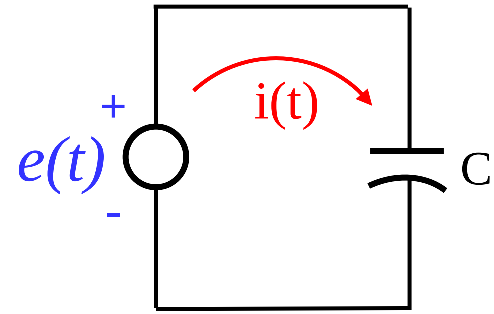
Impedance

$i(t) = Ie^{st}$ $v(t) = Ve^{st}$

$e(t)=E_o e^{\omega t}$
$=E_o Sin(\omega t+\phi)$





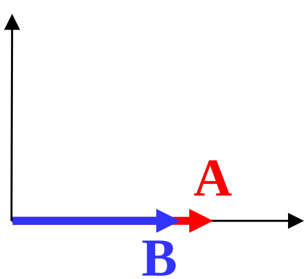


$$R_R = \frac{V}{I_L} = R = cons\tan t$$

$$V = RI$$

R=1Ω
ω=50Hz => R=1Ω
ω=1000Hz => R=1Ω

$$B = R_R A$$

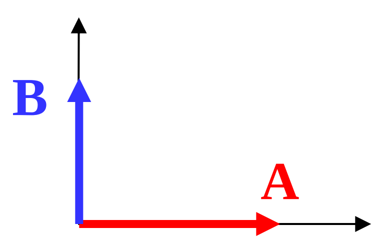


$$R_L = \frac{V}{I_L} = sL = j\omega L \equiv Z_L(\omega)$$

$$V = \left(j\omega L \right) I$$

L=1H
ω=50Hz => Z=50Ω
ω=1000Hz => Z=1000Ω

$$B = j\left|Z_L\right| A$$

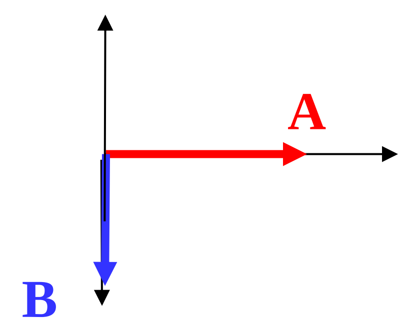


$$R_C = \frac{V}{I_C} = \frac{1}{sC} = \frac{1}{j\omega C} = -j\frac{1}{\omega C} \equiv Z_C(\omega)$$

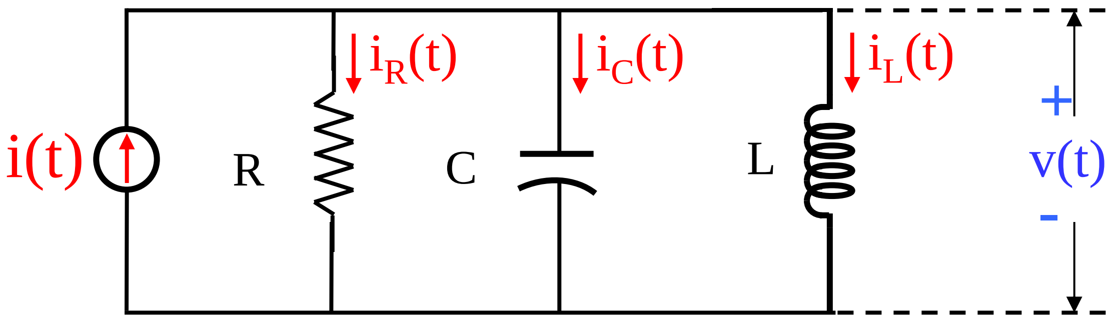
$$V = \left(-j\frac{1}{\omega C} \right) I$$

C=1F
ω=50Hz => Z=0,02Ω
ω=1000Hz => Z=0,001Ω

$$B = -j\left|Z_C\right| A$$



Let's find impedance and admittance of the circuit below by applying Kirchhoff's Current Law (KCL).



Kirchhoff's Current Law (KCL):

$$i(t) = i_R(t) + i_C(t) + i_L(t) = \frac{1}{R}v(t) + C\frac{dv(t)}{dt} + \frac{1}{L}\int v(t)dt$$

If the frequency-dependent values of the circuit elements are replaced :

$$i(t) = (\frac{1}{R} + j\omega C + \frac{1}{j\omega L})Ve^{j\omega t} = Ie^{j\omega t}$$

