Ankara University
Engineering Faculty
Department of Engineering Physics

PEN207

Circuit Design and Analysis

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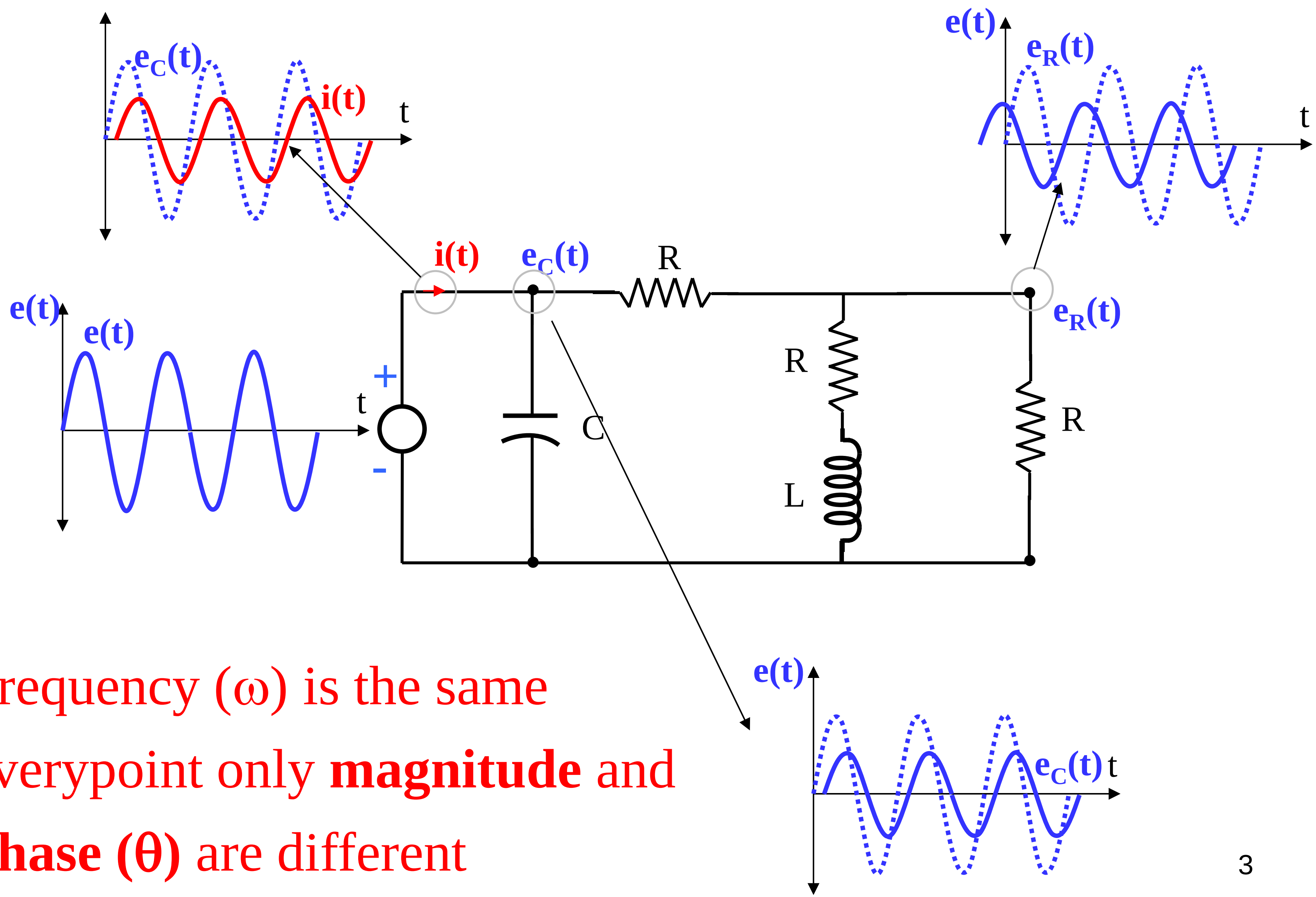
Chapter-5

Steady State AC Circuits (2/2)

Phasor Diagram Methods



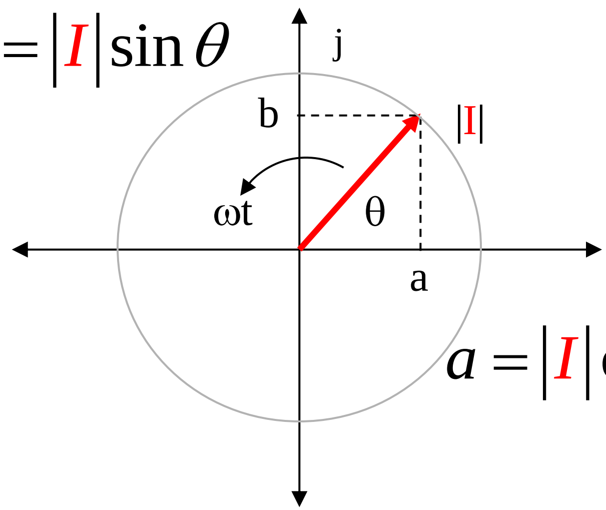
Frequency (ω) is the same everypoint only **magnitude** and **phase (θ)** are different

Phasor Diagram Methods

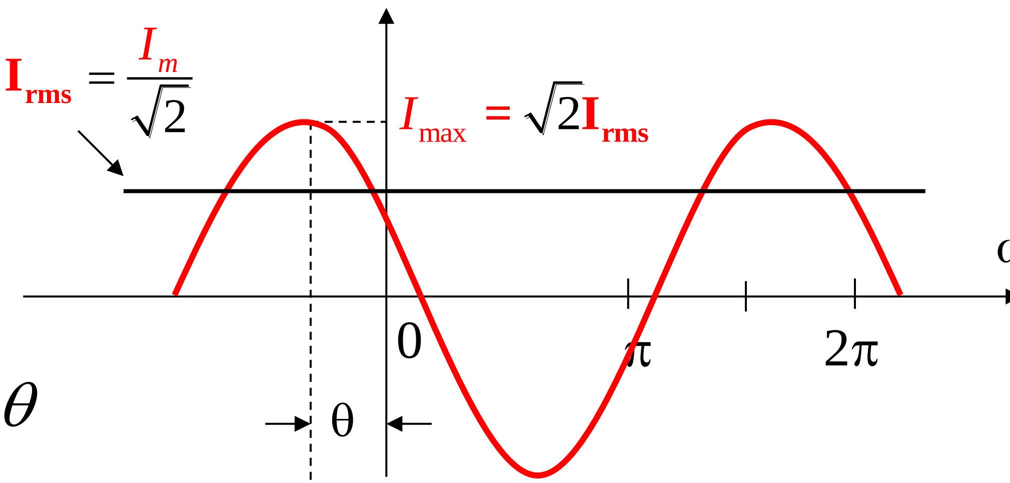
Phasor Diagram Method is a practical method used to analysis circuits when the current and voltage sources in the circuits are all supplying sinusoidal signal with the same frequency.

$$i(t) = \sqrt{2} I_{rms} \cos(\omega t + \theta)$$

$$b = |I| \sin\theta$$



$$a = |I| \cos\theta$$



Any vector can be represented by a complex number

$$\mathbf{I} = a + jb$$

$$|I| = \sqrt{(a + jb).(a - jb)}$$

$$= \sqrt{a^2 + b^2}$$

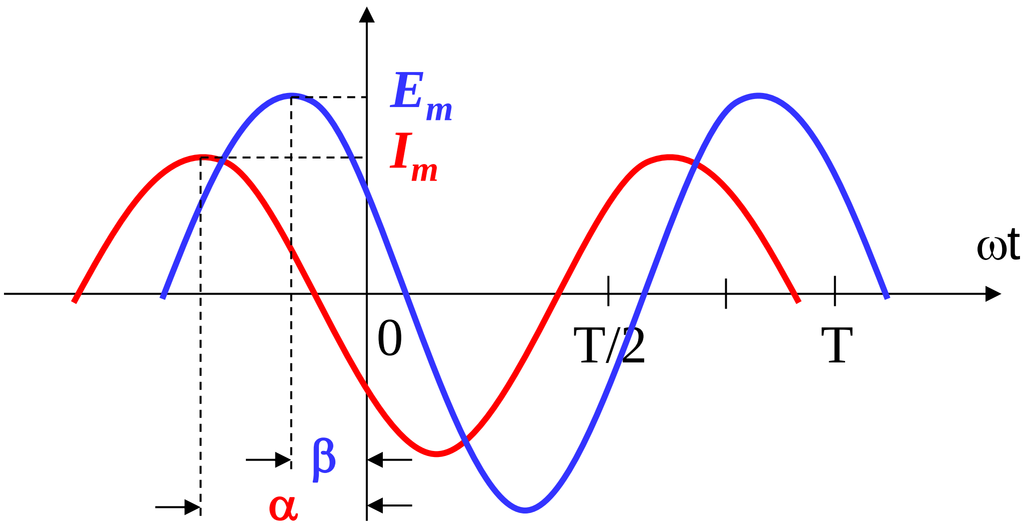
$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$i(t) = \sqrt{2} I_{rms} \cos(\omega t + \theta)$$

$$\mathbf{I} = I \angle \theta$$

$$A = rms \angle phaseangle$$

Representing Current and Voltage on Phasor Diagram



$$i(t) = \sqrt{2} I_{rms} \cos(\omega t + \alpha) \qquad e(t) = \sqrt{2} E_{rms} \cos(\omega t + \beta)$$

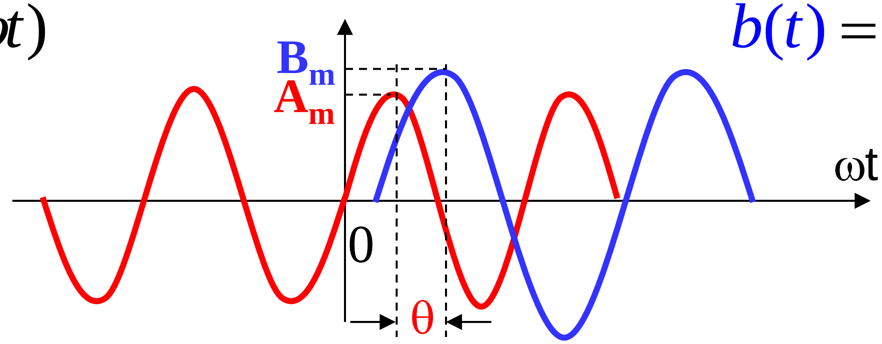
Phasor representation: $\mathbf{I} = I \angle \alpha$ $\qquad \mathbf{E} = E \angle \beta$

Phasor Diagram method uses *I* and *E* quantities to represent sinusoidal currents and voltages; They have magnitudes equal to RMS and an angle equal to the value of the argument (phase angle) at t=0

