Ankara University
Engineering Faculty
Department of Engineering Physics

PEN207

Circuit Design and Analysis

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Chapter-5

Steady State AC Circuits (1/2)

Steady State AC Circuits
Content

- Introduction to Periodic Functions
- Root Mean Square (RMS), Effective Current and Voltage
- Phasor Diagrams
- Circuit Reduction

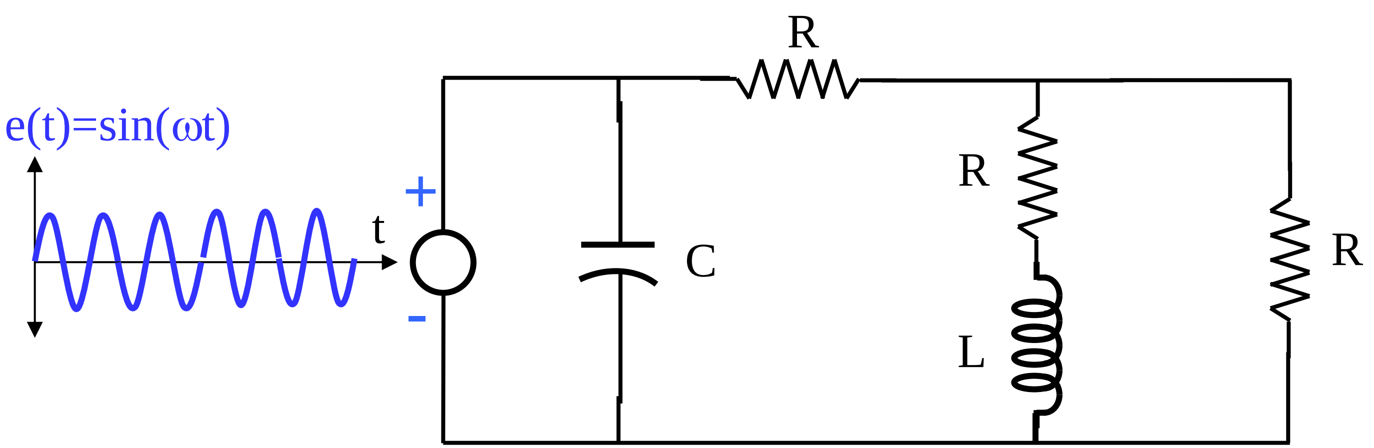
In this chapter,

- Steady state response of circuits to periodic (sinusoidal) stimulation,
- *Phasor diagram* methods to analyse alternating current (AC) circuits,
- *Root Mean Square (rms)*, *effective current* and *effective voltage*,
- Power in ac circuits,

will be learned.

In this chapter

Periodic driving force



Z=? ; I=? ; P=? ;

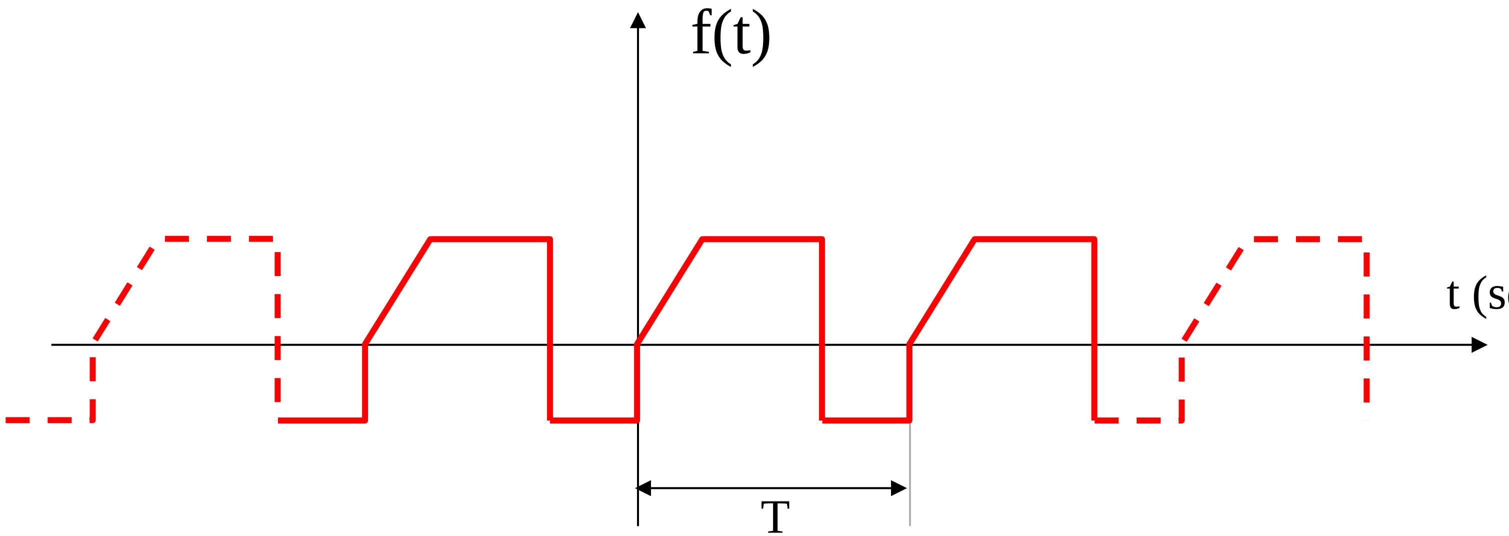
And what'is the Thevenin equivalent circuit?

Introduction to Periodic Functions

If a function f(t) has the following property

$$f(t) = f(t+T)$$

Then, it repeats itself with a period T.



