Ankara University
Engineering Faculty
Department of Engineering Physics

PEN207

Circuit Design and Analysis

Prof. Dr. Hüseyin Sarı

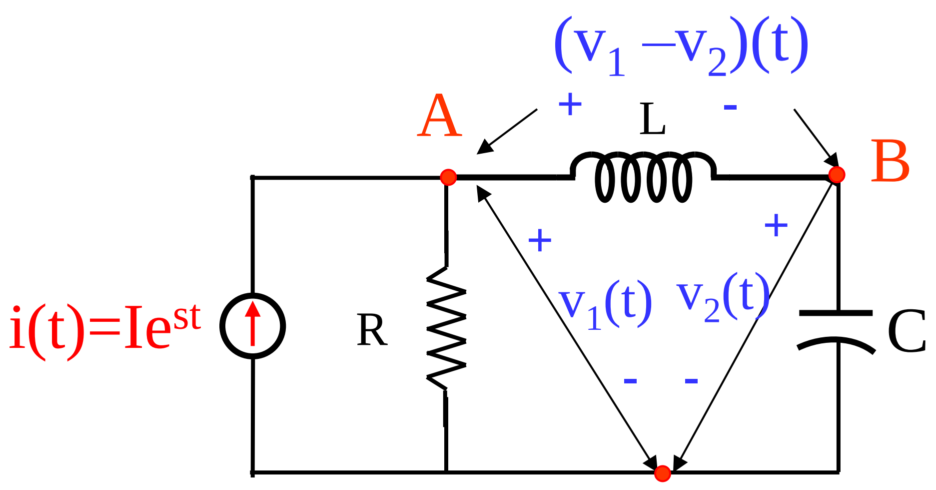
Chapter-4

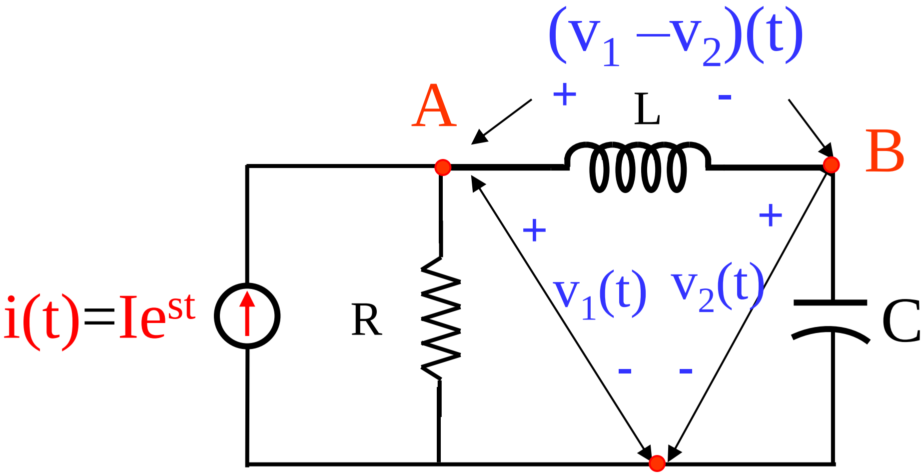
Exponential Input and Transformed Circuits
(2/2)

Transformed Circuits

In the previous sections, the solution of the circuit responses were simple circuits which could be described by a single differential equation. The purpose of this section is to extend these concepts to multi-junction (node) and multi-loop (mesh) circuits that require a common solution of multiple simultaneous equations.

Consider the circuit below. The current source in this circuit is $i(t)=Ie^{st}$. The voltage $v_2(t)$ on the capacitor C is the desired response. Since the circuit has three nodes, the solution requires two nodes (3-1 = 2).





KCL for node A:

$$Gv_1(t) + \frac{1}{L}\int\left(v_1(t) - v_2(t)\right)dt = i(t) = Ie^{st}$$

KCL for node B:

$$-\frac{1}{L}\int\left(v_1(t) - v_2(t)\right)dt + C\frac{dv_2(t)}{dt} = 0$$

If derivative of both side are taken:

$$G\frac{dv_1(t)}{dt}+\frac{1}{L}(v_1(t)-v_2(t))=\frac{di(t)}{dt}=sIe^{st}$$

$$-\frac{1}{L}\left(v_1(t)-v_2(t)\right)+C\frac{d^2v_2(t)}{dt^2}=0$$

equations containing only derivatives are obtained. There are two dependent variables in the equations, $v_1(t)$ ve $v_2(t)$. Solutions are:

$$v_{1f}(t)=V_1e^{st}$$

$$v_{2f}(t)=V_2e^{st}$$

Both components contain exponential terms; only the coefficients are different. Eliminating the exp term:

$$sGV_1e^{st}+\frac{1}{L}(V_1-V_2)e^{st}=sIe^{st}$$

$$-\frac{1}{L}(V_1-V_2)e^{st}+s^2CV_2e^{st}=0$$

Rearanging:

$$V_1(G+\frac{1}{sL})-V_2\frac{1}{sL}=I$$

$$-V_1\frac{1}{sL}+V_2(sC+\frac{1}{sL})=0$$

Two equations, two unknowns (V_1 and V_2) (Assuming that the current I is given).