Impedance

$$Z(\omega) = \frac{V}{I} = \frac{1}{1/R + j\omega C + 1/j\omega L}$$

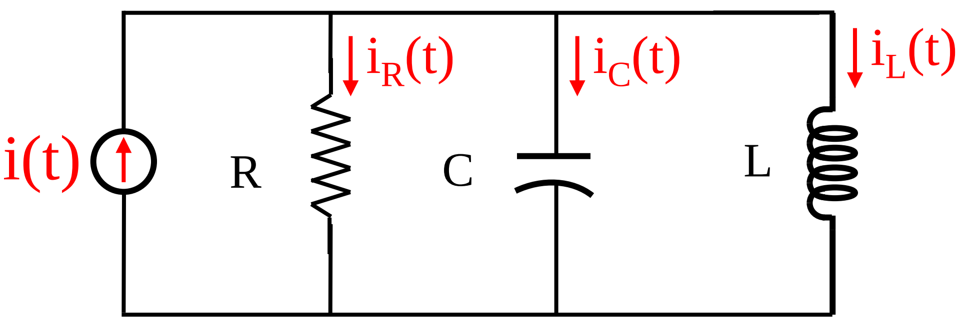
Admittance

$$Y(\omega) = \frac{I}{V} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

By converting a circuit having a periodically changing source into a frequency domain, we can analyze it as a constant excited circuit consisting of only resistors.

Transformed Circuit

In Time Domain



$$i(t) = Ie^{j\omega t}$$

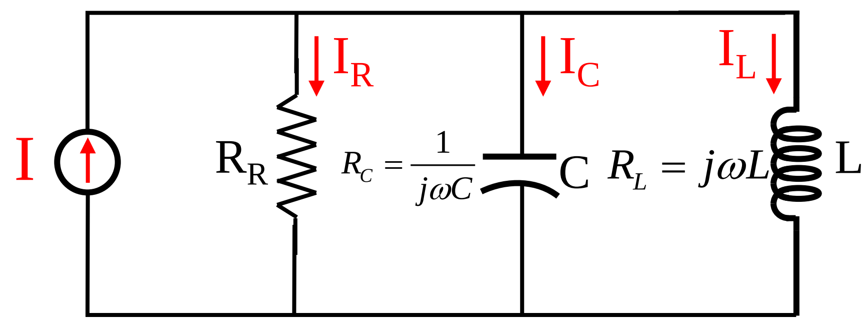
$$v(t) = Ve^{j\omega t}$$

$$i_R(t) = \frac{v(t)}{R}$$

$$i_C(t) = C\frac{dv(t)}{dt}$$

$$i_L(t) = \frac{1}{L}\int v(t)dt$$

In Frequency Domain



$$i(t) = Ie^{j(\omega=0)t} = I$$

$$v(t) = Ve^{j(\omega=0)t} = V$$

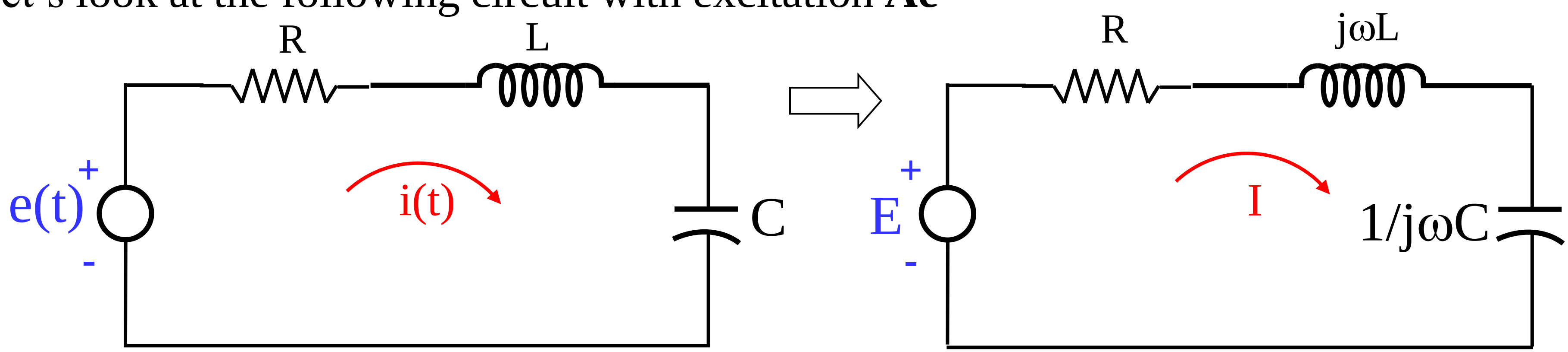
$$I_R = \frac{V}{R}$$

$$I_C = \frac{V}{R_C} = \frac{V}{1/j\omega C}$$

$$I_L = \frac{V}{R_L} = \frac{V}{j\omega L}$$

Parallel Circuit

Let's look at the following circuit with excitation **A**$e^{j\omega t}$



In Time Domain

In Frequency Domain

$$e(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt \Rightarrow E = RI + j\omega LI + (1/j\omega C)I$$

$$I = \frac{E}{R + j\omega L + (1/j\omega C)} = \frac{E}{Z(\omega)}$$

The denominator is equal to the series impedance of the circuit. The current as a function of time can be found by multplyind $e^{j\omega t}$:

$$i(t) = Ie^{j\omega t} = \frac{E}{R + j\omega L + (1/j\omega C)}e^{j\omega t}$$

Tepkiyi bulmak için gerekli basamaklar:

1- Uyarım $\mathbf{Ae^{st}}$ biçiminde tanımlanır.
2- Devre dönüşümü yapılır (zaman bölgesinden frekans bölgesine).
3- Uygun Kirchhoff yasası denklemleri (KAY veya KGY) yazılır ve frekans bölgesindeki tepki bulunur.
4- Frekans bölgesindeki tepki e^{st} ile çarpılarak zaman bölgesindeki tepkiye (zaman fonksiyonu ile çarpılarak) dönüştürülür.

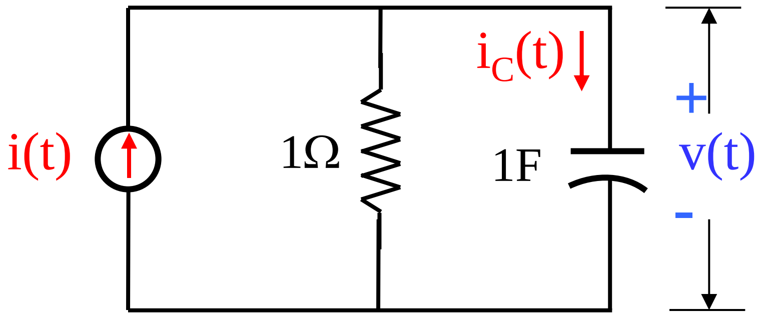
Transformation to Frequency Domain

$$e(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt \Rightarrow E = RI + sLI + (1/sC)I$$

$$i(t) = Ie^{st} = \frac{E}{R + sL + (1/sC)}e^{st} \Leftarrow I = \frac{E}{R + sL + (1/sC)} = \frac{E}{Z(s)}$$

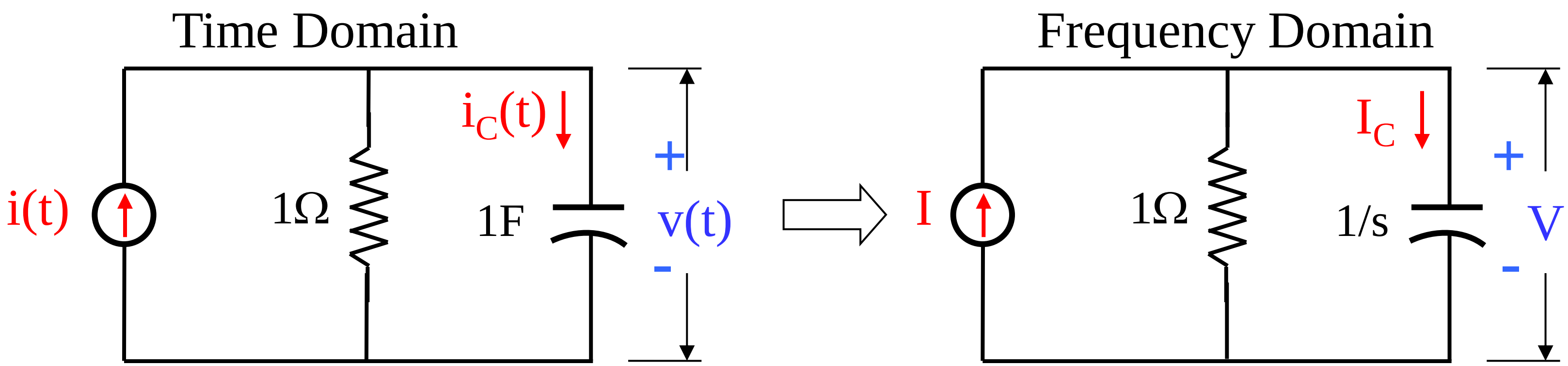
Transformation back to Time Domain

Example-4.2: Find the current through the capacitor $i_C(t)$ for the current source $i(t)$ **a)** $10e^{-2t}$ and **b)** 10A in the circuit below.