Phasor Diagram Method

$a(t)=\sqrt{2}A_{rms}\cos(\omega t)$

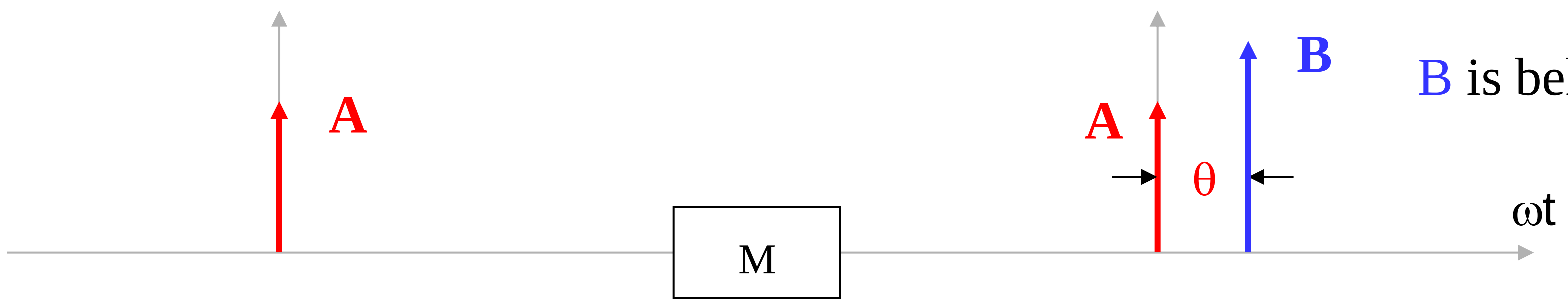
$\vec{\mathbf{A}} = |A|\angle 0^o$



$b(t)=\sqrt{2}B_{rms}\cos(\omega t+\theta)$

$\vec{B}=|B|\angle\left(0^o+\theta\right)$

Time Domain:

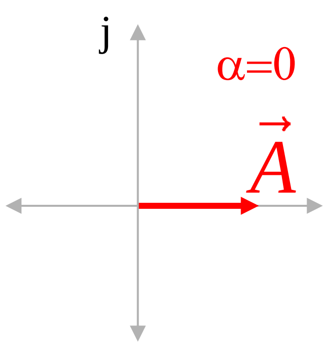


B is behind A

Complex Plane:

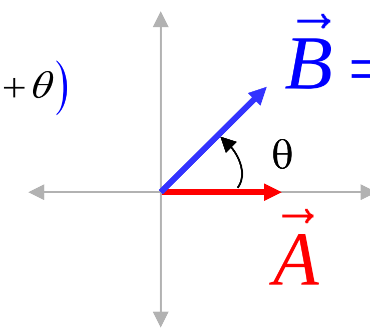
$A = a + jb$

$= a$



$$\vec{B}=M\vec{A}$$

$$\vec{B}=M\vec{A}=M\,|A|\angle 0^o=|B|\angle\left(0^o+\theta\right)$$



$$\vec{B}=M\vec{A}$$

$\vec{\mathbf{A}} = |A|\angle 0^o$

$\vec{B}=|B|\angle\left(0^o+\theta\right)$

Phasor Diagram Method

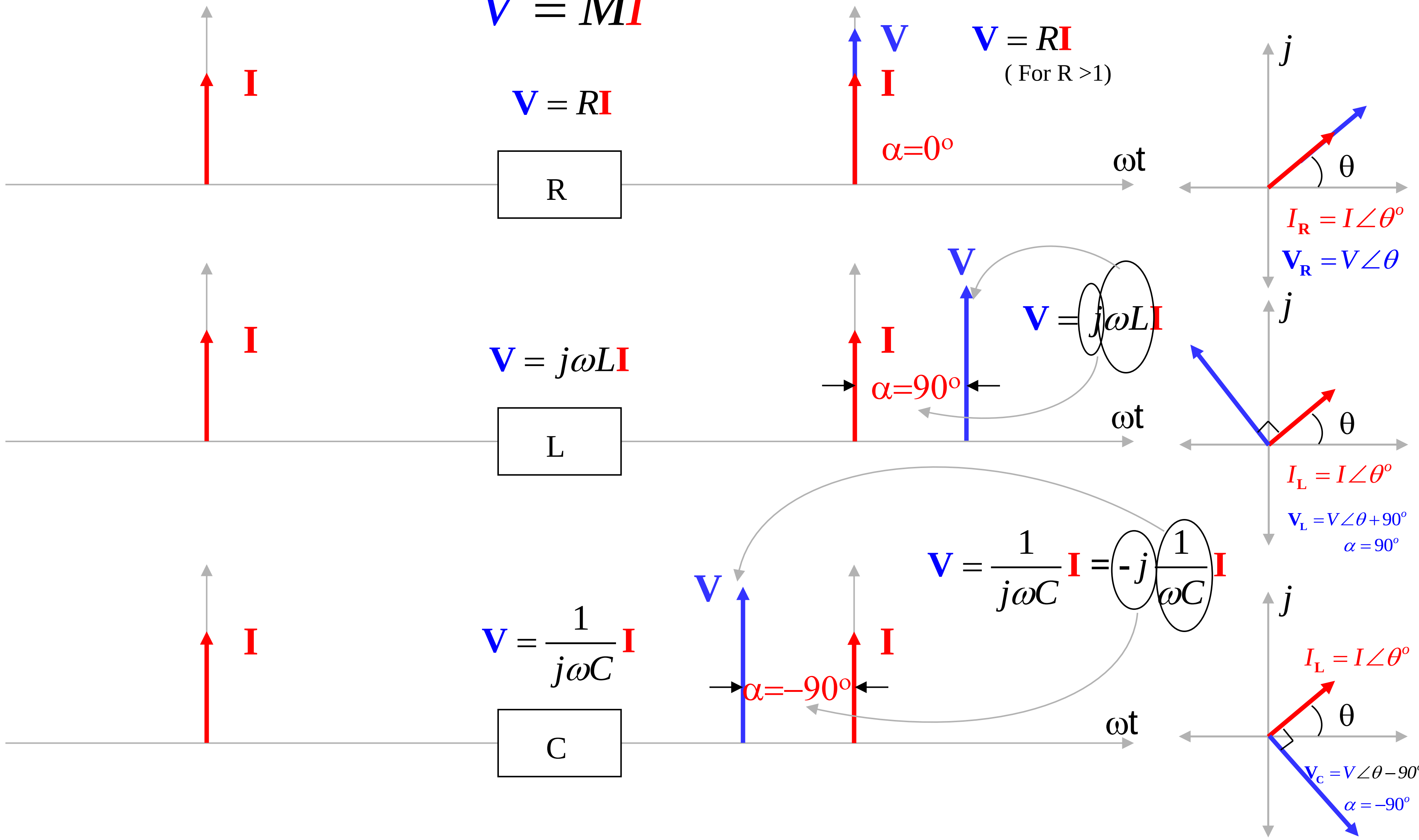
$$i(t) = \sqrt{2} I_{rms} \cos(\omega t + \theta)$$

$$v(t) = \sqrt{2} V_{rms} \cos(\omega t + \theta + \alpha)$$

$$\mathbf{I} = I \angle \theta^o$$

$$\mathbf{V} = V \angle \left(\theta^o + \alpha^o \right)$$

$$\vec{V} = M \vec{I}$$



Phasor Diagram Method-Calculations-1

Conversion from Phasor Diagram notation to Complex Number notation:

$$\mathbf{A} = |A|\angle\theta \Rightarrow \mathbf{A} = |A|\cos\theta + j(|A|\sin\theta) = a + jb$$

Conversion from Complex Number notation to Phasor Diagram notation :

$$\mathbf{A} = a + jb \Rightarrow \begin{cases} |A| = \sqrt{(a+jb).(a-jb)} = \sqrt{a^2+b^2} \\ \theta = \tan^{-1}\left(\frac{b}{a}\right) \end{cases} \Rightarrow \mathbf{A} = |A|\angle\theta$$

Operations:

$$\mathbf{A} = A\angle\theta_A$$

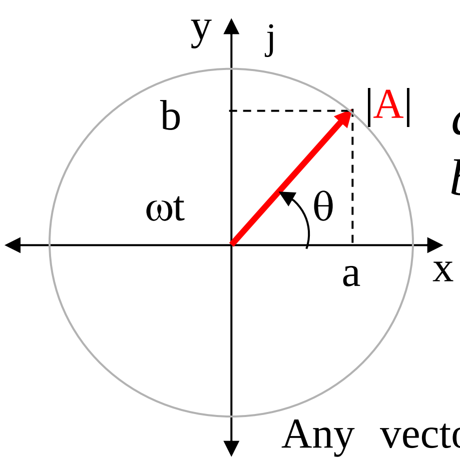
$$\mathbf{B} = B\angle\theta_B$$

Division:

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{|A|\angle\theta_A}{|B|\angle\theta_B} = (|A|/|B|)\angle(\theta_A - \theta_B)$$

Multiplication:

$$\mathbf{A}.\mathbf{B} = (A\angle\theta_A).(B\angle\theta_B) = (A.B)\angle(\theta_A + \theta_B)$$



$$a = |A|\cos\theta$$

$$b = |A|\sin\theta$$

Any vector **A**, can be represented by a complex number.

Addition/Substaction:

$$\mathbf{A} \pm \mathbf{B} = (A\angle\theta_A) \pm (B\angle\theta_B) = (a_1 + a_2 j) \pm (b_1 + b_2 j) = (a_1 \pm b_1) + j(a_2 \pm b_2) \equiv c + jd$$

$$\mathbf{A} \pm \mathbf{B} = \sqrt{c^2 + d^2}\angle\tan^{-1}(d/c)$$

Phasor Diagram Method-Example

Conversion from Phasor Diagram notation to Complex Number notation:

$$\mathbf{A} = 5\angle 30^o \Rightarrow \mathbf{A} = 5\cos(30^o) + j(5\sin(30^o)) = 4.3 + j2.5$$

Conversion from Complex Number notation to Phasor Diagram notation :

$$\mathbf{A} = 3 + j4 \Rightarrow$$

$$|A| = \sqrt{(3+j4).(3-j4)} = \sqrt{3^2 + 4^2} = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^o$$

$$\Rightarrow \vec{B} = M\vec{A}$$

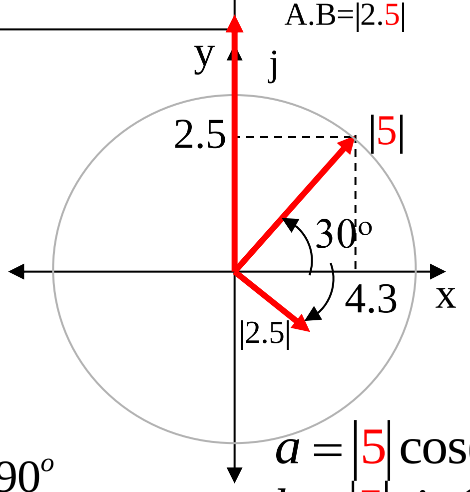
Operations:

$$\mathbf{A} = 5\angle 30^o$$

$$\mathbf{B} = 2\angle 60^o$$

Division:

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{|5|\angle 30^o}{|2|\angle 60^o} = (|5|/|2|)\angle(30^o - 60^o) = 2.5\angle -30^o$$



$$a = |5|\cos(30^o) = 4.3$$

$$b = |5|\sin(30^o) = 2.5$$

Multiplication:

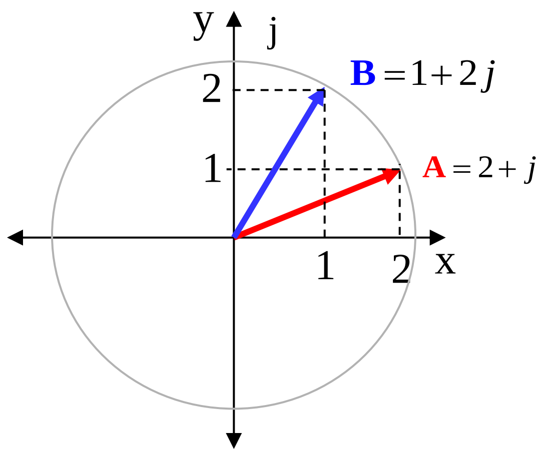
$$\mathbf{A}.\mathbf{B} = (5\angle 30^o).(2\angle 60^o) = (5.2)\angle(30^o + 60^o) = 10\angle 90^o$$
$$= j10$$

Addition/Substaction:

$$\mathbf{A} + \mathbf{B} = (5\angle 30^o) + (2\angle 60^o) = (4.3 + j2.5) + (1 + j1.73) = (4.3 + 1) + j(2.5 + 1.73) \equiv 5.3 + j4.2$$

$$\mathbf{A} + \mathbf{B} = \sqrt{(5.3)^2 + (4.2)^2}\angle\tan^{-1}(4.2/5.3) \cong 6.8\angle 86^o$$

Phasor Diagram and Complex Representation



$$\mathbf{A} = 2 + j \qquad \mathbf{A} = 5\angle 26^o$$

$$\mathbf{B} = 1 + 2j \qquad \mathbf{B} = 5\angle 63^o$$

Multiplication:

Phasor Diagram

$$\mathbf{A.B} = \left(\sqrt{5}\angle 26^o\right).\left(\sqrt{5}\angle 63^o\right) = \left(\sqrt{5}.\sqrt{5}\right)\angle\left(26^o + 63^o\right) = 5\angle 90^o$$

With complex numbers:

$$A.\mathbf{B} = (2+j).(1+j2) = (2+4j+j-2) = (2+4j+j-2) = 5j$$

$$\left|A.\mathbf{B}\right| = \sqrt{(5j).(-5j)} = \sqrt{25} = 5 \qquad \theta = \tan^{-1}\left(\frac{5}{0}\right) = 90^o$$

$$\mathbf{A.B} = 5\angle 90^o$$

Division:

Phasor Diagram

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{|5|\angle 26^o}{|5|\angle 63^o} = \left(|5|/|5|\right)\angle\left(26^o - 63^o\right) = 1\angle -37^o$$

With complex numbers:

$$\frac{A}{\mathbf{B}} = \frac{2+j}{1+j2} = \frac{(2+j).(1-j2)}{(1+j2).(1-j2)} = \frac{2-4j+j+2}{5} = \frac{4-3j}{5} = \frac{4-3j}{5} = 4/5 - 3/5j$$

$$\left|\frac{A}{\mathbf{B}}\right| = \sqrt{(4/5)^2 + (-3/5)^2} = 1 \qquad \theta = \tan^{-1}\left(\frac{-3/5}{4/5}\right) = -37^o$$

$$\frac{\mathbf{A}}{\mathbf{B}} = 1\angle -37^o$$

Phasor Diagram Method

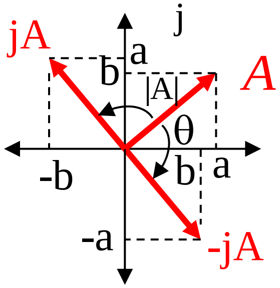
The relations between current and voltage in circuit elements, such as resistance, inductance and capacitance, are adapted to the equations that define transformed circuits. Relations involving impedance and admittance parameters can be given:

Resistance: $\mathbf{V} = R.\mathbf{I}$

Inductance: $\mathbf{V} = j\omega L\mathbf{I}$

Capacitance: $\mathbf{V} = \frac{1}{j\omega C}\mathbf{I} = -j\frac{1}{\omega C}\mathbf{I}$

Complex number



$$A = a + jb$$

$$jA = j(a + jb) = -b + ja$$

$$(-j)A = (-j)(a + jb) = b - ja$$

By multiplyin a vector by j (-j) is the rotating it by 90º (-90º).

In circuit elements such as inductance and capacitance, there is a 90° phase difference between the current and the voltage (the voltage in the coil is in front of the current and the voltage in the capacitor is behind the current)..

For general case: $\mathbf{V} = Z\mathbf{I}$ **Z**: Impedance

(Depending on the value of Z, the **voltage** may be in front of or behind the **current**)

Resistor

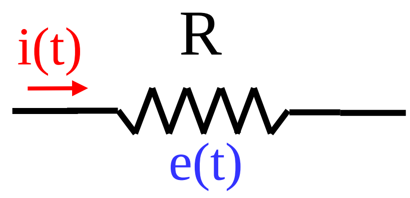
When the current vector I is multiplied by a scalar number R (positive) the new vector V will be in the same direction as I only its magnitude has changed. This means that the current and voltage on the resistor are in the same phase.

$$\mathbf{V} = R.\mathbf{I}$$

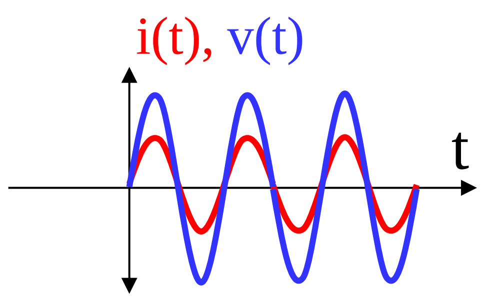
$$i(t) = I_m \sin(\omega t + \theta)$$

$$i(t) = \sqrt{2} I \sin(\omega t + \theta)$$

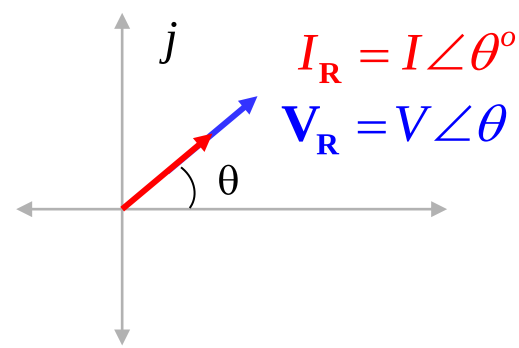
$$I \equiv I_{rms}$$



$$e(t) = R i(t) = E_m \sin(\omega t + \theta)$$



$$\mathbf{V_R} = R\mathbf{I} = RI\angle\theta^o$$

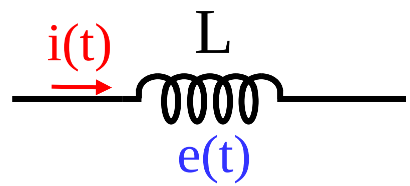


Voltage and current are in the same phase - where one is maximum and the other is maximum. (Plotted for R>1)

Inductance

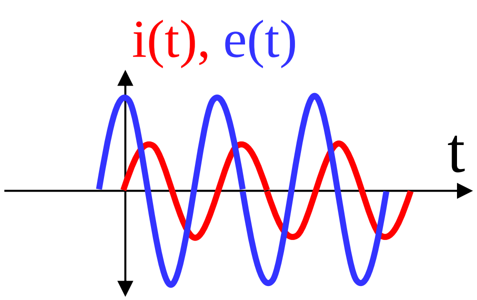
When the current vector I is multiplied by a imaginary number (jωL) the new vector V will be 90° ahead of I vector, the magnitude has changed by ωL. This means that the current and voltage on the inductor are out of phase (current is behind the voltage).

$$i(t) = I_m \sin(\omega t + \theta)$$

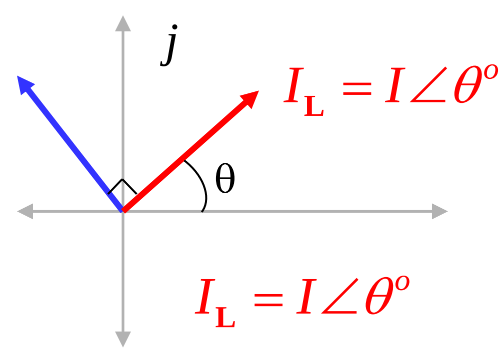


$$\mathbf{V} = j\omega L\mathbf{I}$$

$$e(t) = L\frac{di(t)}{dt} = E_m \cos(\omega t + \theta) = E_m \sin(\omega t + \theta + 90^o)$$



$$\mathbf{V} = j\omega L\mathbf{I} = \omega L\angle\theta + 90^o$$



$$I_\mathbf{L} = I\angle\theta^o$$

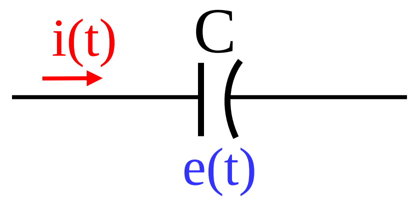
$$\mathbf{V_L} = V\angle\theta + 90^o$$

Voltage v(t) is 90° ahead of current i(t)

Capacitance

When the current vector I is multiplied by a imaginary number (1/jωC) the new vector V will be 90º ahead of I vector, the magnitude has changed by 1/ωC. This means that the current and voltage on the inductor are out of phase (current is ahead of the voltage).