***Period* (T)**, time required to repeat itself; unit is second (s)

***Frequency* (f)**, is the number of times the wave repeats itself per unit time; unit: Hertz (1/s), or shortly (Hz)

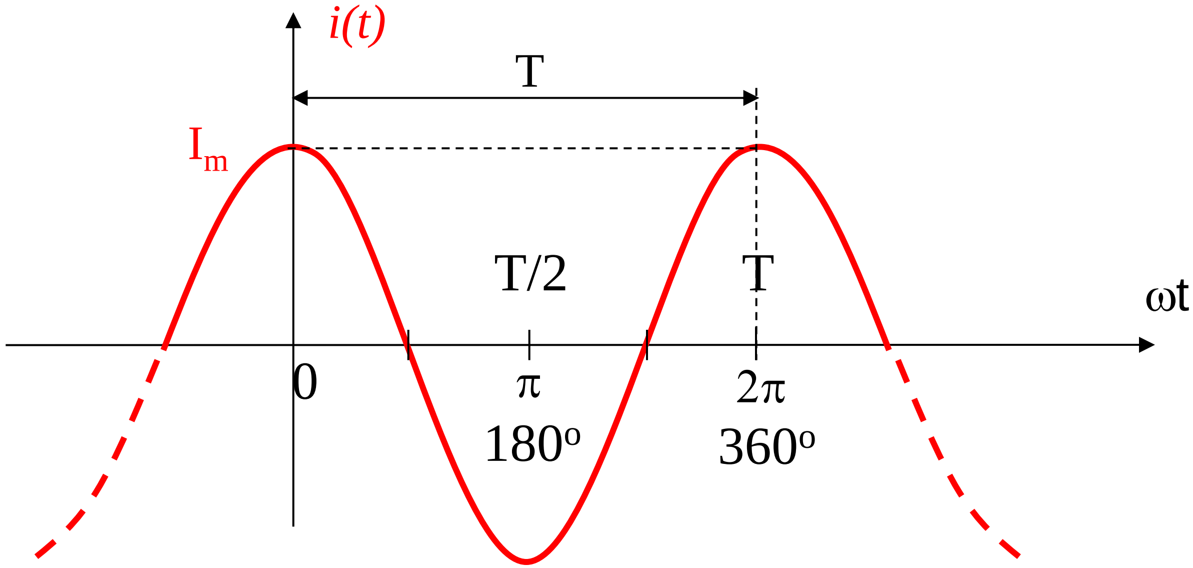
Relation between period (T) and frequency (f): $$f = \frac{1}{T}$$

Sine or cosine wave is a special periodic waveform used in electricity.

Do not confuse!

$$\omega = 2\pi f$$

ω: Angular frequency (rad/s)
f: Frequency (1/s)



$$i(t) = I_m \cos(\omega t)$$

Instant value of current: ***i(t)***

Maximum value of current (Amplitude): I_m

Frequency: $f = \dfrac{1}{T}$ (1/s) (# of ocsillation in a unit time)

Angular frequency: $\omega = \dfrac{2\pi}{T} = 2\pi f$ (rad/s) (Angle swept in a unit time)

$$i(t) = I_m \cos(\omega t) = I_m \cos(2\pi f t) = I_m \cos(\frac{2\pi}{T} t)$$

The maximum of a periodic function may not always be at t=0. In most electrical circuits, current and voltage do not simultaneously pass through zero or reach maximum; they follow each other with a phase angle.

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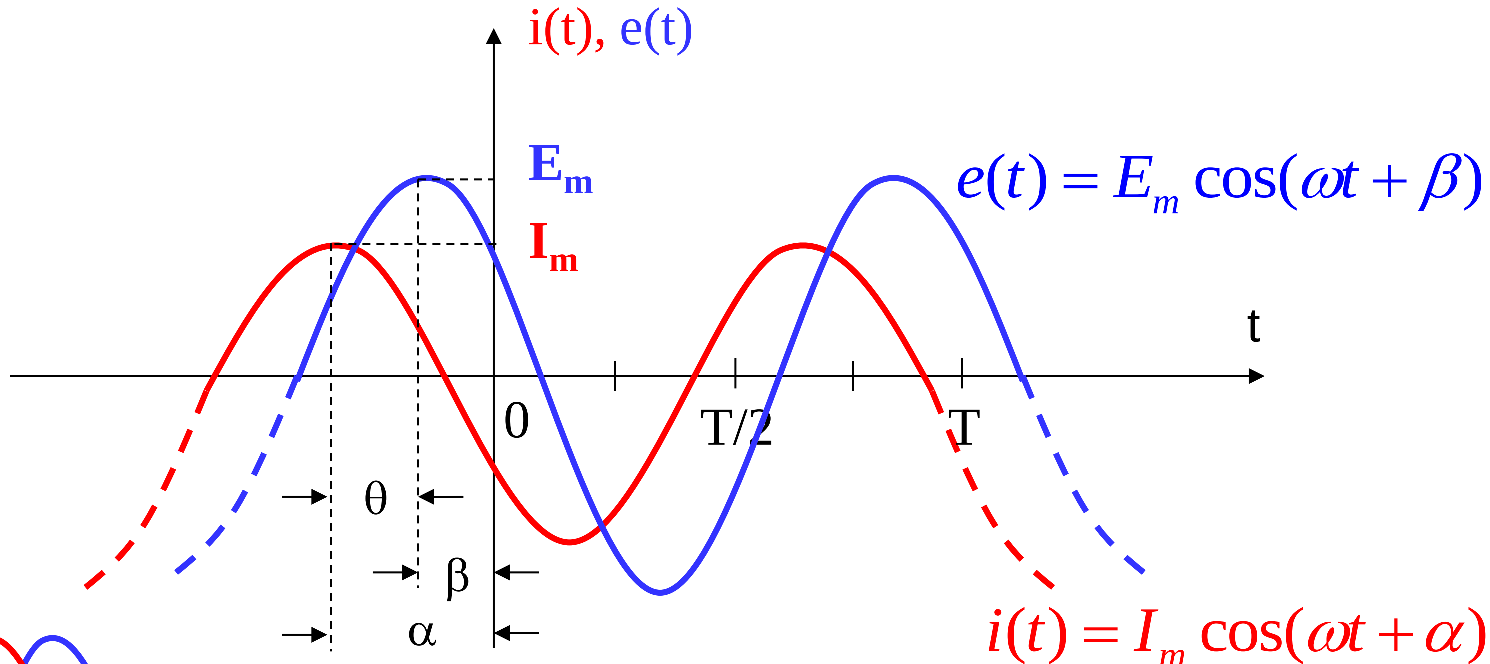
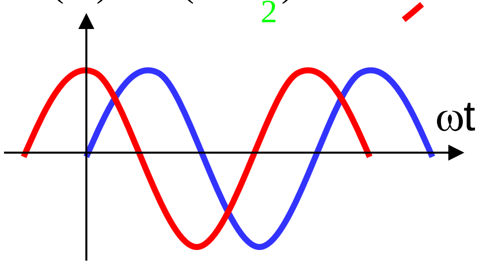