Common solution for V_2/I :

$$G\equiv\frac{1}{R}$$

$$\frac{V_2}{I}=\frac{1}{s^2LCG+sC+G} \Rightarrow V_2=\frac{I}{s^2LCG+sC+G}$$

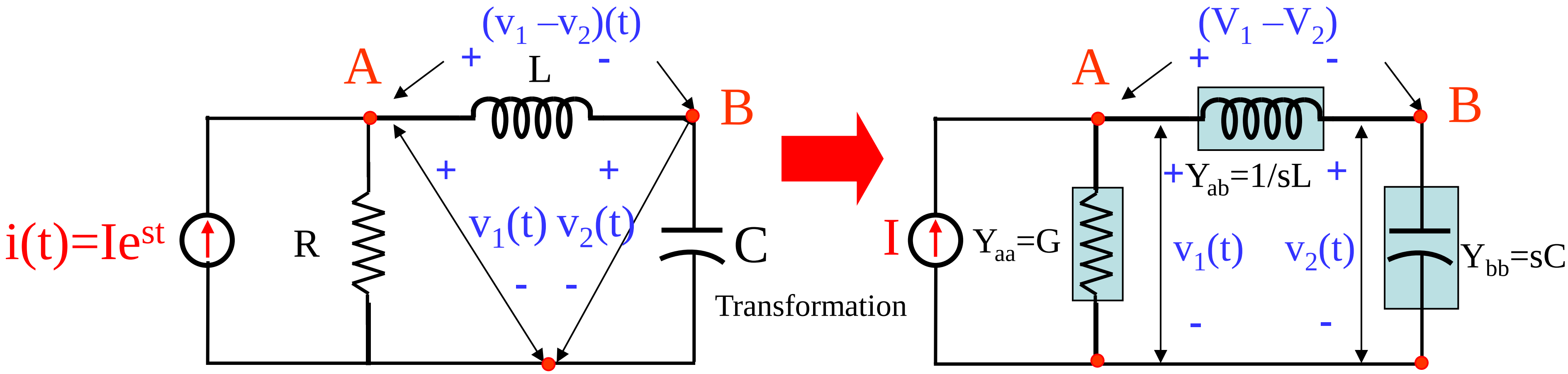
Solution:

$$v_{2f}(t)=V_2e^{st}$$

$$v_{2f}(t)=\frac{Ie^{st}}{s^2LCG+sC+G}$$

obtained.

The solution of the circuit can also be understood directly by a transformation of this circuit into the frequency domain. The transformed circuit is shown in the figure below.



Conductivity (G), inductance (L) and capacitance (C) were converted to admittance (Y). In Chapter 2, the Node-Voltage Method can be applied directly by replacing the conductivity (G) with the admittance (Y) in the transformed circuit.

KCL Equations:

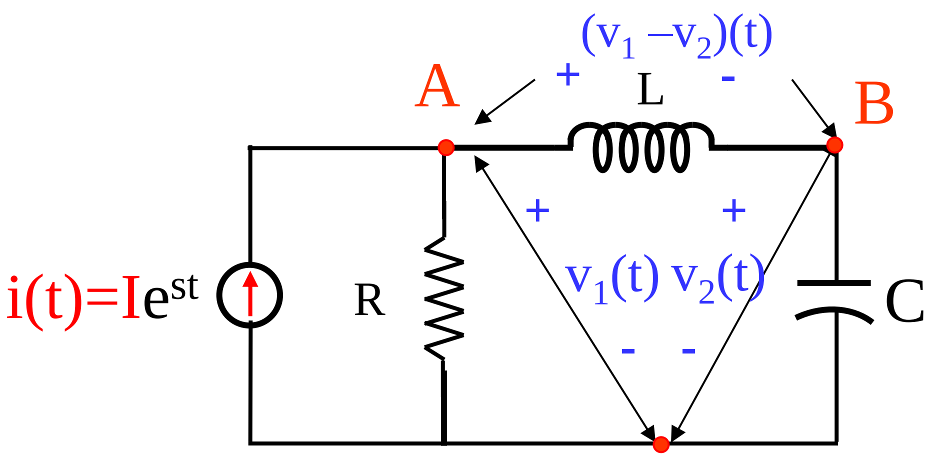
For node A:

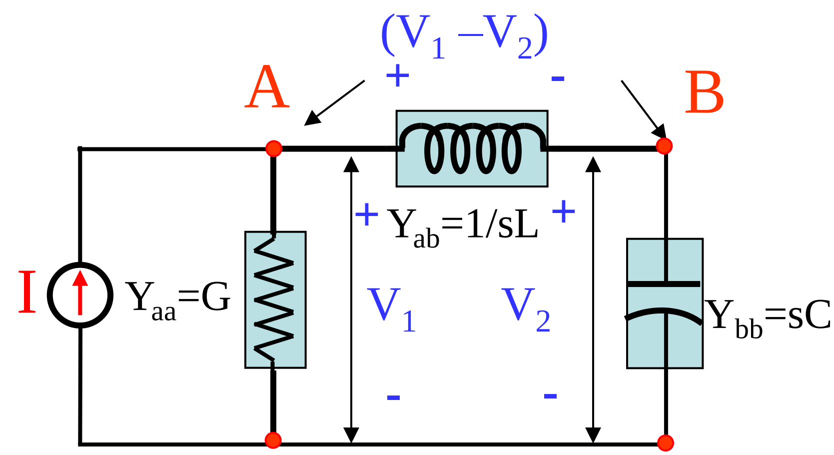
$$V_1\left(G+\frac{1}{sL}\right)-V_2\frac{1}{sL}=I$$

For node B:

$$-V_1\frac{1}{sL}+V_2\left(sC+\frac{1}{sL}\right)=0$$

The equations of the circuit were expressed with the differential equations in time domain (independent variable time). The transformed circuit and the algebraic equations corresponding to the circuit are converted into s or frequency domain. The effect of the transformation makes it possible to find solutions with algebraic equations instead of differential equations. Thus, all methods of resistance circuits obtained in Chapter 2 can be used for transformed circuits.