Solution-4.2:

(a) $i(t)=10e^{-2t}$



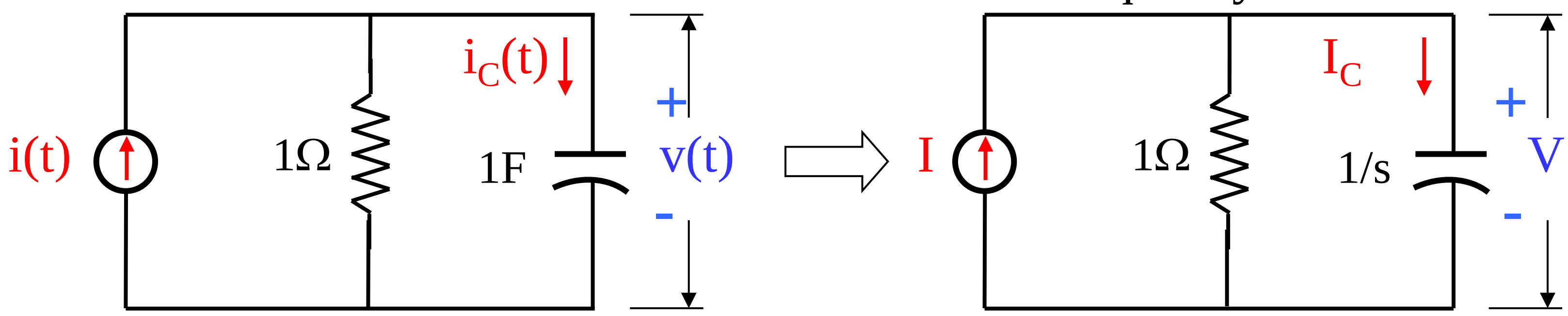
$$I=(1mho)V+sV \Rightarrow V=\frac{I}{1+s}$$

Current on capacitor: $I_C=sV=\dfrac{s}{1+s}I$

$$\left.\begin{aligned} i(t)&=10e^{-2t}=Ie^{st} \\ I&=10;\ s=-2 \end{aligned}\right\} \Rightarrow I_C=\frac{s}{1+s}I=\frac{-2}{1-2}(10)=20 \Rightarrow \boxed{i_C(t)=20e^{-2t}}$$

(b) i(t)=10 A

Time Domain　　　　Frequency Domain

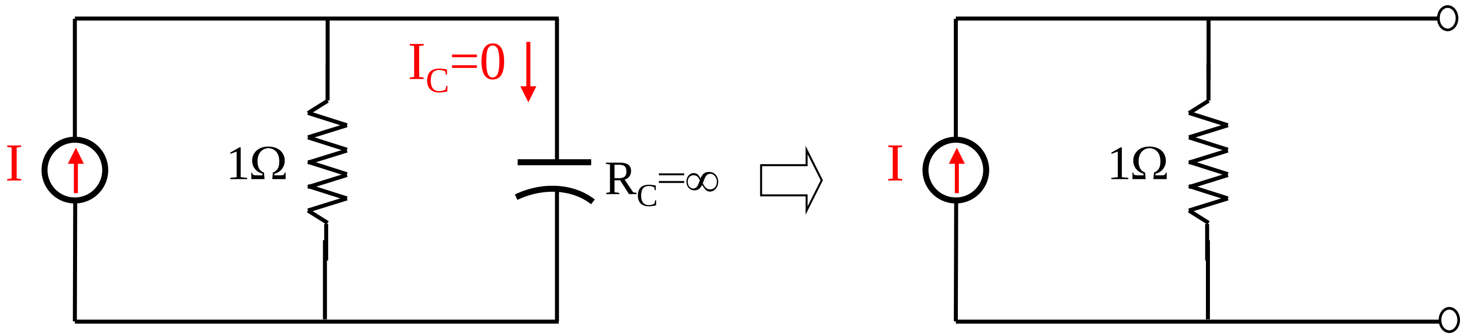


$$I = (1mho)V + sV \Rightarrow V = \frac{I}{1+s}$$

Current on capacitor: $I_C = sV = \dfrac{s}{1+s} I$

$$\left.\begin{array}{l} i(t) = 10e^{st} = 10 \\ I = 10; s = 0 \end{array}\right\} \Rightarrow I_C = sV = \frac{s}{1+s} I = 0 \Rightarrow \boxed{i_C(t) = 0}$$

Capacitor acts as an open circuit in steady state (impedance (Z) for s=0 goes to infinity)