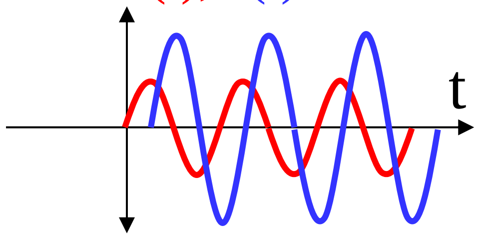
$$i(t) = I_m \sin(\omega t + \theta)$$
$$i(t) = \sqrt{2} I_{rms} \sin(\omega t + \theta)$$



$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I} = -j\frac{1}{\omega C}\mathbf{I}$$

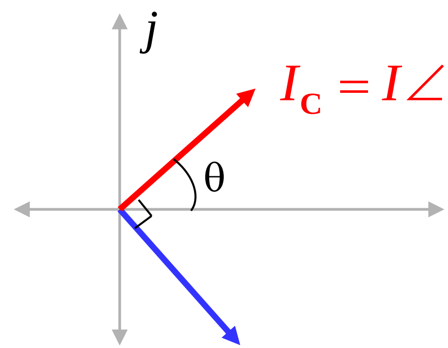
$$e(t) = \frac{1}{C}\int i(t)dt = -E_m \cos(\omega t + \theta) = E_m \sin(\omega t + \theta - 90^o)$$

i(t), e(t)



Voltage v(t) is 90º behind of current i(t)

$$I_C = I\angle\theta^o$$
$$\mathbf{V}_C = V\angle\theta - 90^o$$



$$I_C = I\angle\theta^o$$

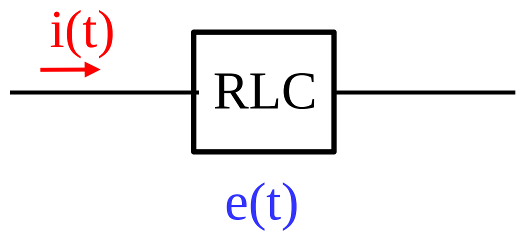
$$\mathbf{V}_C = j\omega L\mathbf{I}_C = \omega L\angle\theta - 90^o$$

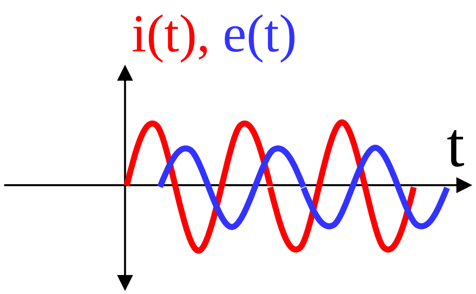
General Case

When the current vector I is multiplied by a complex number ***Z*** the new vector V will be θ ahead or behind of I vector.

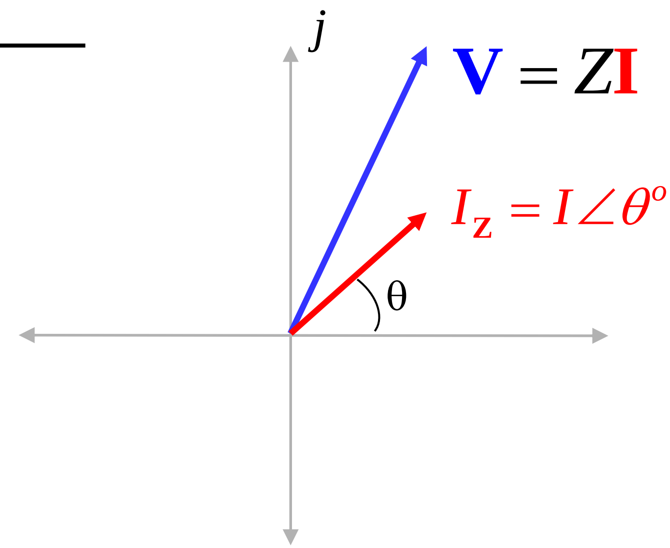
$$\mathbf{V} = Z\mathbf{I}$$

$$i(t) = I_m \sin(\omega t + \theta)$$



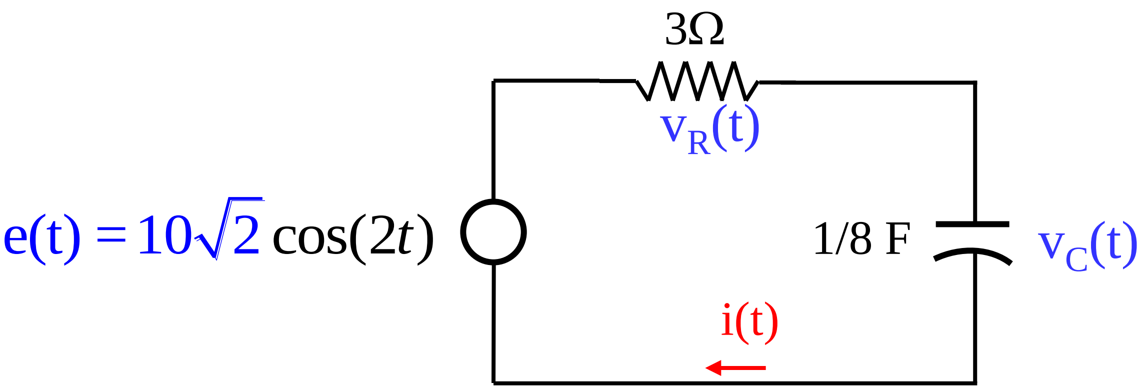


In general there is θ° phase difference between voltage v(t) and current i(t)

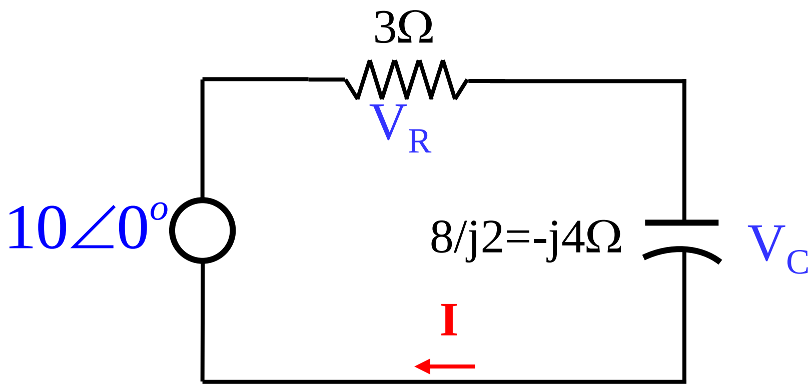


Z complex number defines how much V vector will rotate belirler.

Example-5.2: Find the current i(t) in the circuit below. Use phasor diagram method and draw phasor diagram showing the circuit voltages and current.



Solution-5.2:



$$\omega = 2 \;\; rad/s$$

$$e(t) = 10\sqrt{2}\cos(2t) \Rightarrow \mathbf{V} = 10\angle 0^o$$

Resistor: $\mathbf{V} = 3\mathbf{I}$

capacitor : $\mathbf{V} = \dfrac{1}{j\omega C}\mathbf{I} = \dfrac{1}{j2(\frac{1}{8})}\mathbf{I} = -j4\mathbf{I}$

Kirchhoff's Voltage Law (KVL): $\mathbf{V_R} + \mathbf{V_C} = 10\angle 0^o$

$$3\mathbf{I} - j4\mathbf{I} = 10\angle 0^o \Rightarrow \mathbf{I}(3 - j4) = 10\angle 0^o$$

$$\mathbf{I} = \frac{10\angle 0^o}{|(3 - j4)|}$$

$$(3 - j4) = |(3 - j4)| \angle - \tan^{-1}(b/a)^o$$

$$|(3 - j4)| = \sqrt{(3 - j4).(3 + j4)} = \sqrt{(3^2 + 4^2)} = 5$$

$$\angle - \tan(b/a)^o = \angle - \tan^{-1}(-4/3)^o = -53.1^o$$

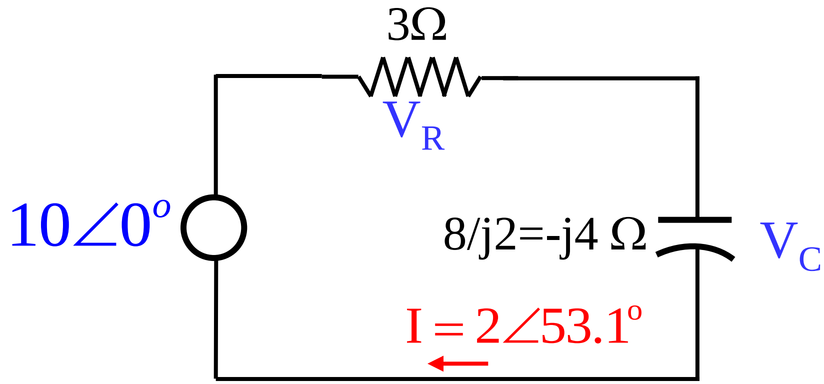
$$(3 - j4) = 5\angle - 53.1^o$$

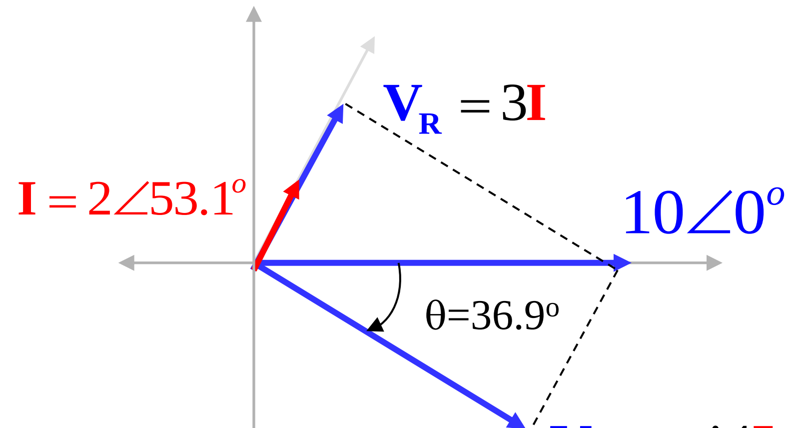
$$\mathbf{I} = \frac{10\angle 0^o}{5\angle - 53.1^o} = 2\angle 53.1^o$$

$$\boxed{\mathbf{I} = 2\angle 53.1^o}$$

$$i(t) = 2\sqrt{2}\cos(2t + 53.1^o)$$

Phasor Diagram Method Current-Voltage Relations

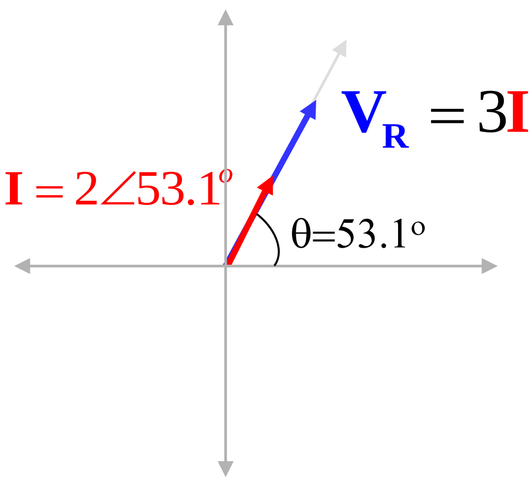




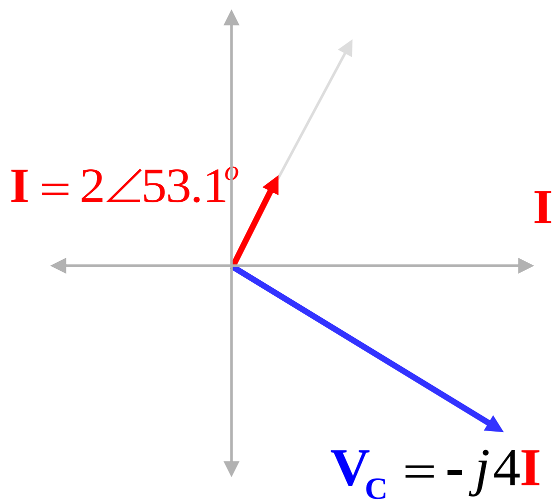
Resistor

Capacitor

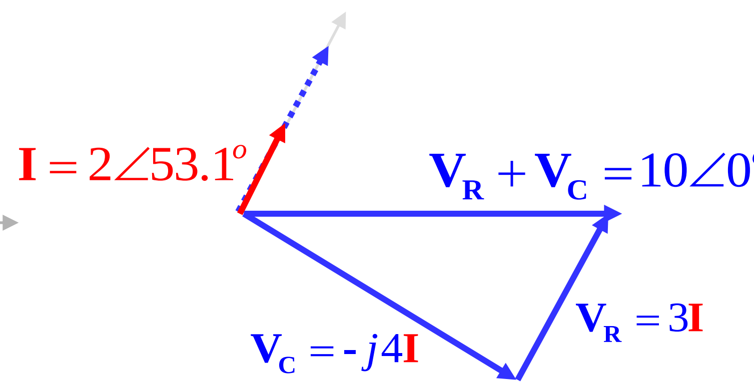
RC



Current and Voltage in phase.