$\cos(\omega t) = \sin(\omega t - \frac{\pi}{2})$

$\sin(\omega t) = \cos(\omega t + \frac{\pi}{2})$



Cosine, is ahead of sine.
How much? t=90°/ω

$$e(t) = E_m \cos(\omega t + \theta)$$

Phase angle $\theta = \alpha - \beta$

In the above case, the voltage e(t) is behind the current i(t) by the phase angle θ (the voltage becomes maximum **after** time t=θ/ω).

Since ωt in current and voltage is expressed in radians, the phase angle must also be expressed in radians.

$$e(t) = E_m \cos(\omega t + \theta)$$

Phase angle (radian)

$$e(t) = 155\cos(377t + \frac{\pi}{6})$$

In practice, however, the phase angle is express in degrees. for example:

$$e(t) = 155\cos(377t + 30^o)$$

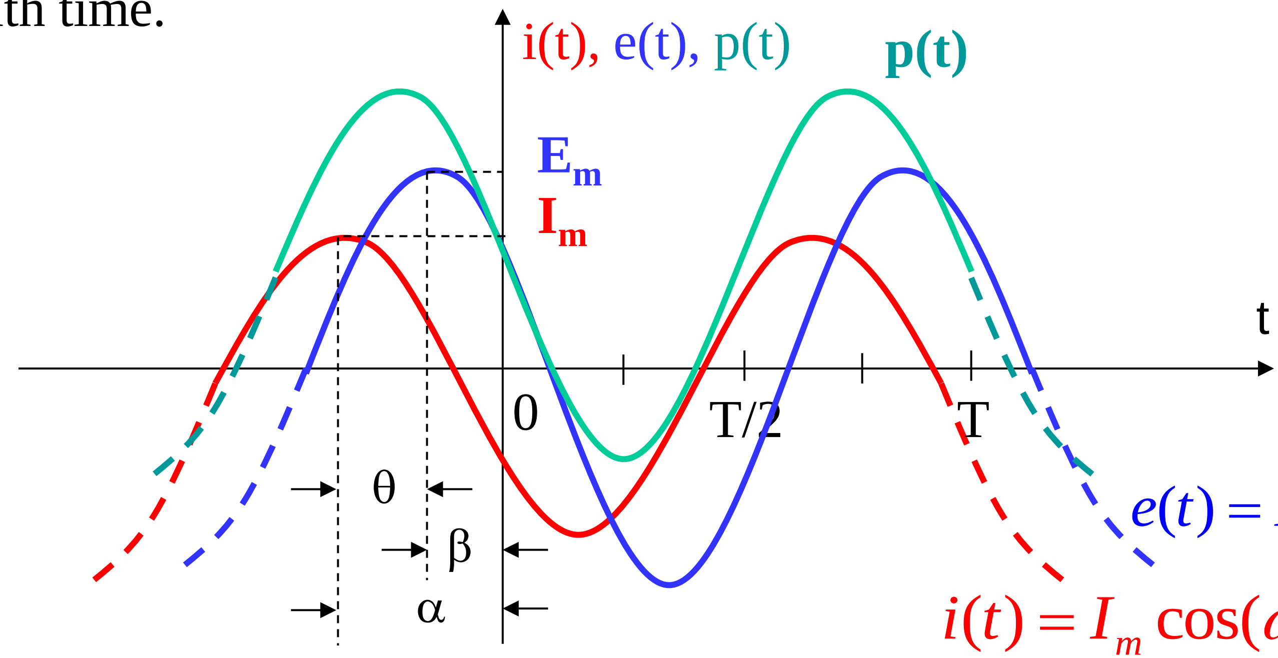
Phase angle (degree)

However, when calculating the instantaneous value of the wave, the terms in the argument (ωt and θ) must be in the same unit

Circuit that driven by sinusoidal current (or voltage) is called ***alternative current (a.c) circuit***

Circuit that driven by constant current (or voltage) is called ***direct current (d.c)*** circuit

In **a.c** circuits since **current** and **voltage** changes with time **instant power** is also changes with time.



$$e(t) = E_m \cos(\omega t + \theta)$$

$$i(t) = I_m \cos(\omega t)$$

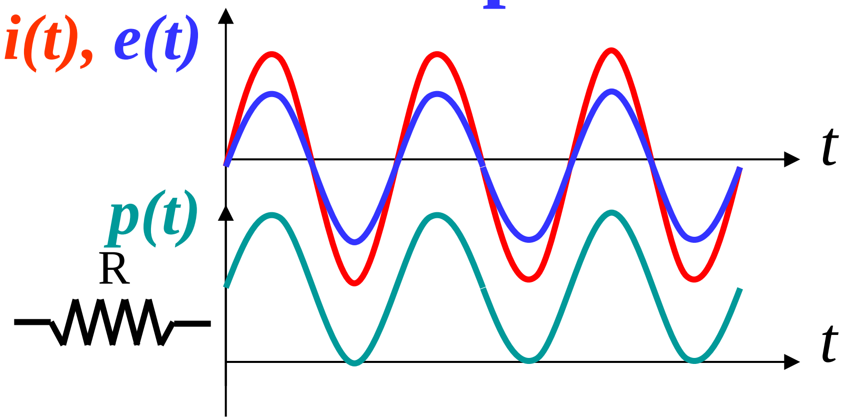
$$p(t) = i(t).e(t)$$

Since **power (p(t))** is the product of current and voltage, power is positive (except where θ= 90°) even when e(t) and i(t) are both negative.