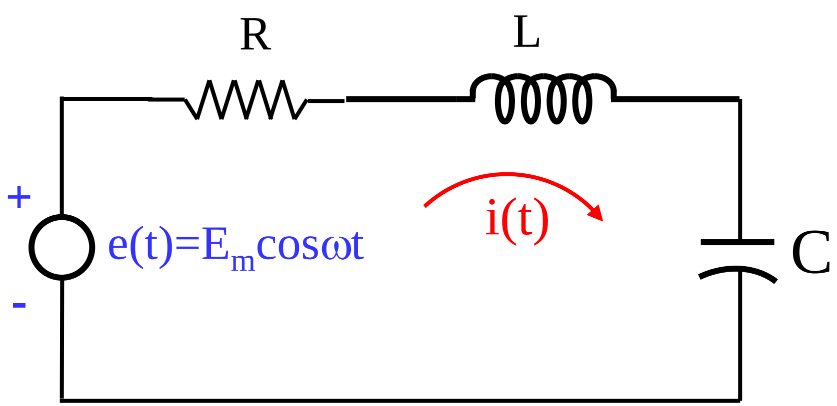


Sinusoidally Varying Sources

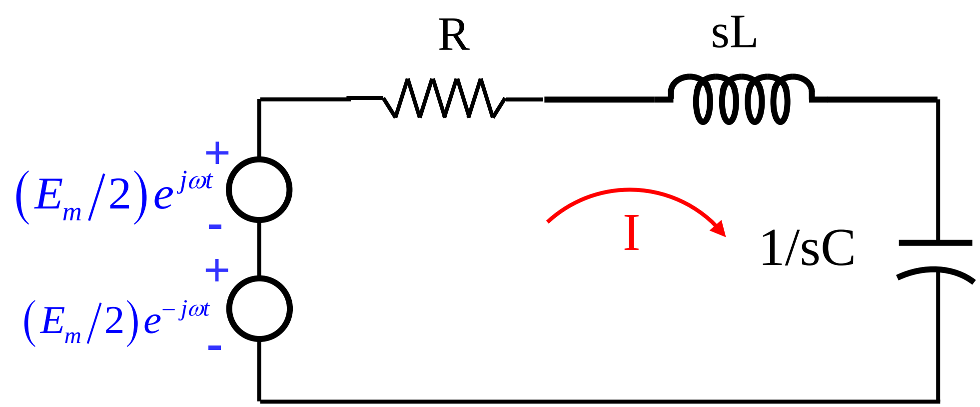
In this section sinusoidally varying source (sinωt, cosωt) will be investigated

biçimindeki bir kaynak gerilimi ile uyarılan aşağıdaki devreyi düşünelim. Consider the following circuit which is stimulated by a voltage source in the form of $e(t)=E_m\cos\omega t$.

Source voltage can be defined as the sum of two exponential functions.



In time domain



In frequency domain

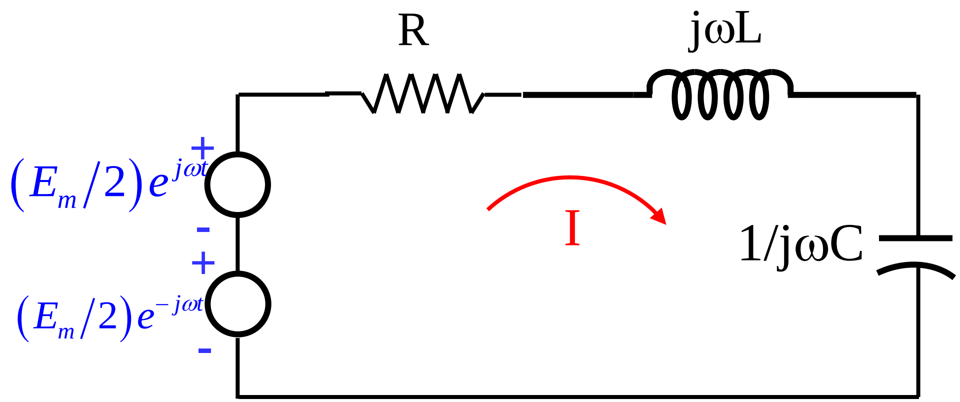
$$e(t)=E_m\cos(\omega t+\phi) \Rightarrow e(t)=\frac{E_m}{2}\left[e^{j(\omega t+\phi)}+e^{-j(\omega t+\phi)}\right]$$

$$e(t)=E_1e^{j(\omega t)}+E_2e^{-j(\omega t)} \Rightarrow$$

$$s_1=j\omega \qquad E_1\equiv\frac{E_m}{2}e^{j\phi}$$

$$s_2=-j\omega \qquad E_2\equiv\frac{E_m}{2}e^{-j\phi}$$

Sinusoidally Varying Sources



In frequency domain

$$E = (R)I + (sL)I + (1/sC)I \quad \Rightarrow \quad I = \frac{E}{R + j\omega L + 1/j\omega C}$$

Response can be calculated using the superposition principle :

$$s = j\omega \qquad\qquad s = -j\omega$$

$$I_1 = \frac{E_m/2}{R + j\omega L + 1/j\omega C} \qquad\qquad I_2 = \frac{E_m/2}{R - j\omega L + 1/(-j\omega C)}$$

Currents: $I_1 = \dfrac{E_m/2}{R + j\omega L + 1/j\omega C}$ $\quad I_2 = \dfrac{E_m/2}{R - j\omega L + 1/(-j\omega C)}$

Currents can be represented by exponential functions:

$$\mathbf{I}_1 = |I_1| e^{j\theta_1} \qquad \mathbf{I}_2 = |I_2| e^{j\theta_2}$$

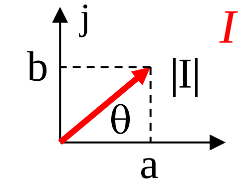
$$I_{1,2} = \frac{E_m/2}{R \pm j(\omega L + 1/\omega C)}$$

$$|I_{1,2}| = \left(\frac{1}{(a \pm jb)} \cdot \frac{(a \mp jb)}{(a \mp jb)} \right)^{1/2} = \frac{1}{\sqrt{a^2 + b^2}}$$