Current is 90° behind voltage



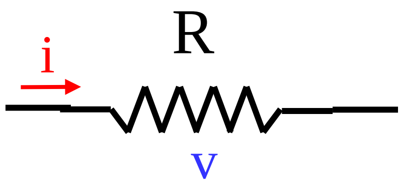
Current is 53.1° behind voltage

The phasor diagram representation shows the phase vector of three voltages and one current. Note that the sum of V_R and V_C constitutes the source voltage and vector. V_C is 90° behind I; V_R is in the current direction (in phase).

Phasor Diagram Current-Voltage Relations

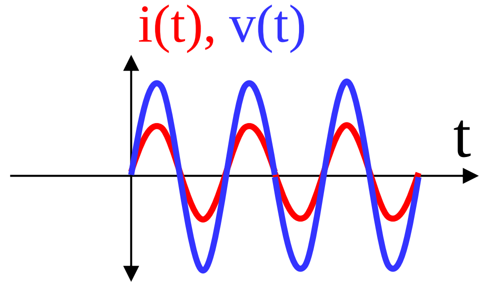
Phasor Diagram Representation

Resistor

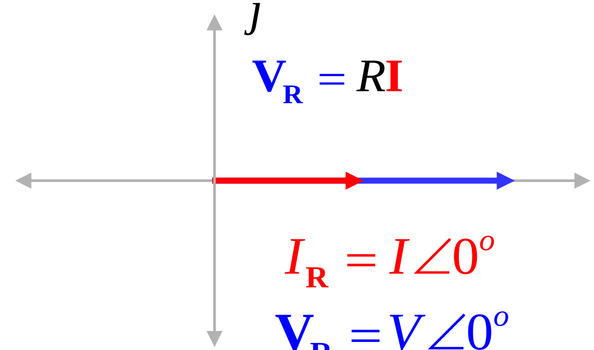


$i(t) = I_m \sin(\omega t)$

$e(t) = R i(t) = E_m \sin(\omega t)$



Voltage and **current** are in phase

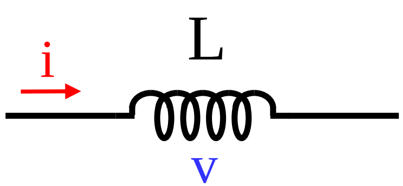


$\mathbf{V_R} = R\mathbf{I}$

$I_\mathbf{R} = I\angle 0^o$

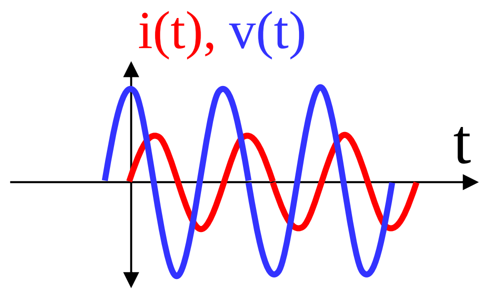
$\mathbf{V_R} = V\angle 0^o$

Inductor

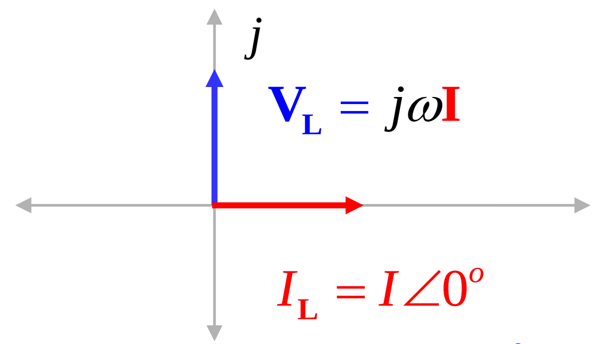


$i(t) = I_m \sin(\omega t)$

$e(t) = L\dfrac{di(t)}{dt} = E_m \cos(\omega t)$



Voltage is 90^o ahead of **current**

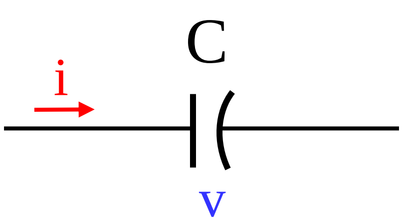


$\mathbf{V_L} = j\omega\mathbf{I}$

$I_\mathbf{L} = I\angle 0^o$

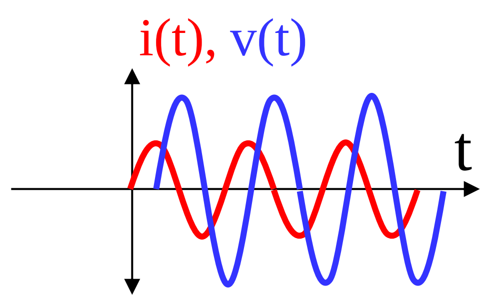
$\mathbf{V_L} = V\angle 90^o$

Capacitor

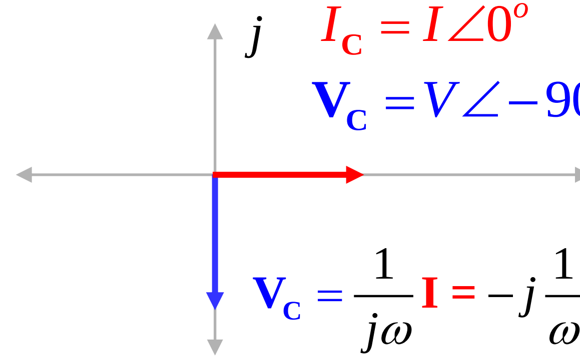


$i(t) = I_m \sin(\omega t)$

$e(t) = \dfrac{1}{C}\int i(t)dt = -E_m \cos(\omega t)$



Voltage is 90^o behind of **current**

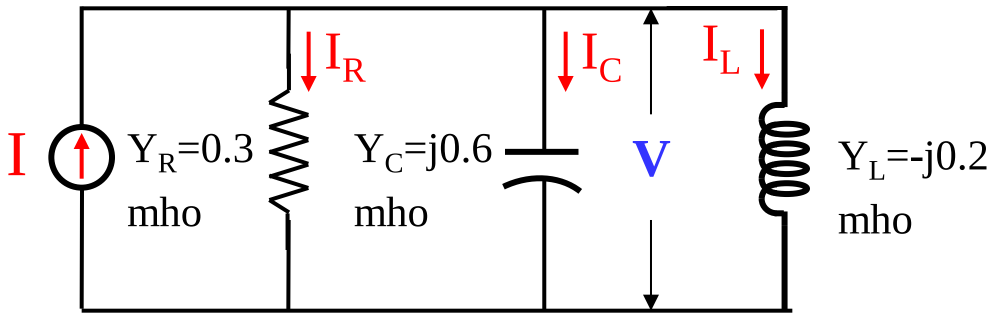


$I_\mathbf{C} = I\angle 0^o$

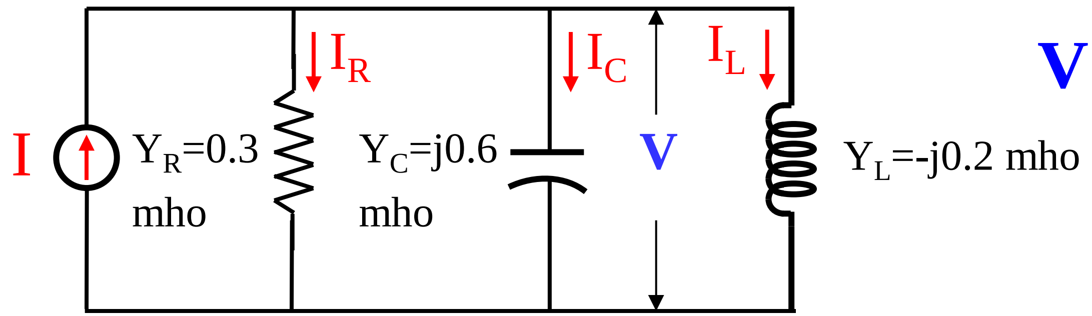
$\mathbf{V_C} = V\angle -90^o$

$\mathbf{V_C} = \dfrac{1}{j\omega}\mathbf{I} = -j\dfrac{1}{\omega}\mathbf{I}$

Example-5.3: In the following circuit, find the value of the current value of source required for a voltage of V=10∟0° V using the phasor diagram method.



Solution-5.3:



$$\mathbf{V} = \mathbf{10}\angle 0^o$$

Resistor: $\mathbf{I_R} = 0.3\mathbf{V} = 3\angle 0^o$

$j6 = 6\angle -\tan(6/0)^o = 6\angle 90^o$

Capacitor: $\mathbf{I_C} = j0.6\mathbf{V} = j6 = 6\angle 90^o$

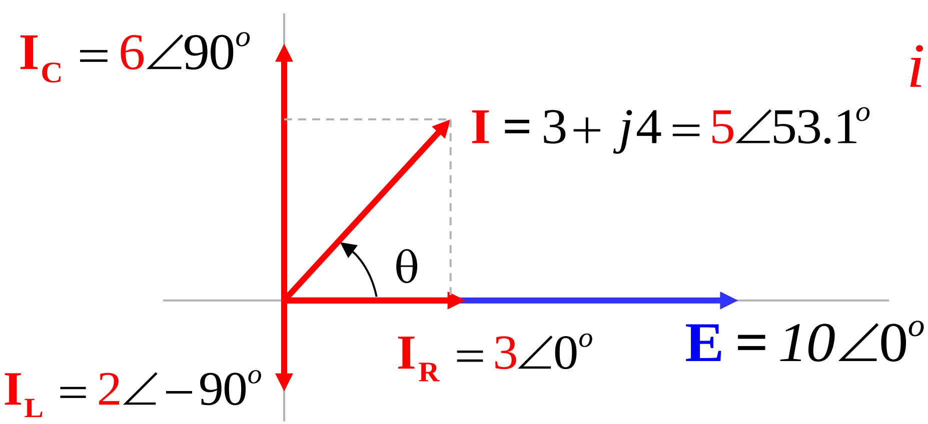
$-\tan^{-1}(6/0)^o = -\tan^{-1}(\infty)^o = 90^o$