Since power is different from zero in alternating current and voltage, energy transfer can be effectively done using alternating current and voltage.

In the case of power transmission, the direction of the energy flow reverses when the power is negative depending on the phase angle. This undesirable situation is solved by the use of transformers.

Power Dissapitad on resistor, capacitance and coil

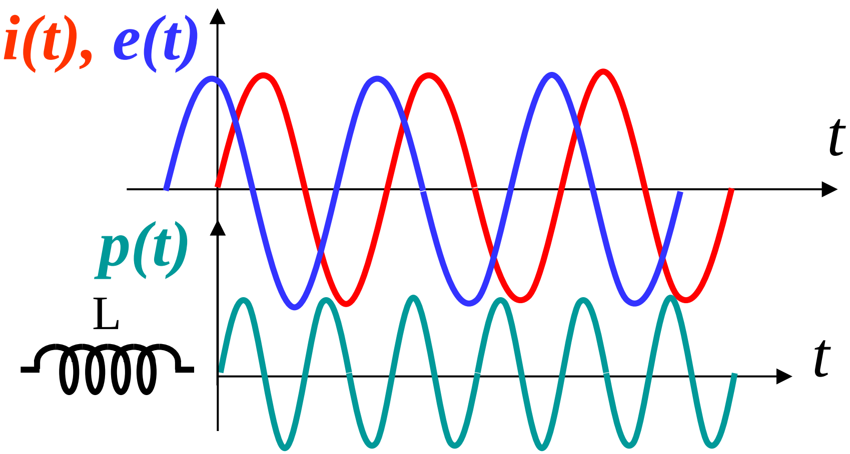


$$i(t) = I_m \sin(\omega t)$$

$$e(t) = Ri(t) = E_m \sin(\omega t)$$

$$p(t) = e(t).i(t) = P_m \sin^2(\omega t)$$

<P> ≥ 0

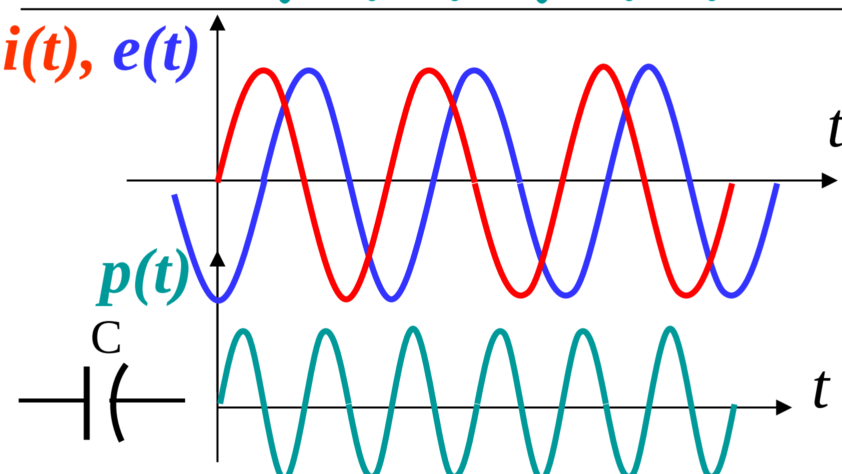


$$i(t) = I_m \sin(\omega t)$$

$$e(t) = L\frac{di(t)}{dt} = E_m \cos(\omega t)$$

$$p(t) = e(t).i(t) = P_m \sin(2\omega t)$$

<P> = 0



$$e(t) = E_m \sin(\omega t)$$

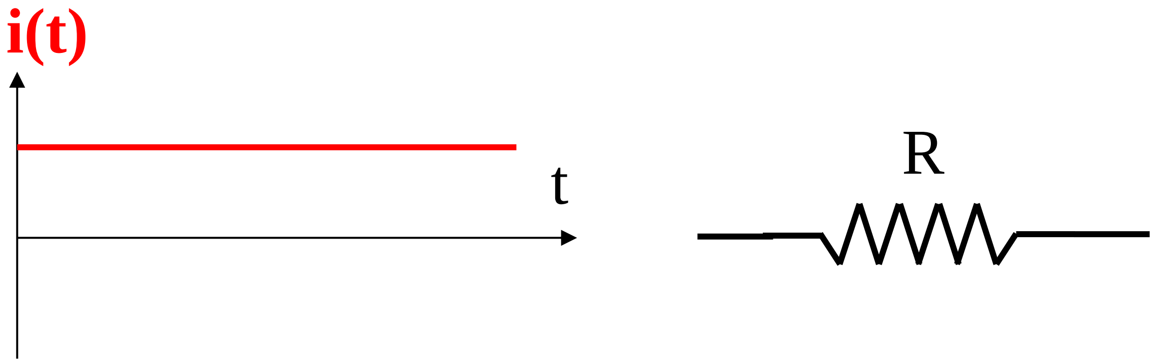
$$i(t) = C\frac{de(t)}{dt} = I_m \cos(\omega t)$$