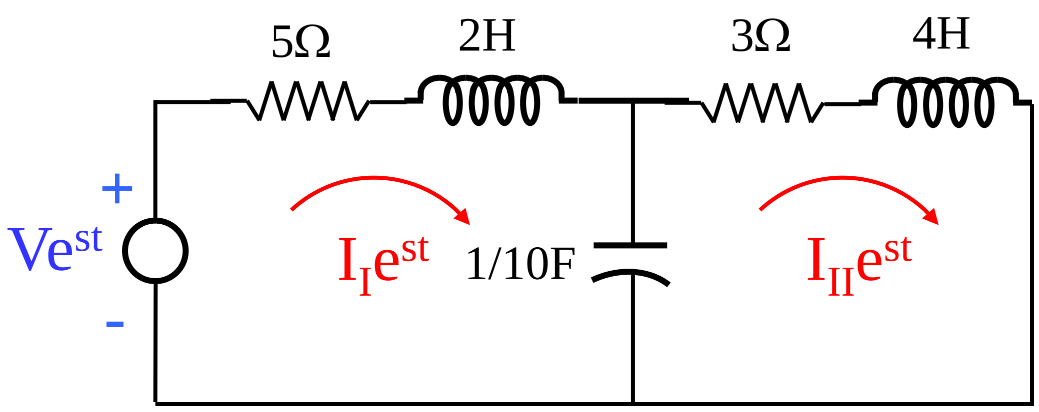
For node A:

$$Gv_1(t)+\frac{1}{L}\int\left(v_1(t)-v_2(t)\right)dt=i(t) \quad \Rightarrow \quad V_1\left(G+\frac{1}{sL}\right)-V_2\frac{1}{sL}=I$$

For node B:

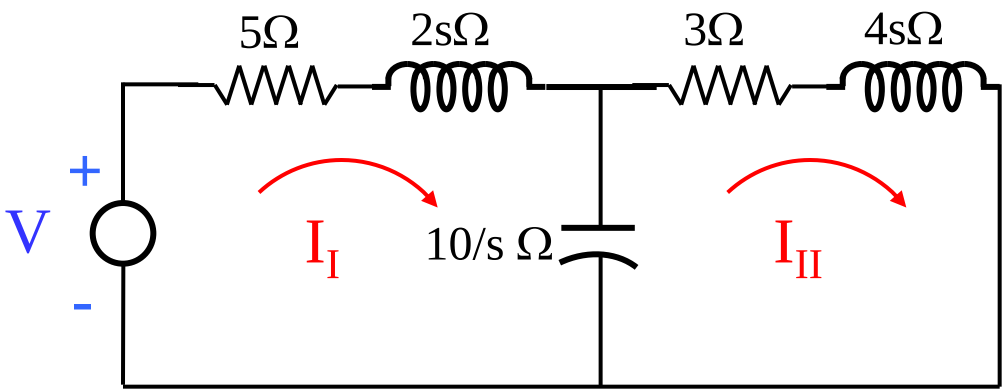
$$-\frac{1}{L}\int\left(v_1(t)-v_2(t)\right)dt+C\frac{dv_2(t)}{dt}=0 \quad \Rightarrow \quad -V_1\frac{1}{sL}+V_2\left(sC+\frac{1}{sL}\right)=0$$

Example-4.5: Convert the following circuit using the impedance parameters.



Solution-4.5:

The transformed circuit using the impedance values is shown in the figure below.



Mesh Current equations:

$$I_I(5+2s+\frac{10}{s})-I_{II}\frac{10}{s}=V$$

$$-I_I\frac{10}{s}+I_{II}(3+4s+\frac{10}{s})=0$$

Two equations, two unknowns (I_I ve I_{II}) (assuming that voltage V is given).

Common solution for I_{II}/V:

$$\frac{I_{II}}{V}=\frac{1}{8s^3+26s^2+76s+80} \Rightarrow I_{II}=\frac{V}{8s^3+26s^2+76s+80}$$

Current expression in time domani: $i_{II}(t)=I_{II}e^{st}$

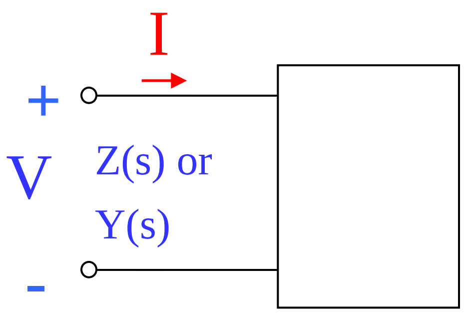
$$i_{II}(t)=\left(\frac{V}{8s^3+26s^2+76s+80}\right)e^{st}$$ obtained