Amplitudes: $|I_1| = |I_2| = \dfrac{E_m/2}{\sqrt{R^2 + (\omega L + 1/\omega C)^2}} \equiv I$

Phase: $\theta_1 = -\theta_2 = -\tan^{-1}\left(\dfrac{\omega L - 1/\omega C}{R}\right)$

$$I = a + jb$$



$$|I| = \sqrt{(a + jb).(a - jb)}$$

$$= \sqrt{a^2 + b^2}$$

$$\theta = -\tan^{-1}\left(\frac{b}{a}\right)$$

Currents in time domain: $i_1(t) = |I_1| e^{j\theta_1} e^{j\omega t}$ $\quad i_2(t) = |I_2| e^{-j\theta_1} e^{-j\omega t}$

Total response can be expressed as a superposition of two currents: $i(t) = i_1(t) + i_2(t) = I\left[e^{j(\omega t + \theta_1)} + e^{-j(\omega t + \theta_1)} \right]$

Using the amplitudes and phases relation current in time domain can be written:

$$i(t) = \frac{E_m/2}{\sqrt{R^2 + (\omega L + 1/\omega C)^2}} \cos(\omega t + \theta_1) = I_m \cos(\omega t + \theta_1)$$

Total response can be determined by amplitude I_m and phase angle θ_1

Impedance Z(jω) and admittance Y(jω) are complex number if the driving force is periodic function of time.

impedance

$$\mathbf{Z} = R + jX$$

resistor ***reactance***

admitdance

$$\mathbf{Y} = G + jB$$

conduction ***susceptance***

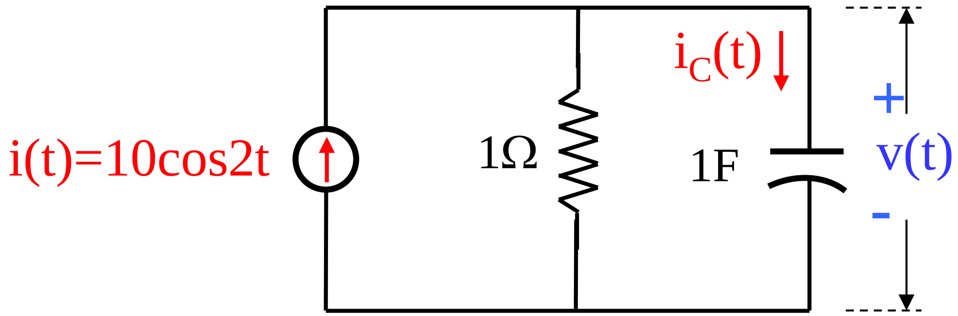
R and G are the real components of **Z**(jω) and **Y**(jω) and are referred to as ***resistance*** and ***conductivity***, respectively.

X and B are the imaginary components of **Z**(jω) and **Y**(jω) and are referred to ***reactance*** and ***susceptance,*** respectively.

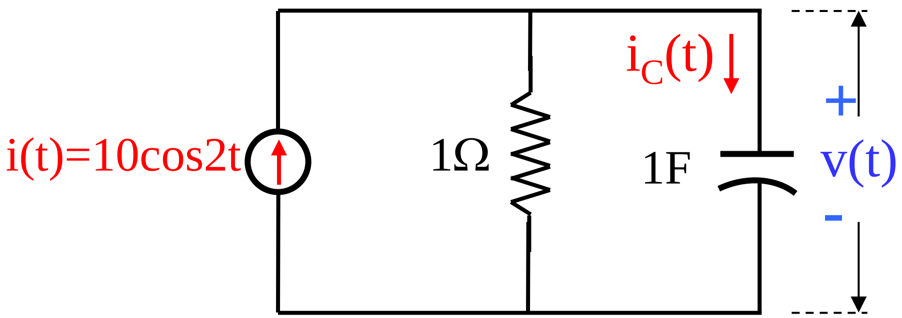
Inductance and capacitive reactances: $X_L = \omega L \quad X_C = \frac{1}{\omega C}$

Inductance and capacitive suseptances: $B_L = -\frac{1}{\omega L} \quad B_C = \omega C$

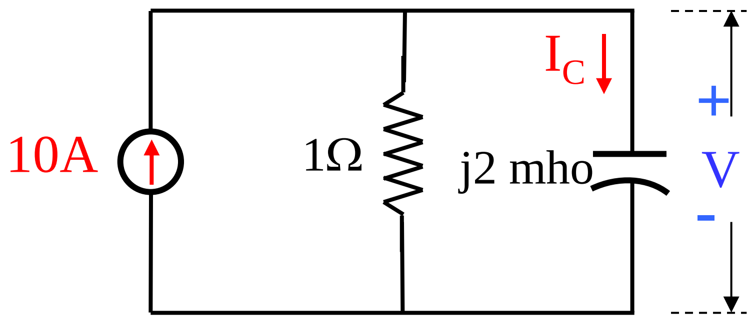
Example-4.3: Find the current in the capacitor $i_C(t)$ for the current source $i(t)=10\cos 2t$ in the circuit below.