$$p(t) = e(t).i(t) = P_m \sin(2\omega t)$$

<P> = 0

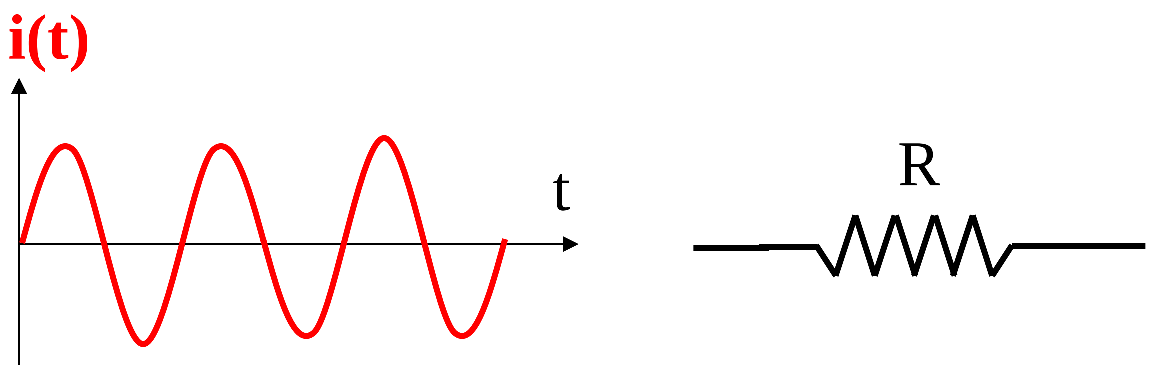
Root Mean Square (or RMS) Value of Current & Voltage

The power generated by the direct current I on the resistor is constant and is given as I^2R. This effect is manifested as heat on the resistance.



$$P = IV = I^2R$$

The power generated by current ***i*** whose value changes over time on a resistor is i^2R, which is a function of time.

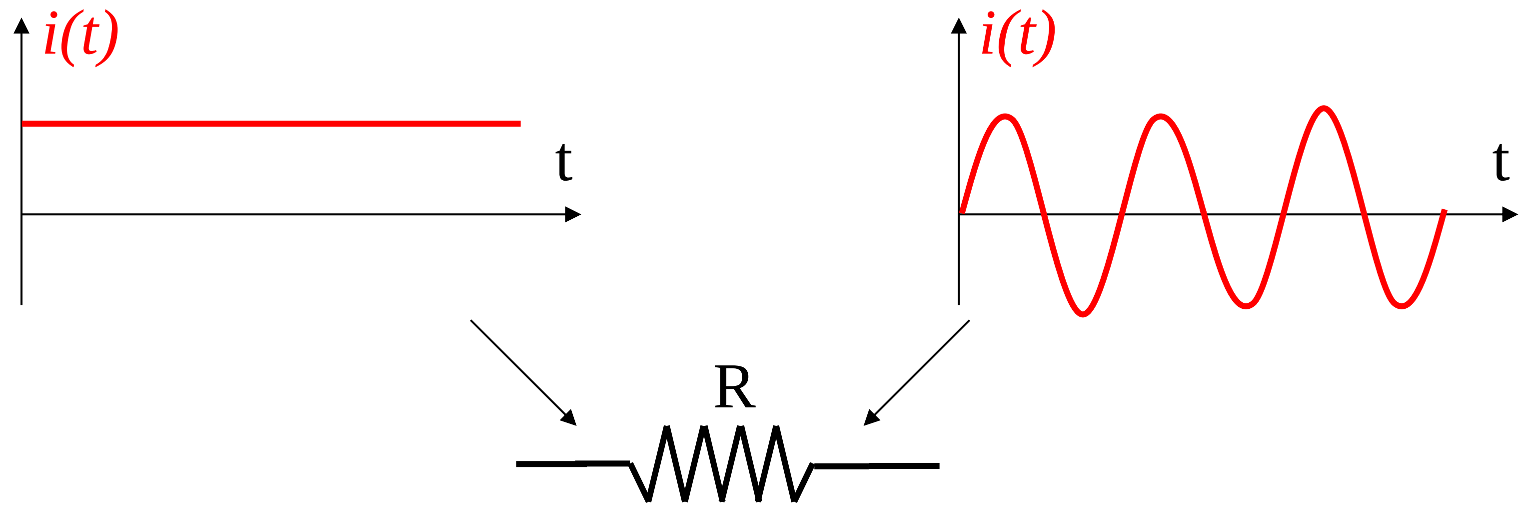


$$p(t) = i^2(t)R$$

If the "heating" effects of a periodic current and direct current are compared, **the average heating power of the periodic current must be taken into account**.

Root Mean Square (or RMS) Value of Current & Voltage

It may be necessary to compare the effects of alternating current (or voltage) with those of direct current (or voltage). For this purpose, let us look at the effects of alternating current (or voltage).



If it is desired to compare the effects of a alternating current and direct current, the heating effect of these two waves on a resistor can be examined.

The heat produced by the direct current on a resistor R is constant. (independent of time)

$$P = IV = I^2 R$$

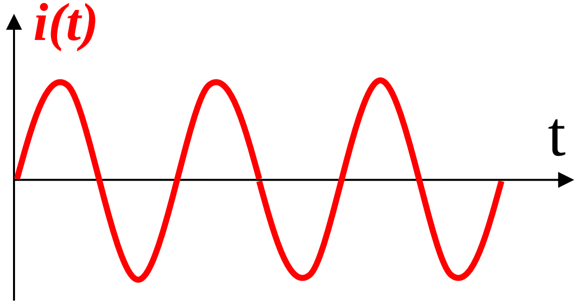
$$p(t) = i^2(t) R$$

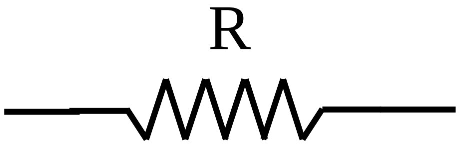
The heat produced by the alternating current on a resistor R depends on time.