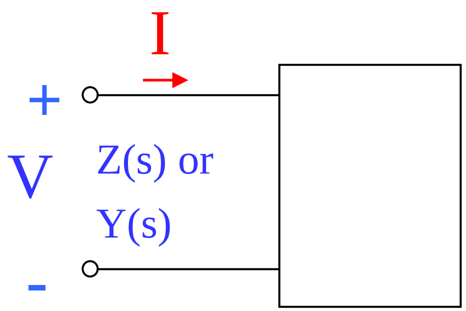
Input Impedance and Admittance

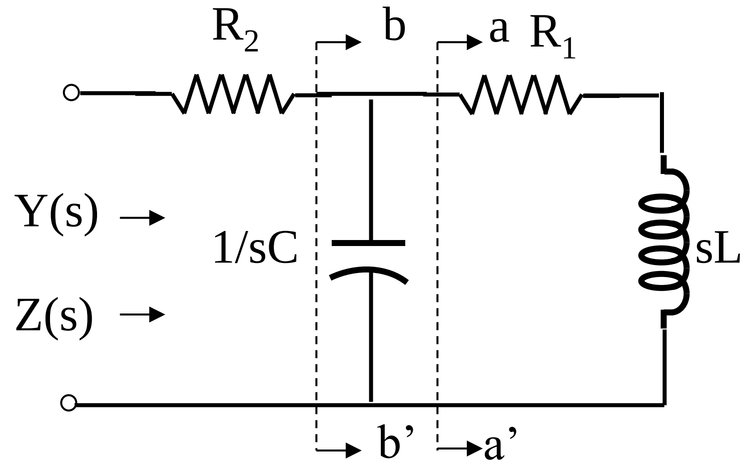
Many electrical circuits have a pair of input terminals to which the source is applied. Such circuits are called *two-terminal circuits* or *single-input circuits*.

For these circuits, the ratio of voltage to current (V / I) is called input impedance.

Such a circuit with input impedance or admittance Y(s) can be shown as a two-closed box. The impedance or admittance functions fully describe the behavior of the circuit at its endpoints. These values can be determined by the circuit reduction method described in Section 2.5.







Let's look at the circuit above:

$$Z_{aa'}(s) = R_1 + sL \qquad Y_{aa'}(s) = \frac{1}{Z_{aa'}(s)} = \frac{1}{R_1 + sL}$$

$$Y_{bb'}(s) = sC + Y_{aa'}(s)$$

$$Y_{bb'}(s) = sC + \frac{1}{R_1 + sL} = \frac{s^2LC + sR_1C + 1}{R_1 + sL}$$

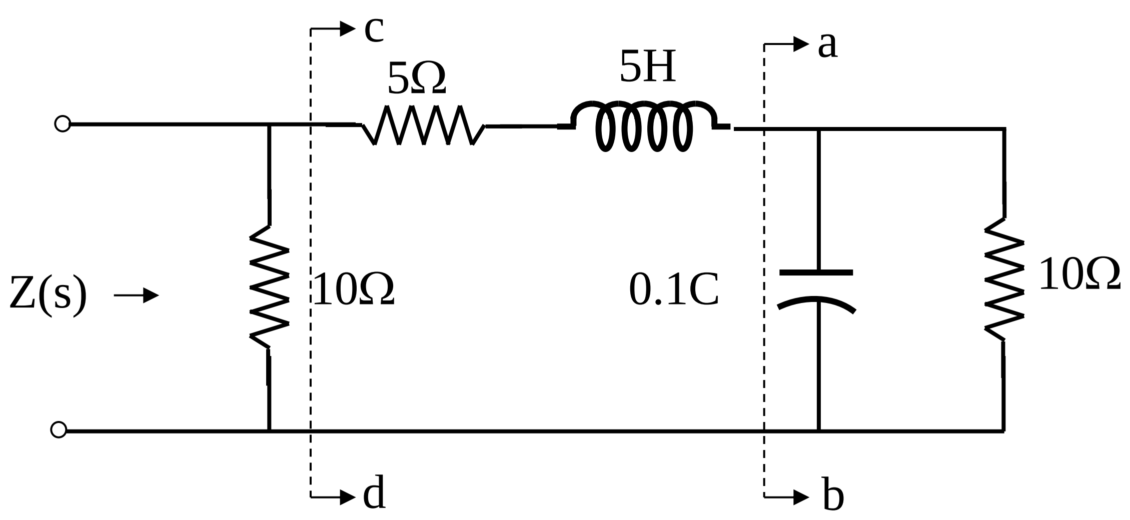
Input impedance:

$$Z_{bb'}(s) = \frac{1}{Y_{bb'}(s)} = \frac{R_1 + sL}{s^2LC + sR_1C + 1} \qquad Z(s) = R_2 + \frac{R_1 + sL}{s^2LC + sR_1C + 1}$$

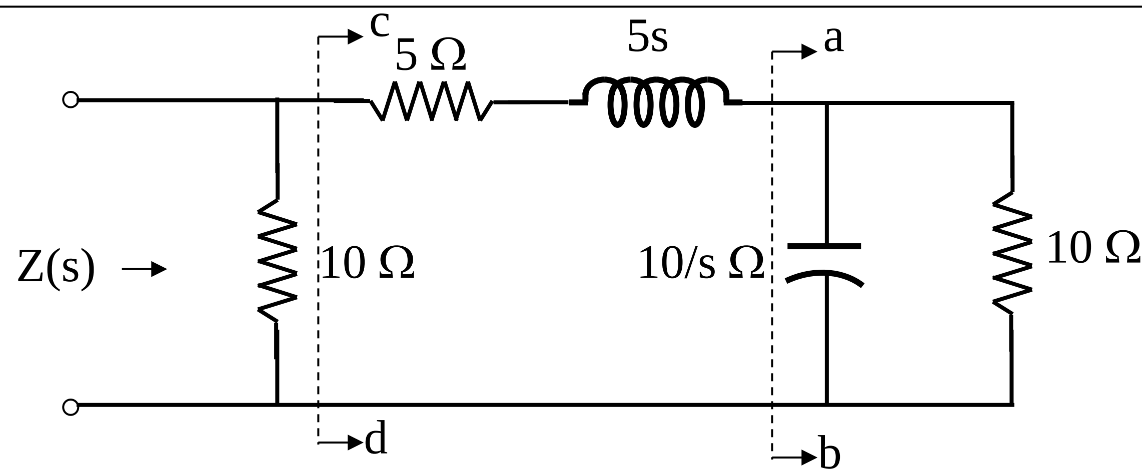
Input admittance is the inverse of the input impedance:

$$Z(s) = \frac{s^2R_2LC + s(R_1R_2C + L) + R_1 + R_2}{s^2LC + sR_1C + 1} \qquad Y(s) = \frac{1}{Z(s)} = \frac{s^2LC + sR_1C + 1}{s^2R_2LC + s(R_1R_2C + L) + R_1 + R_2}$$