Solution-4.3:



Time Domain



Frequency Domain

$$i(t) = Ie^{st} \qquad s = 2j \qquad I = 10$$

$$I = \frac{V}{1\,\Omega} + \frac{V}{R_C} \Rightarrow V = \frac{1}{1+2j} I$$

$$R_C = \frac{1}{sC} = \frac{1}{2j(1F)} = \frac{1}{2j}$$

$$I_C = 2jV \Rightarrow I_C = \frac{2j}{1+2j} I$$

$$G_C = 2j(1F) = 2j\ mho$$

Amplitude: $I = 10$

$$I_C = 8 + 4j$$

$$I_C = \frac{20j}{1+2j} = \frac{20j(1-2j)}{(1+2j)(1-2j)} = \frac{40+20j}{5} = 8+4j$$

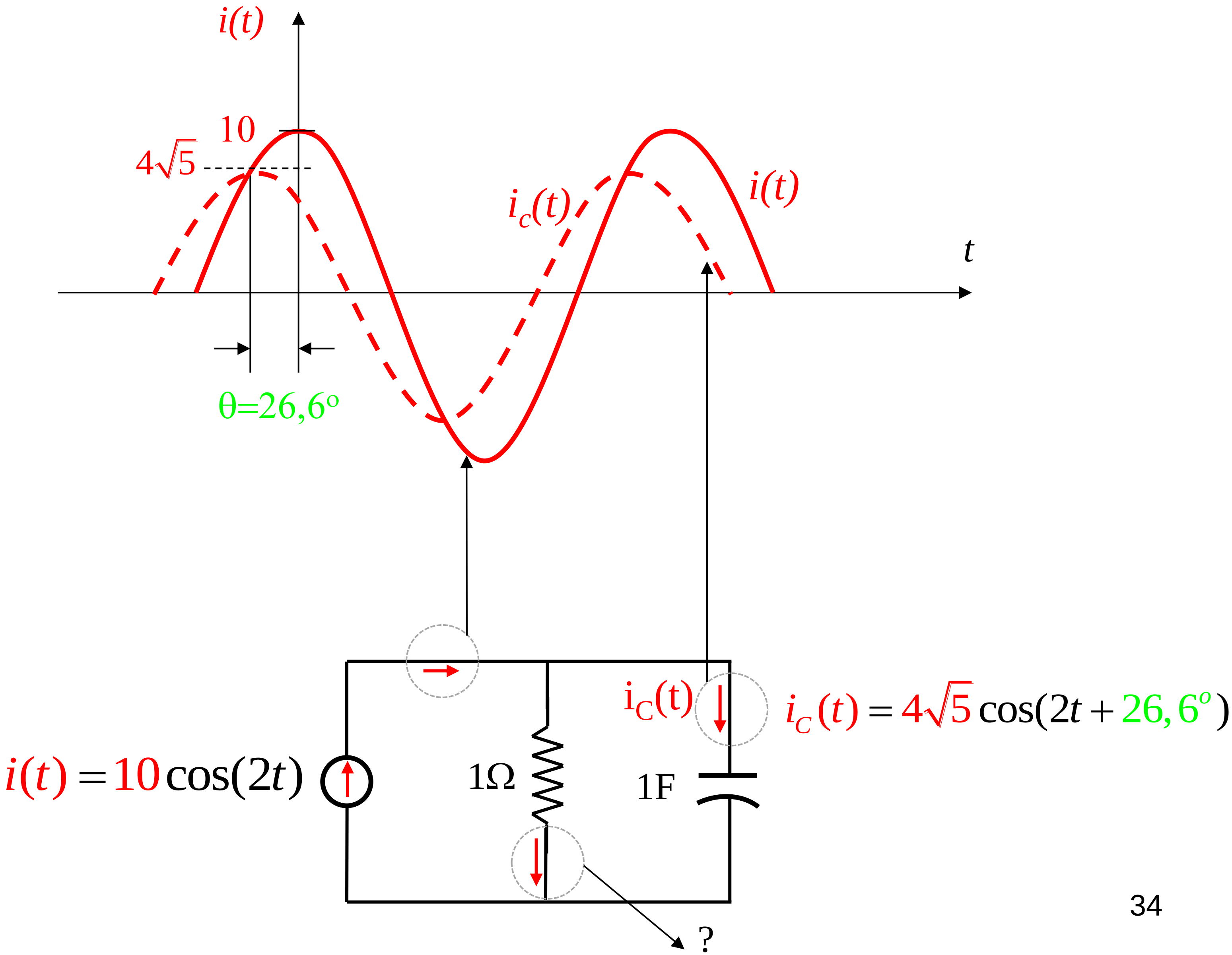
$$|I_C| = \sqrt{(8+4j)(8-4j)} = \sqrt{80} = 4\sqrt{5}$$