$-\tan^{-1}(-2/0)^o = -\tan^{-1}(\infty)^o = -90^o$

Inductor: $\mathbf{I_L} = -j0.2\mathbf{V} = -j2 = 2\angle -90^o$

Kirchhoff's Current Law (KCL): $\mathbf{I} = \mathbf{I_R} + \mathbf{I_C} + \mathbf{I_L}$

$$\mathbf{I} = 3 + j6 + (-j2) = 3 + j4 = 5\angle 53.1^o$$

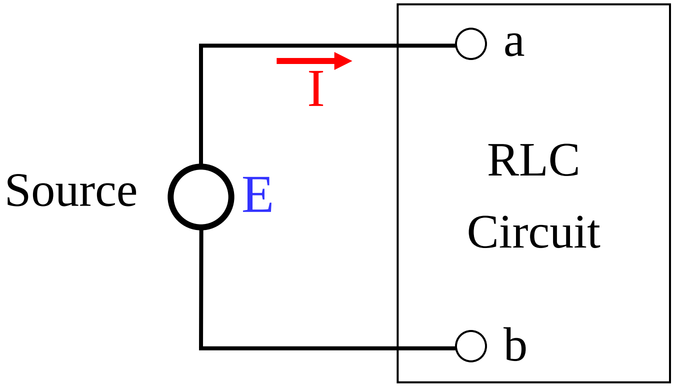


$$i(t) = 5\sqrt{2}\cos(\omega t + 53.1^o)$$

There is θ=53.1° phase shift between applied current and voltage

Circuit Reduction

Like resistive circuits, complex a.c circuits can also be reduced to equivalent resistive circuits. Let's consider the circuit below. The equivalent impedance of the circuit from points a-b:



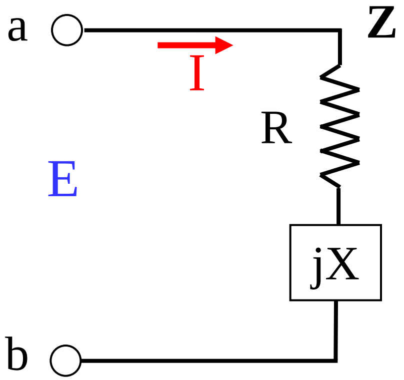
$$\mathbf{Z} = \frac{\mathbf{E}}{\mathbf{I}} \qquad \mathbf{Y} = \frac{\mathbf{I}}{\mathbf{E}} = \frac{1}{\mathbf{Z}}$$

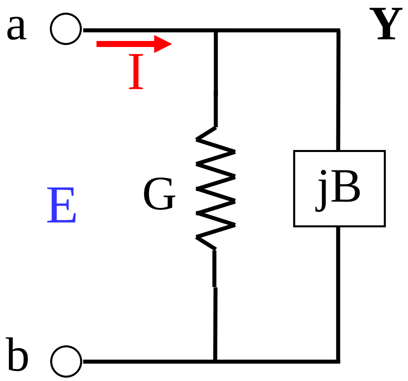
$$\mathbf{Z} = R + jX$$

$$\mathbf{Y} = G + jB$$

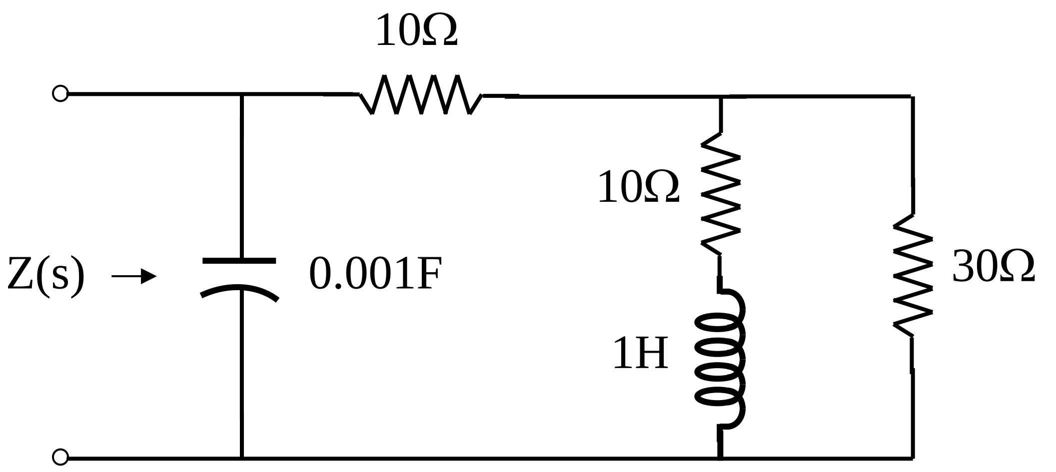
$$\mathbf{I} = \mathbf{E}\mathbf{Y} = G\mathbf{E} + jB\mathbf{E}$$

$$\mathbf{E} = \mathbf{I}\mathbf{Z} = R\mathbf{I} + jX\mathbf{I}$$

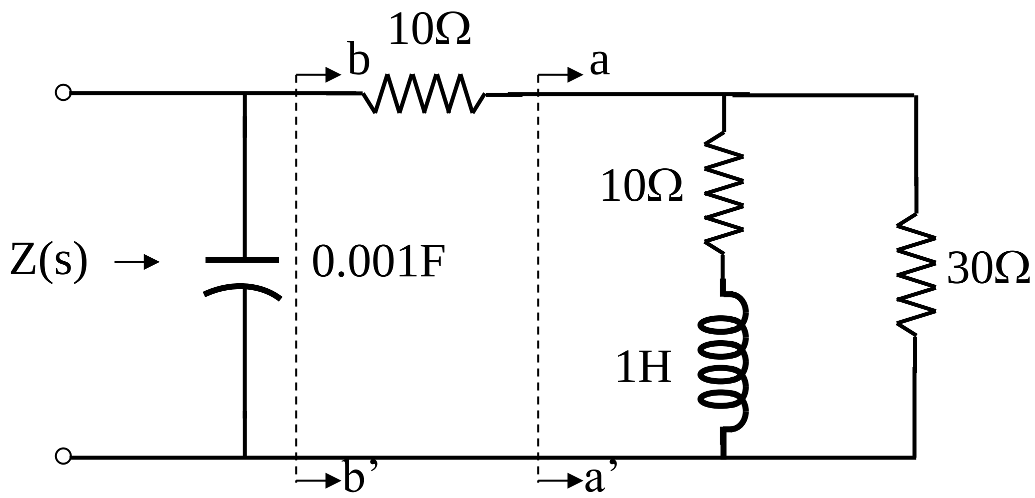




Example-5.4: Angular frequency of the driving source is 10 rad/s, find the input impedance of the following circuit.



Solution-5.4:



To the right of line aa', the 30Ω resistor is parallel to the 10Ω resistor and the series arrangement of the 1H coil. Impedance of series:

$\omega = 10\ rad/s$

$$10 + j\omega 1.0 = 10 + j10 = 14.14\angle 45^o$$

Admittance: $$\frac{1}{14.14\angle 45^o} = 0.05 - j0.05$$

The total admittance of the circuit elements to the right of aa is:

$$Y_a = 0.05 - j0.05 + \frac{1}{30\Omega} = 0.0833 - j0.05 = 0.0970\angle -31^o$$

Impedance of the right side of the line bb'

$$Z_b = 10 + \frac{1}{\mathbf{Y_a}} = 10 + \frac{1}{0.0970\angle -31^o} \Rightarrow Z_b = 18.83 + j5.30 = 19.60\angle 15.7^o$$

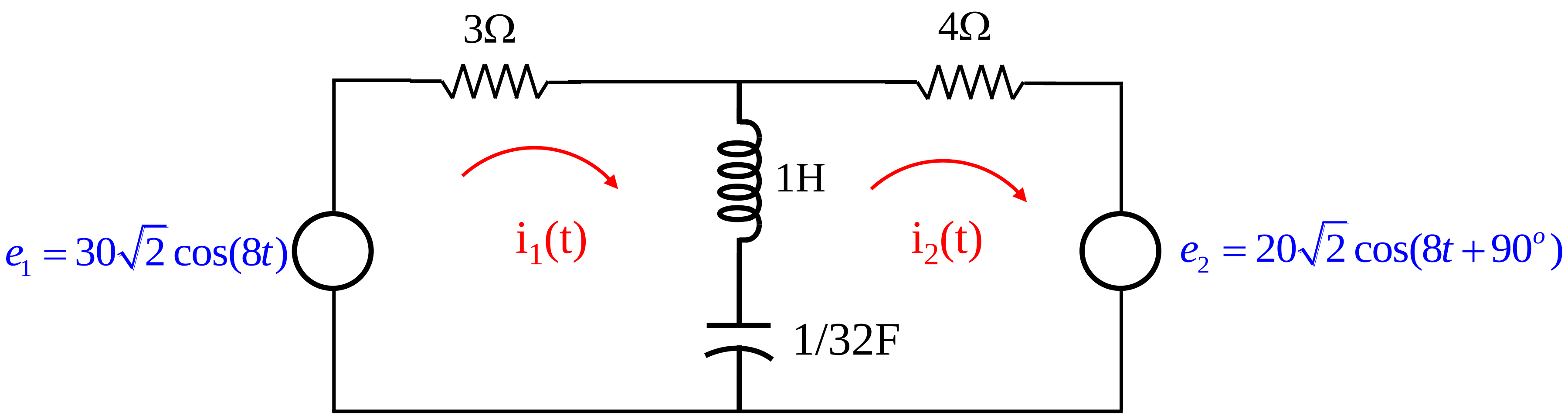
Admittance of capacitor: $Y_C = j\omega(0.001) = j0.01$

Input Admittance: $\mathbf{Y} = j0.01 + \frac{1}{\mathbf{Z_b}} = j0.01 + \frac{1}{19.60\angle 15.7^o} = 0.0492 + j0.0038$