Example-4.6: Find the input impedance Z (s) of the circuit below.



Solution-4.6:



Let us find the admittance of the parallel RC circuit to the right of ab line.

$$Y_{ab}(s) = \frac{1}{10} + \frac{s}{10} = \frac{s+1}{10}$$

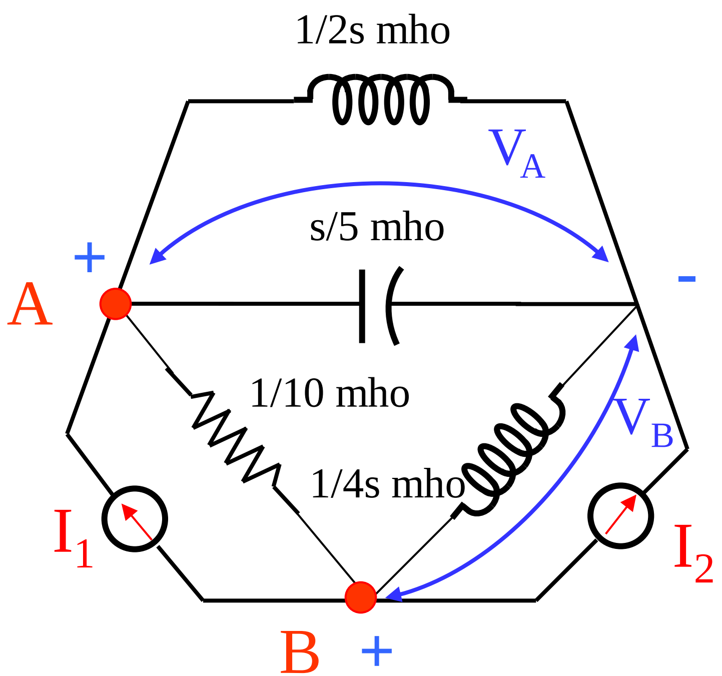
Add the impedance of the serial connected RL to $Z_{ab}(s)$

$$Z_{ab}(s) = \frac{1}{Y_{ab}(s)} = \frac{10}{s+1} \qquad Z_{cd}(s) = 5 + 5s + \frac{10}{s+1} = \frac{5s^2 + 10s + 15}{s+1}$$

$$Y(s) = \frac{1}{10} + \frac{1}{Z_{cd}(s)} \qquad Y(s) = \frac{1}{10} + \frac{s+1}{5s^2 + 10s + 15} = \frac{5s^2 + 20s + 25}{50s^2 + 100s + 150}$$

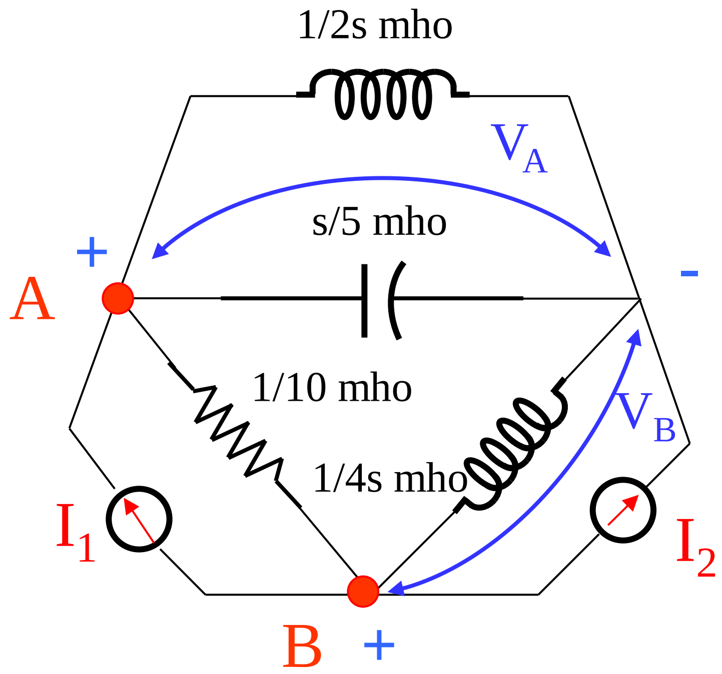
$$Y(s) = \frac{1}{10} + \frac{1}{Z_{cd}(s)} \qquad Z(s) = \frac{1}{Y(s)} = 10\frac{s^2 + 2s + 3}{s^2 + 4s + 5}$$

Example-4.7: Find the voltage V_A in the circuit below.



Solution-4.7:

Since the desired quantity V_A is the voltage, its negative end will be selected as the reference point, the points A and B and the voltage V_B will be defined as follows.