$$\tan^{-1}(4/8) = 26,6^o$$

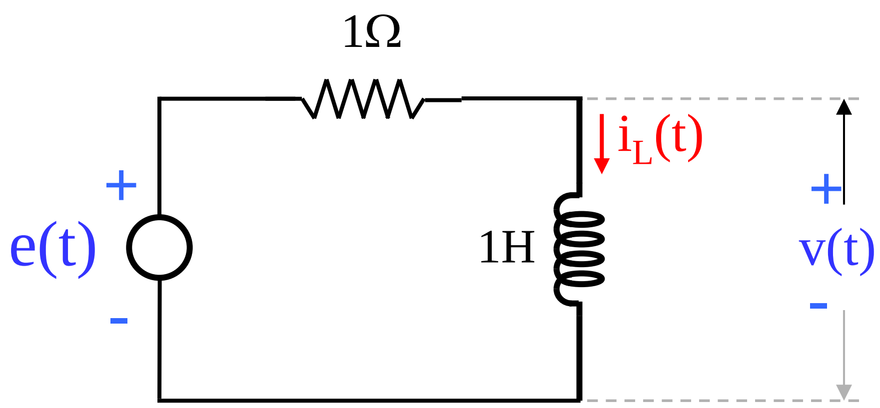
$$I_C = 4\sqrt{5} e^{j(26,6^o)}$$

$$i_C(t) = I_m \cos(2t + \theta^o) \Longrightarrow i_C(t) = 4\sqrt{5}\cos(2t + 26,6^o)$$

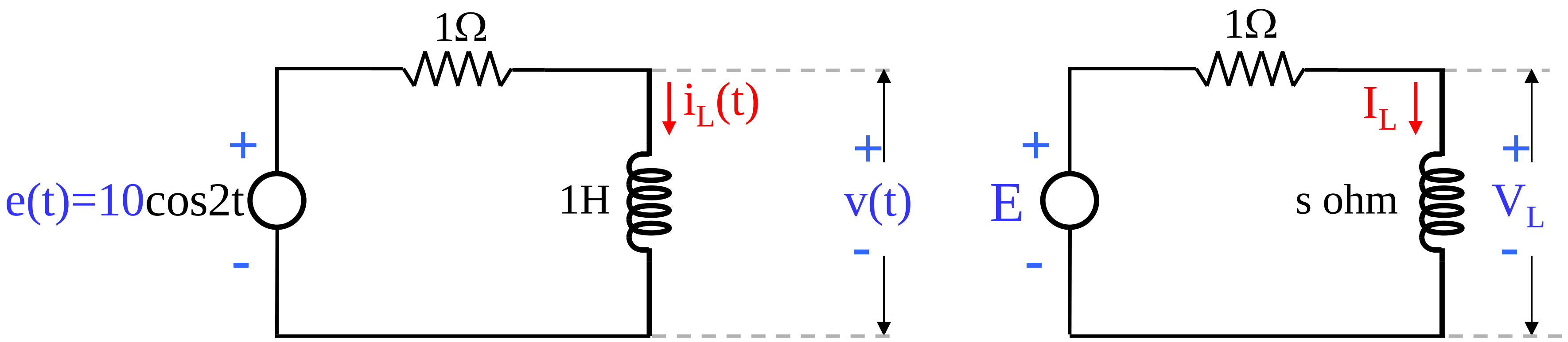
The current passing through the capacitor $i_C(t)$ leg 26.6° behind the current $i(t)$ that stimulates the circuit.



Example-4.4: Find the voltage on the inductor for the direving voltage a) $e(t)=10\cos(2t)$ and b) 10V in the circuit below



Solution-4.4: (a)



$$s = 2j \qquad E = 10$$

$$E = 1\Omega I + sI = (1+s)I \Rightarrow I = \frac{E}{1+s}$$

$$V_L = \frac{s}{1+s}E$$

$$V_L = \frac{20j}{1+2j} = 4\sqrt{5}e^{j26,6^o}$$

$$v_L(t) = 4\sqrt{5}\cos(2t + 26,6^o)$$

(b) Constant voltage (10V):

The constant voltage is can be represented as $10e^{0t}$. By placing s=0, $V_L=0$ is obtained ($V_L(t)=0$).

In steady state, the inductance acts as a short circuit. For s=0, the impedance of the inductance appears to be zero. Zero impedance means short circuit.