If it is necessary to compare the heating effects of a periodic current with a direct current, the average heating power of the periodic current should be taken into account.

Root Mean Square (or RMS) Value of Current & Voltage

How can the alternating current have the equivalent effect to give the same power as the direct current?





Average power (P_{avr}) (heating effect) generated by a periodic signal on a resistance (R) over a period of time:

$$P_{avr} = i^2(t)R = \left(\frac{1}{T}\int_0^T i^2(t)dt\right).R = I_{ort}^2 R$$

If this power is compared to the power I^2R generated by the direct current, the current expression is:

$$I_{avr} \equiv \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

The current value giving this effect is called the *Effective Current* or rms (**r**oot **m**ean **s**quare) value. *Effective Current* or rms of a periodic signal gives the equivalent effect as direct current gives

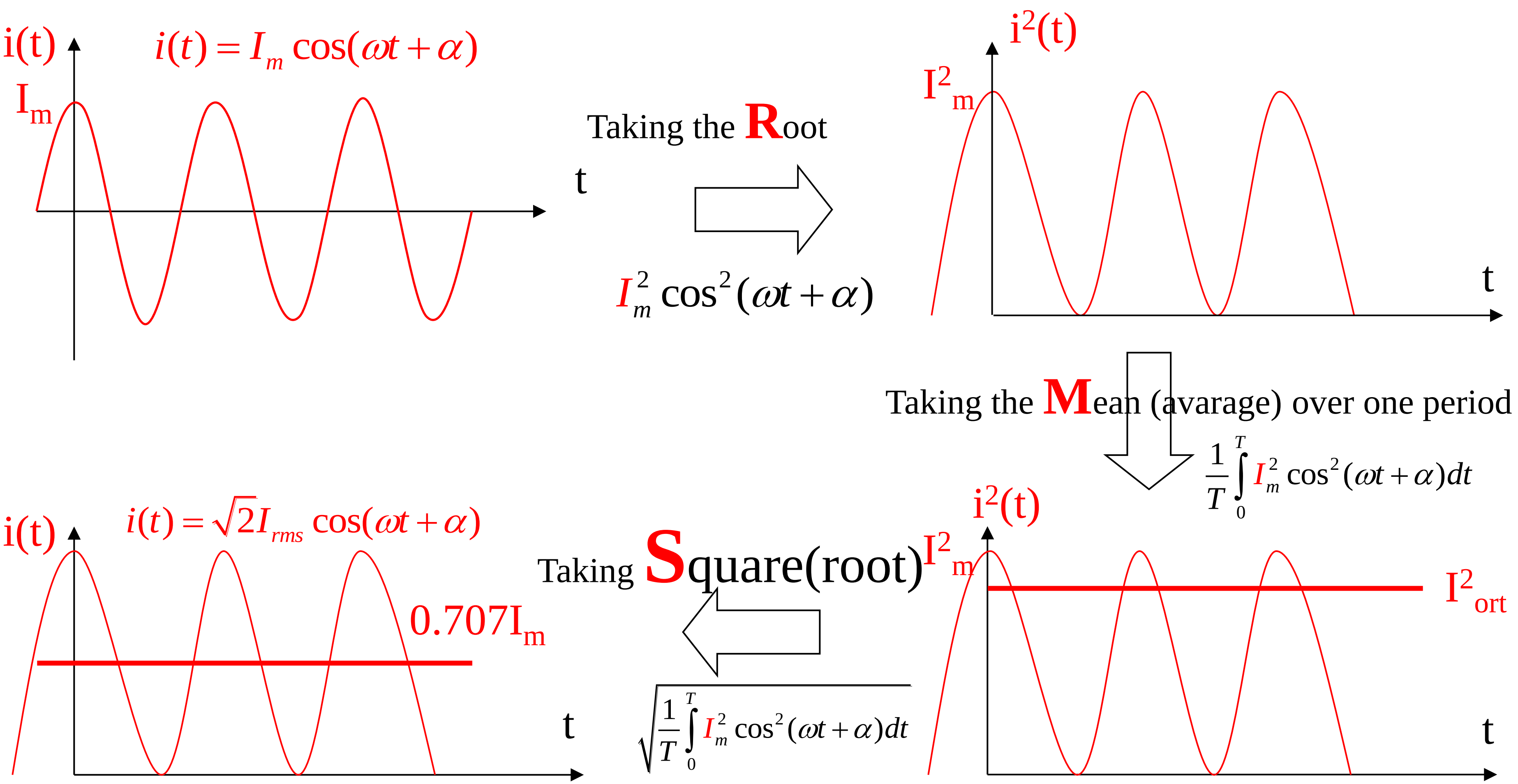
$$I_{rms} \equiv \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

RMS value of alternating current

RMS

RMS (**R**oot **M**ean **S**quare) or (Effective value) calculation:

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(2\pi t/T)dt} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$



Effective (RMS) Current and Voltage

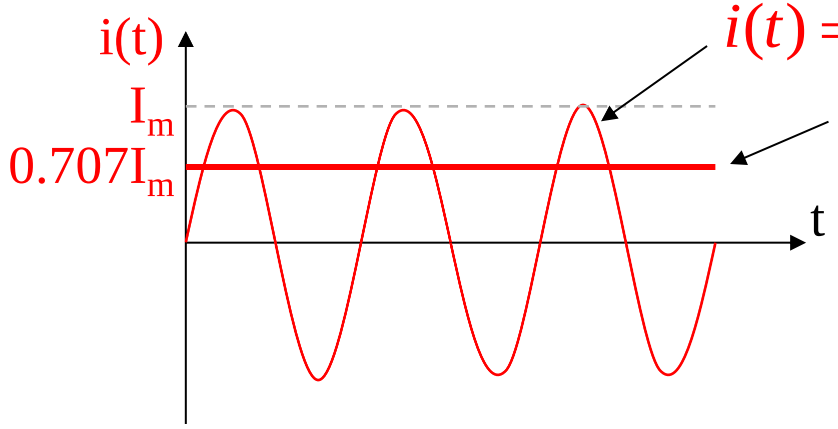
Effective value of sinusoidal current:

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(2\pi t/T)dt} = \frac{I_m}{\sqrt{2}} = 0,707 I_m$$

$$i(t) = I_m \cos(\omega t)$$

$$\omega = 2\pi / T$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$



$$i(t) = I_m \cos(\omega t + \alpha)$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Instantaneous current in terms of effective (rms) value

$$i(t) = \sqrt{2} I_{rms} \cos(\omega t + \alpha)$$

The subscript is removed because the rms value is widely used.

$$I_{rms} \rightarrow I$$

$$I_m = \sqrt{2} I_{rms} = \sqrt{2} I$$

$$i(t) = \sqrt{2} I \cos(\omega t + \alpha)$$

Effective (RMS) Current and Voltage

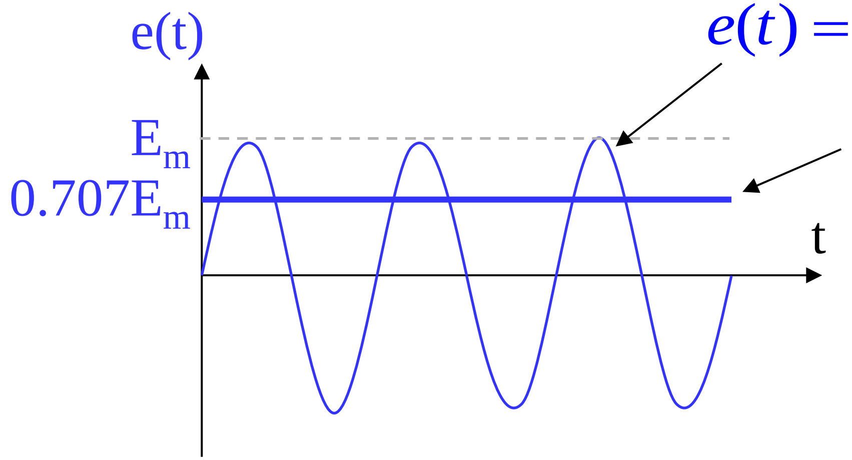
Similarly, the same steps can be done for voltage.

Effective value of sinusoidal voltage: $E_{rms} = \sqrt{\frac{1}{T}\int_0^T E_m^2 \cos^2(2\pi t/T)dt} = \frac{E_m}{\sqrt{2}} = 0.707E_m$

$e(t) = E_m \cos(\omega t)$

$\omega = 2\pi / T$

$E_{rms} = \frac{E_m}{\sqrt{2}}$



$e(t) = E_m \cos(\omega t + \beta)$

$E_{rms} = \frac{E_m}{\sqrt{2}}$

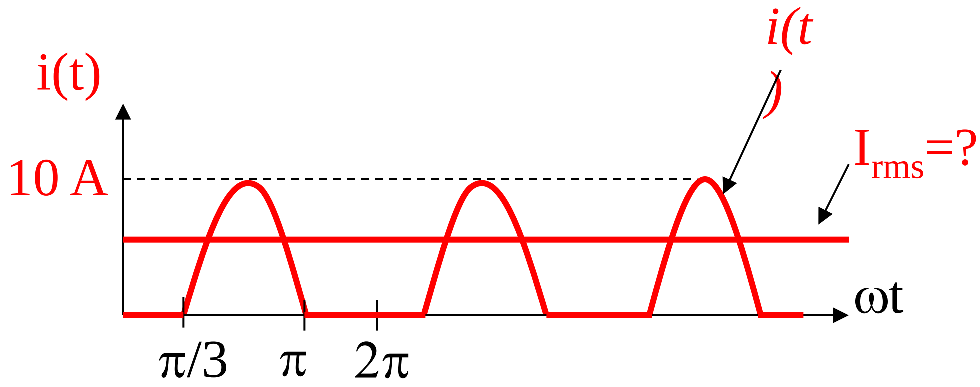
Instantaneous voltage in terms of effective (rms) value: $e(t) = \sqrt{2}E_{rms} \cos(\omega t + \beta)$

The subscript is removed because the rms value is widely used.

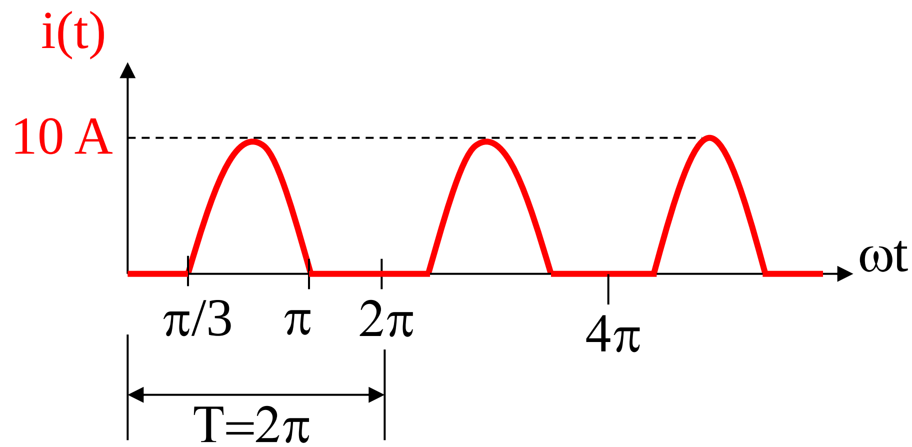
$E_{rms} \rightarrow E$

$$e(t) = \sqrt{2}E \cos(\omega t + \beta)$$

Example-5.1: The waveform of the current in a rectifier is given below. The wave is sinusoidal between the radians π/3 and π and is zero at the other times. Find the effective (rms) value of the current.