KCL equations:

For node A: $$V_A\left(\frac{1}{10}+\frac{s}{5}+\frac{1}{2s}\right)-V_B\frac{1}{10}=I_1$$

For node B: $$-V_A\frac{1}{10}+V_B\left(\frac{1}{10}+\frac{1}{4s}\right)=-(I_1+I_2)$$

Such equations can be easily solved by using the Cramer rule

$$\left(\frac{2s^2+s+5}{10s}\right)V_A - \frac{1}{10}V_B = I_1$$

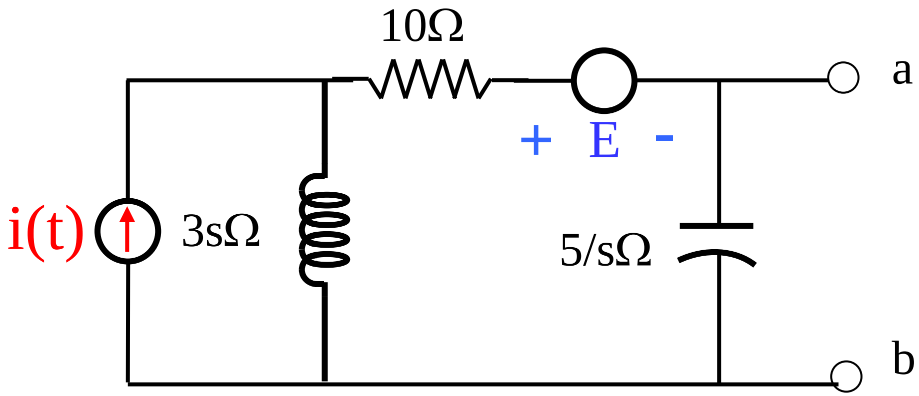
$$-\frac{1}{10}V_A + \left(\frac{2s+5}{20s}\right)V_B = -(I_1+I_2)$$

$$V_A = \frac{\begin{vmatrix} I_1 & -\frac{1}{10} \\ -(I_1+I_2) & \frac{2s+5}{20s} \end{vmatrix}}{\begin{vmatrix} \frac{2s^2+s+5}{10s} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{2s+5}{20s} \end{vmatrix}} \quad \Rightarrow \quad V_A = \frac{50sI_1 - 50s^2 I_2}{4s^3+10s^2+15s+25}$$

Note that the first of the two terms in V_A's includes I_1 as a multiplier and the second as I_2. This is the result of the total circuit response resulting from the effect of two sources, the sum of the individual responses, the superposition principle that determines that these responses are independent of each other.

$$V_A = \frac{50sI_1}{4s^3+10s^2+15s+25} + \frac{-50s^2 I_2}{4s^3+10s^2+15s+25}$$

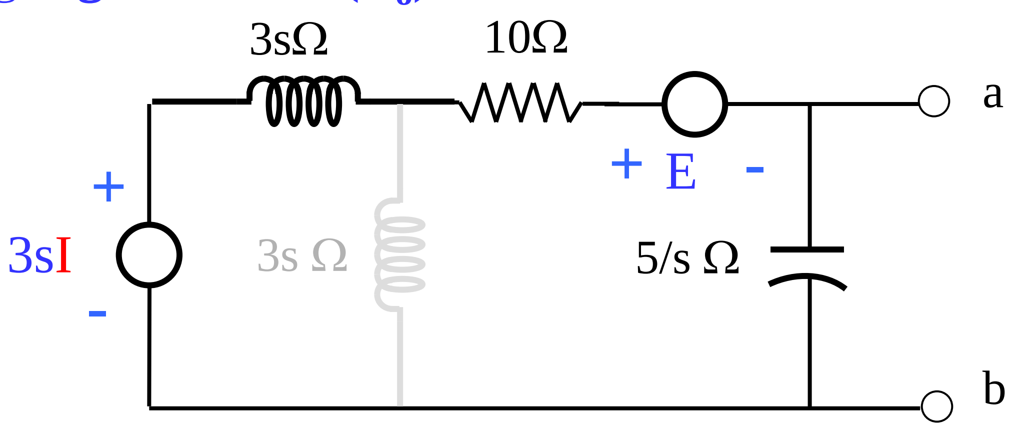
Example-4.8: Find the Thevenin equivalent circuit seen from ab terminals in the circuit below.



Solution-4.8:

The equivalent Thevenin circuit can be found by source xonverting and circuit reduction methods. First, the parallel I source and the 3sΩ impedance are converted into a series connected voltage source 3sI and 3s Ω impedance. The result of this transformation is shown in the circuit below. Series connected impedance and series voltage sources are combined as follows

Thevenin eşdeğer geriliminin (E_o) bulunması:



Thevenin equivalent voltage E_o can be found in two steps:

Current in the circuit:

$$I = \frac{3sI - E}{3s + 10 + 5/s} = \frac{s(3sI - E)}{3s^2 + 10s + 5}$$

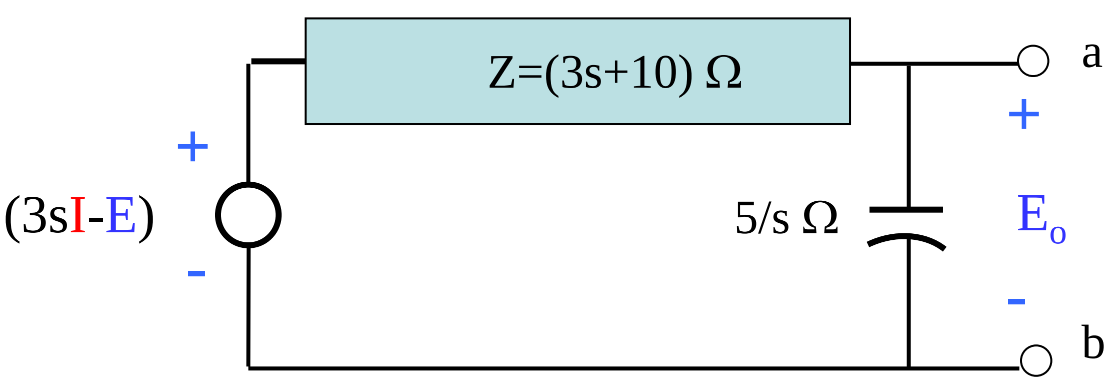
$V = IZ$

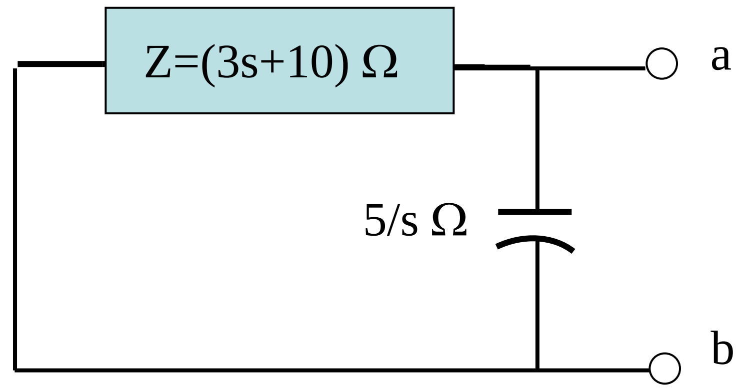
Voltage between ab terminals (voltage on condancator) :

$$E_o = I\left(\frac{5}{s}\right) = \frac{15sI - 5E}{3s^2 + 10s + 5}$$

Thevenin equivalent impedance:

If the power supply is deactivated (short-circuited) in the circuit below, the impedance Z and the 5 / s Ω resistor are connected in parallel when viewed from ab ends.

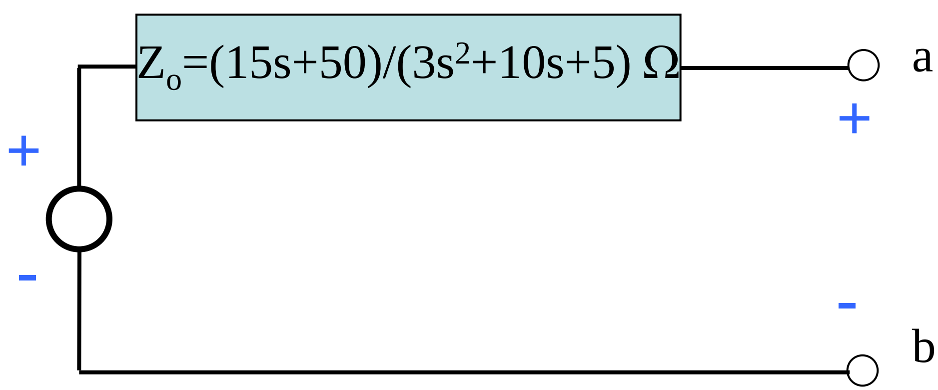




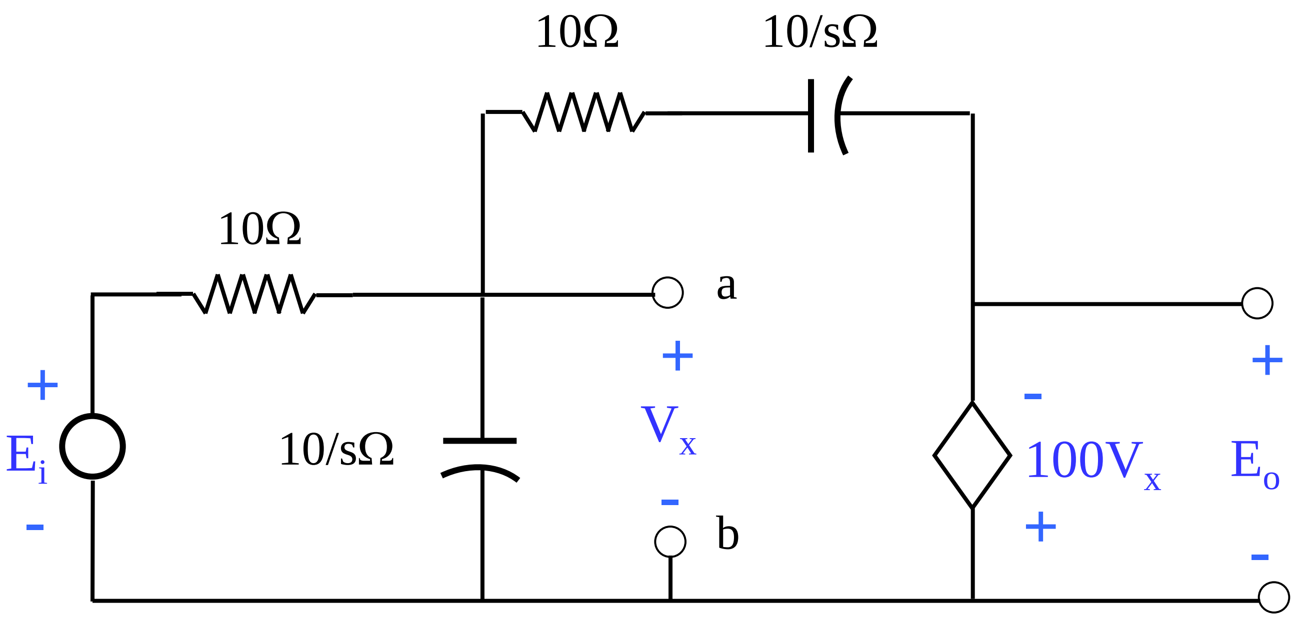
$$Z_o = \frac{(3s+10)(5/s)}{(3s+10)+5/s} = \frac{15s+50}{3s^2+10s+5}$$

Thevenin equivalent circuit:

$$E_o = \frac{15sI-5E}{3s^2+10s+5}$$

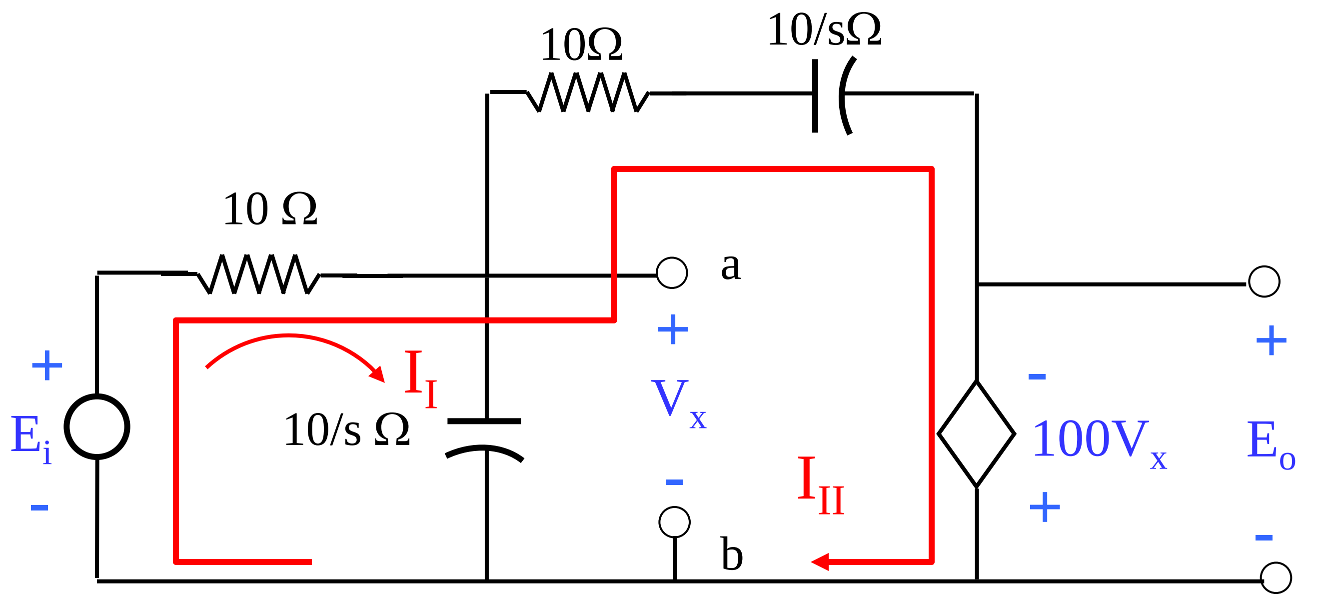


Example-4.9: The following circuit is a simple amplifier model with feedback. What is the ratio of the output voltage to the input voltage (E_o/E_i)



Solution-4.9:

The circuit equations are four, including the equations defining E_o ve V_x, as well as two KVL equations. A different form of mesh current method will be used when writing KVL equations. Note that although the direction of I_I is chosen as usuall manner, the direction pf the current I_{II} is selected such that it emerges exactly around the outer loop. The result of this selection is the sum of the currents I_I and I_{II} at the 10Ω resistor connected in series with E_i, and therefore the algebraic signs of the coefficients in the KVL equation should reflect this fact.



KVL equations:
around loop I_I :

$$(10+\frac{10}{s})I_I - 10I_{II} = E_i$$

Around outher mesh (I_{II}):

$$-10I_I + (10+10+\frac{10}{s})I_{II} = E_i + 100V_x$$

Equations that defines voltages

$$V_x = \frac{10}{s} I_I$$

$$E_o = -100 V_x$$

An important consequence of the unexpected selection of current variables is that the V_x voltage is the function of I_I alone, where it would previously be proportional to the two current differences as a result of the selection.

$$-10 I_I + (20 + \frac{10}{s}) I_{II} = E_i + 100 \frac{10}{s} I_I \quad \Longrightarrow \quad -(10 + \frac{1000}{s}) I_I + (20 + \frac{10}{s}) I_{II} = E_i$$

$$(10 + \frac{10}{s}) I_I - 10 I_{II} = E_i \qquad \text{First equation}$$

$$-(10 + \frac{1000}{s}) I_I + (20 + \frac{10}{s}) I_{II} = E_i \qquad \text{Second equation}$$

Common solution: $$I_I = \frac{s E_i (s+1)}{10 s^2 + 1030 s + 10}$$

From voltage equivalent: $$V_x = \frac{10}{s} I_I = \frac{(s+1)}{s^2 + 103 s + 1} E_i \quad \Longrightarrow \quad E_o = -100 V_x = -\frac{100(s+1)}{s^2 + 103 s + 1} E_i$$

Ratio of the voltages: $$\frac{E_o}{E_i} = -\frac{100(s+1)}{s^2 + 103 s + 1}$$ found.