Solution-5.1:



To find the effective (rms) value of the periodic i(t), the average value is found in a one period interval (2π).

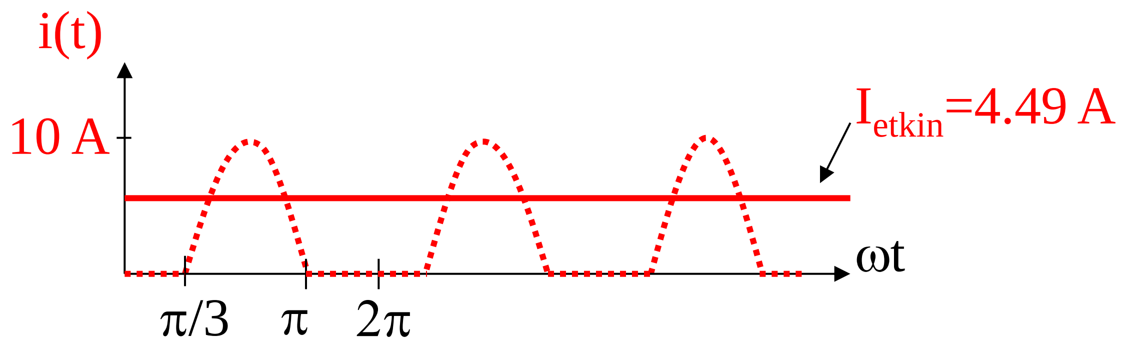
$$I = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(t)d(\omega t)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi/3} i^2(t)d(\omega t) + \int_{\pi/3}^{\pi} i^2(t)d(\omega t) + \int_{\pi}^{2\pi} i^2(t)d(\omega t)\right]}$$

Since the value of i(t) is zero between 0-π/3 and π-2π , the above expression:

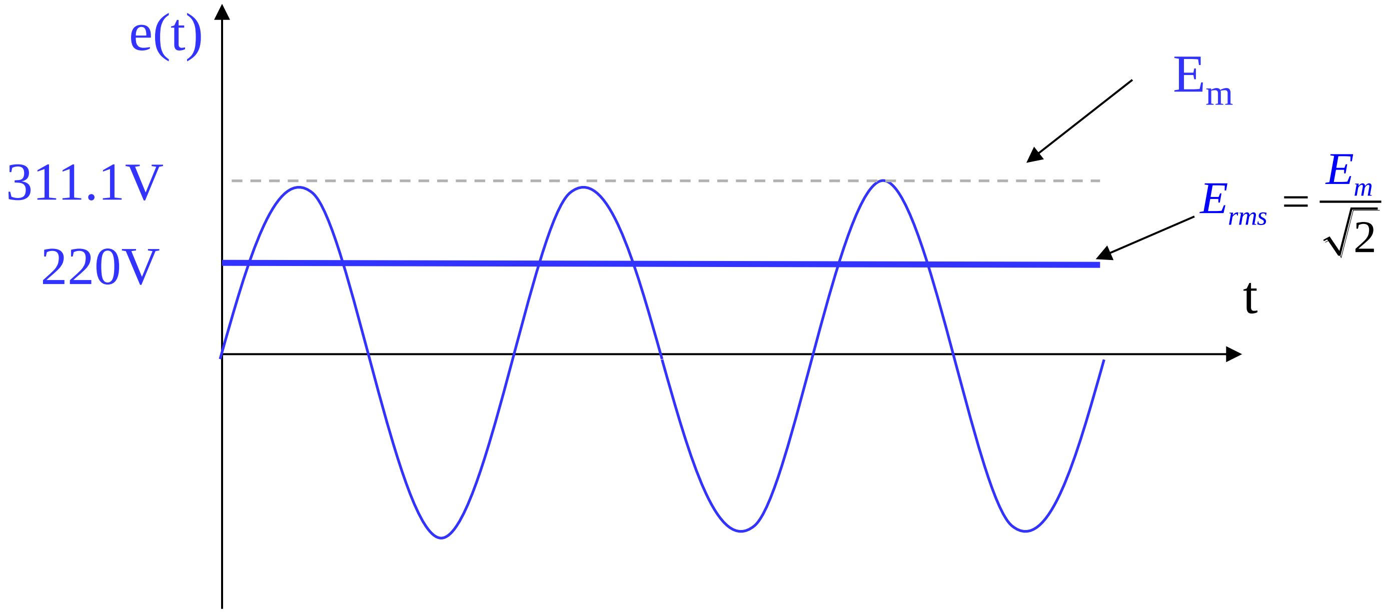
$$I = \sqrt{\frac{1}{2\pi}\left[0 + \int_{\pi/3}^{\pi} i^2 d(\omega t) + 0\right]} = \sqrt{\frac{1}{2\pi}\left[\int_{\pi/3}^{\pi} \left(10\sin(\omega t)\right)^2 d(\omega t)\right]} = 4.49 \ A$$

$$i(t) = I\sin(\omega t) \begin{cases} i(t) = 0 & 0 - \pi/3 \\ i(t) = 10\sin\omega t & \pi/3 - \pi \\ i(t) = 0 & \pi - 2\pi \end{cases}$$

Effective (rms) curent value:



Grid Voltage



$$e(t) = \sqrt{2}E\cos(\omega t + \beta)$$

$$E_{rms} = \frac{E_m}{\sqrt{2}}$$

$0.707E_m$