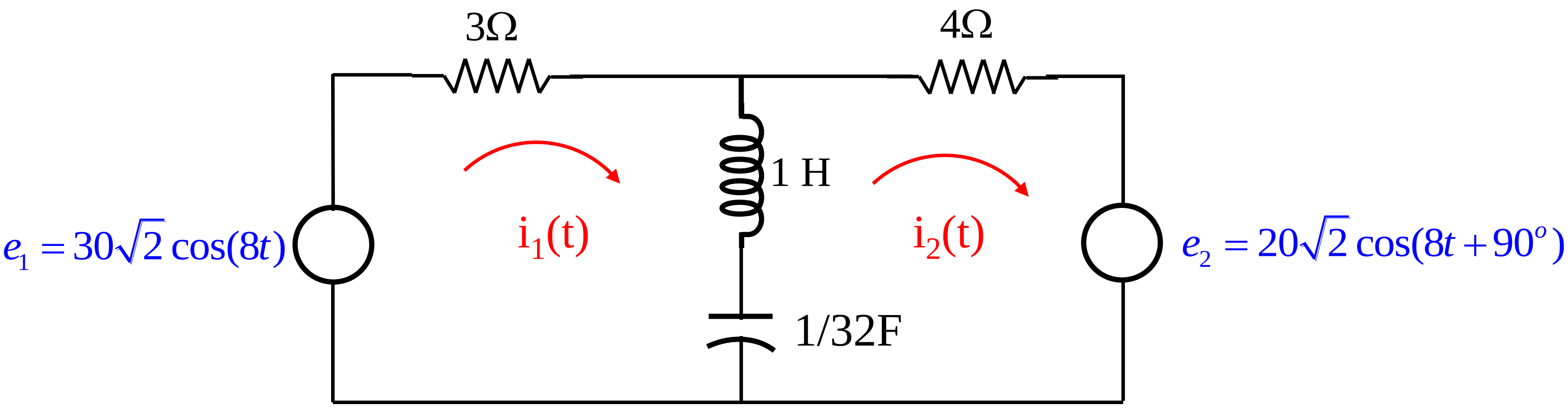
$$\mathbf{Y} = 0,0492 + j0,0038 = 0,0493\angle -4,4^o$$

Input impedance: $\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{0.0493\angle -4.4^o} = 20.2\angle 4.4^o$

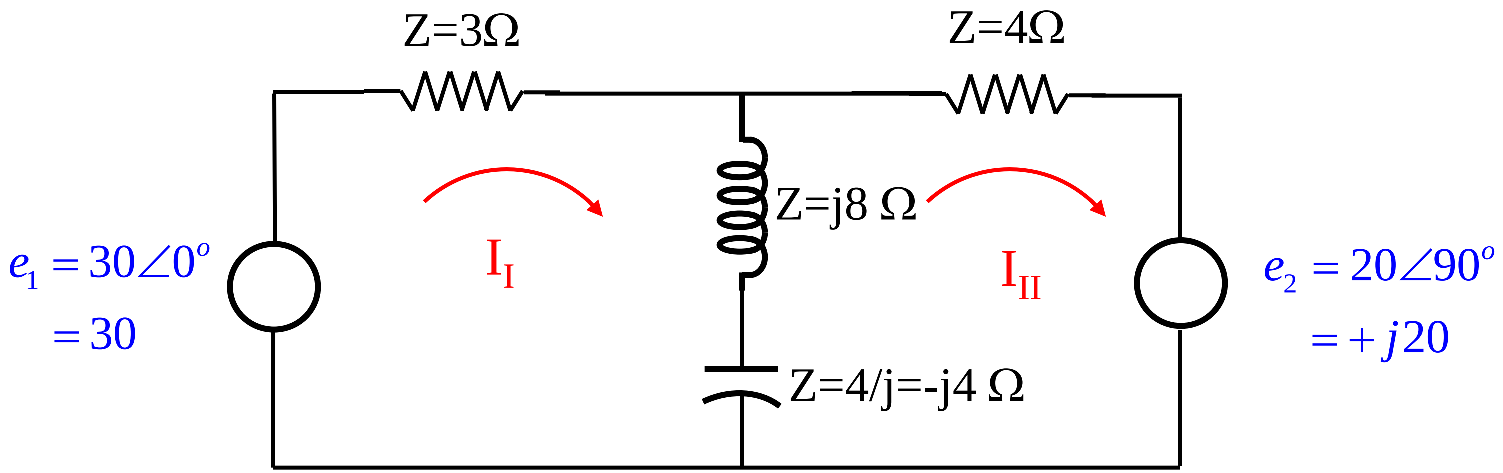
Example-5.5: Find the current $i_1(t)$ of the following circuit.



Solution-5.5:



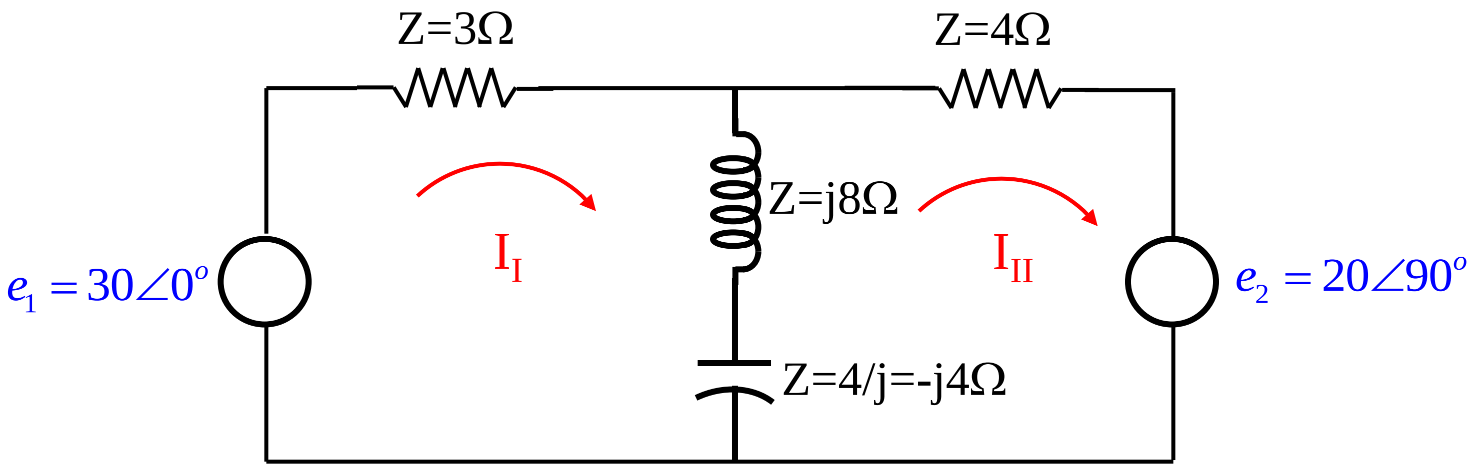
Transformed circuit in frequency domain:



Mesh Current equations:

$$(3 + j8 - j4)\mathbf{I}_I - (j8 - j4)\mathbf{I}_{II} = 30$$

$$-(j8 - j4)\mathbf{I}_I + (4 + j8 - j4)\mathbf{I}_{II} = -j20$$



Mesh Current equations:

$$(3+j8-j4)\mathbf{I}_I - (j8-j4)\mathbf{I}_{II} = 30$$
$$-(j8-j4)\mathbf{I}_I + (4+j8-j4)\mathbf{I}_{II} = -j20$$

After simplification:

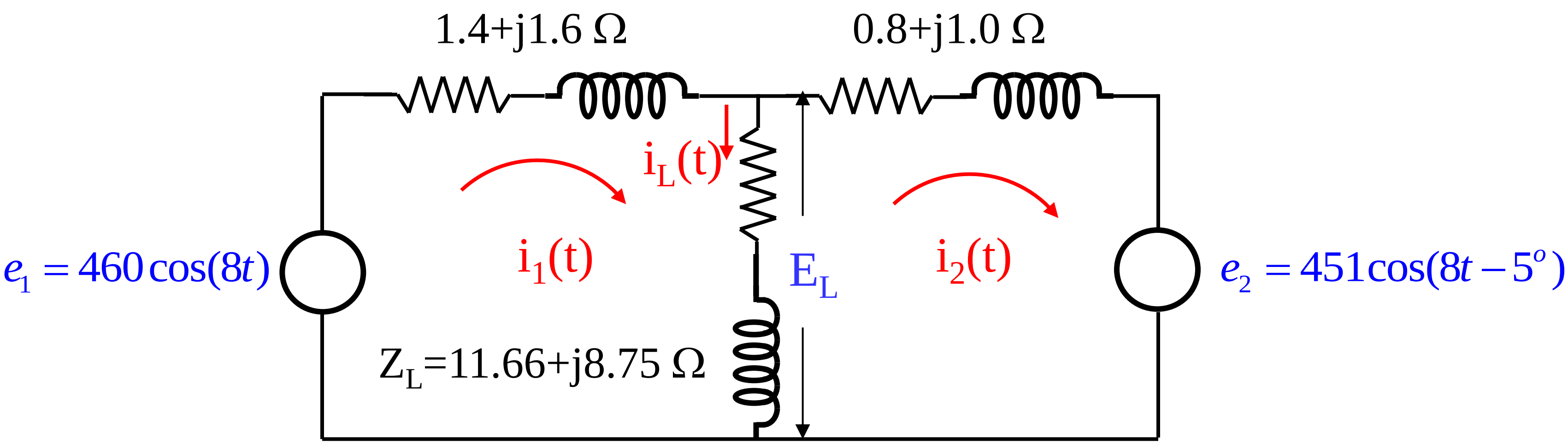
$$(3+j4)\mathbf{I}_I - j4\mathbf{I}_{II} = 30$$
$$-j4\mathbf{I}_I + (4+j4)\mathbf{I}_{II} = -j20$$

$\mathbf{I}_1$ **mesh** current $\mathbf{I}_1 = \mathbf{I}_I = 7.65\angle -35.8^o$

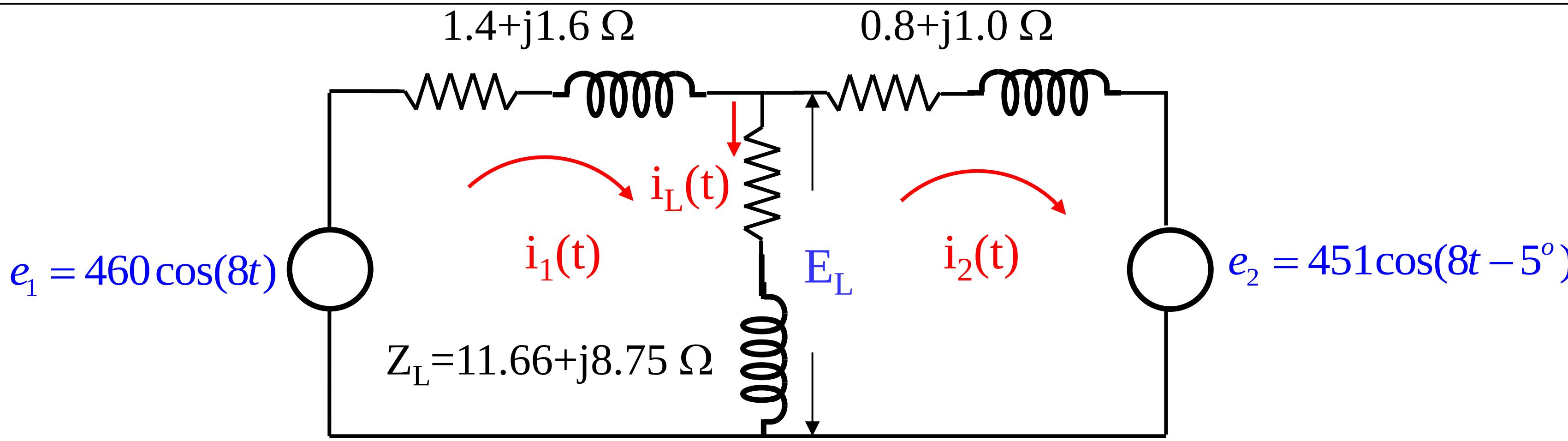
Current as a function of time:

$$i_1(t) = 7.65\sqrt{2}\cos(8t - 35.8^o)$$

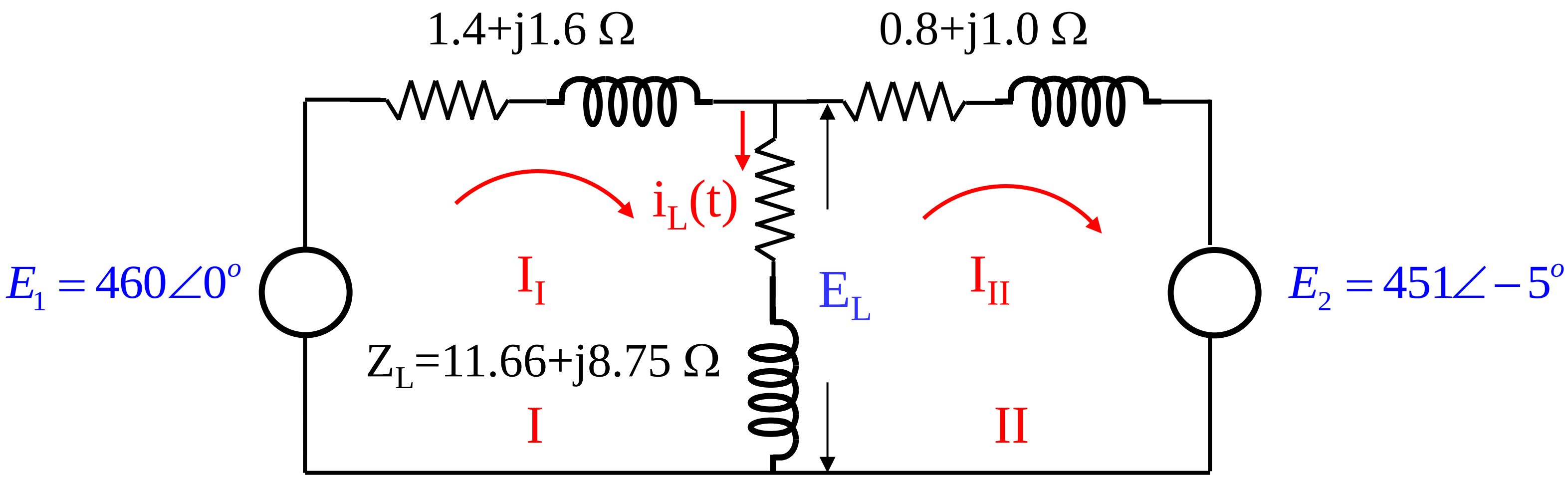
Example-5.6: The following circuit is the equivalent circuit of a load fed from two sources. The sources are considered as ideal voltage sources, load and driver's impedances are given in the figure. The voltages of the 1st and 2nd sources are 460V and 451V, respectively. The voltage source-2 follows 5^o behind the voltage source-1 from. Using the Mesh Current Method, find the load voltage, load current, and current supplied by each source.



Solution-5.6:



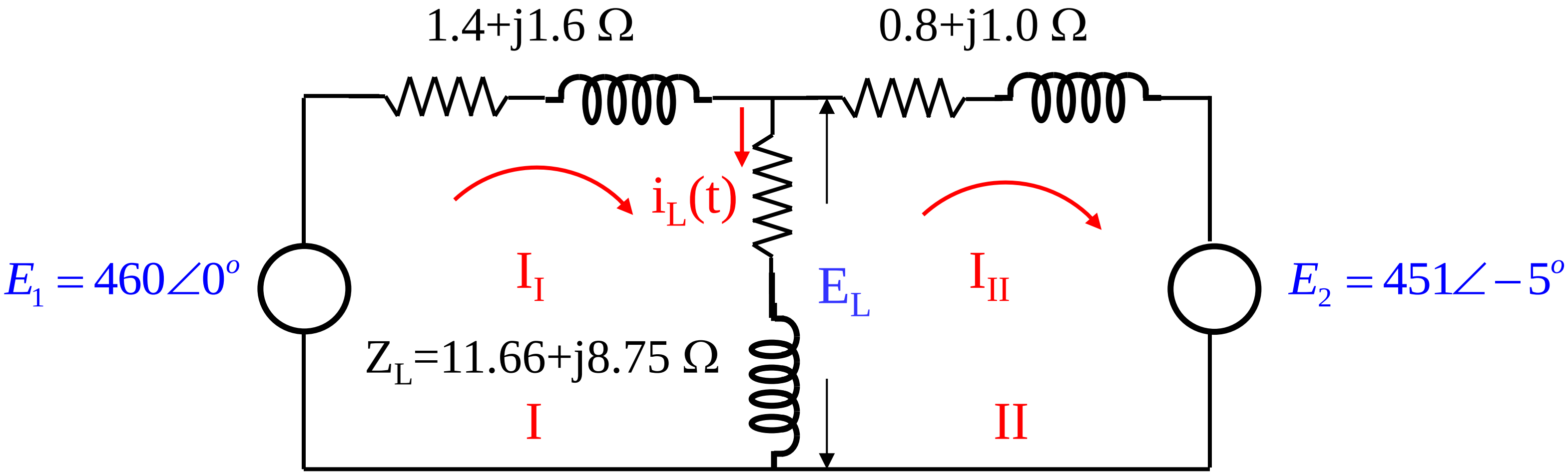
Transformed circuit in frequency domain:



Mesh current equations:

I. Mesh $$[(1.4 + j1.6) + (11.6 + j8.75)]\mathbf{I}_I - (11.66 + j8.75)\mathbf{I}_{II} = 460\angle 0^o$$

II. Mesh $$-(11.66 + j8.75)\mathbf{I}_I + [(11.6 + j8.75) + (0.8 + j1.0)]\mathbf{I}_{II} = -451\angle -5^o$$



Eşitlikler basitleştirilirse

$$(13.6 + j10.35)\mathbf{I}_I - (11.66 + j8.75)\mathbf{I}_{II} = 460\angle 0^o$$
$$-(11.66 + j8.75)\mathbf{I}_I + (12.46 + j9.75)\mathbf{I}_{II} = -451\angle -5^o$$

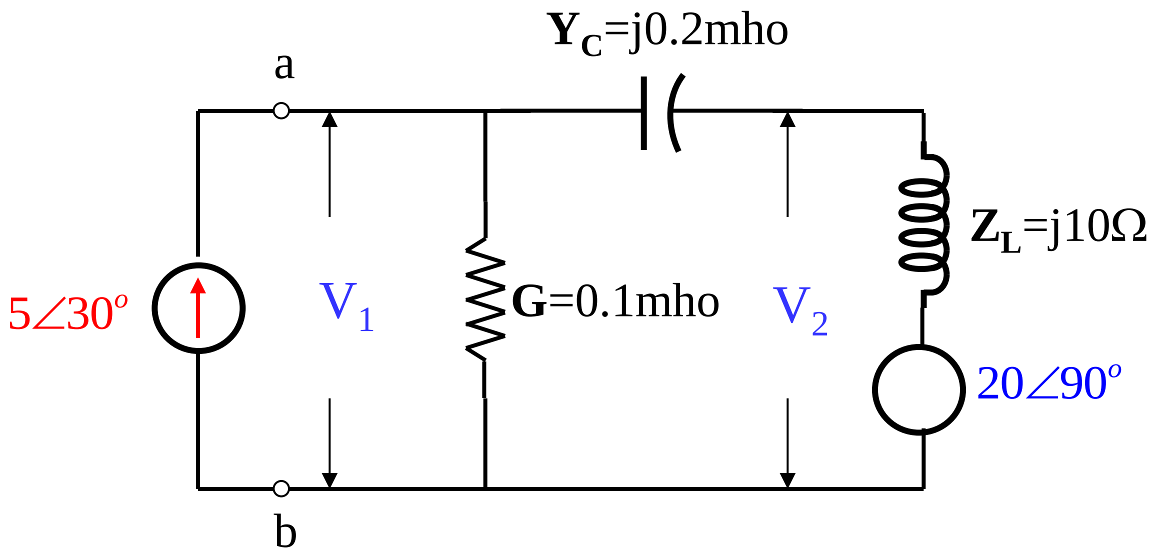
Mesh currents $\mathbf{I}_1 = \mathbf{I}_I = 19.5\angle -5.2^o \quad \mathbf{I}_2 = \mathbf{I}_{II} = -17.8\angle -76.4^o$

Loac current: $\mathbf{I}_L = \mathbf{I}_I - \mathbf{I}_{II} = 30.4\angle -38.9^o$

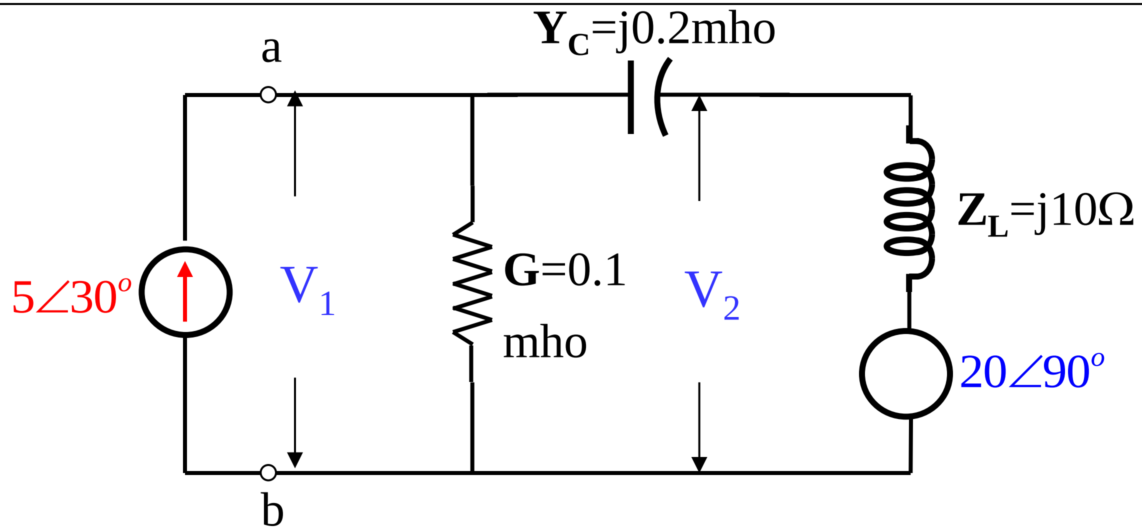
Load voltage: $\mathbf{E}_L = \mathbf{I}_L \mathbf{Z}_L = 30.4\angle -38.9^o (11.66 + j8.75) = 444\angle -2^o$

The load voltage is 444V and the load current is 30.4A. The generator-1 provides 19.5A to load-1; Generator-2 provides a current of 17.8A

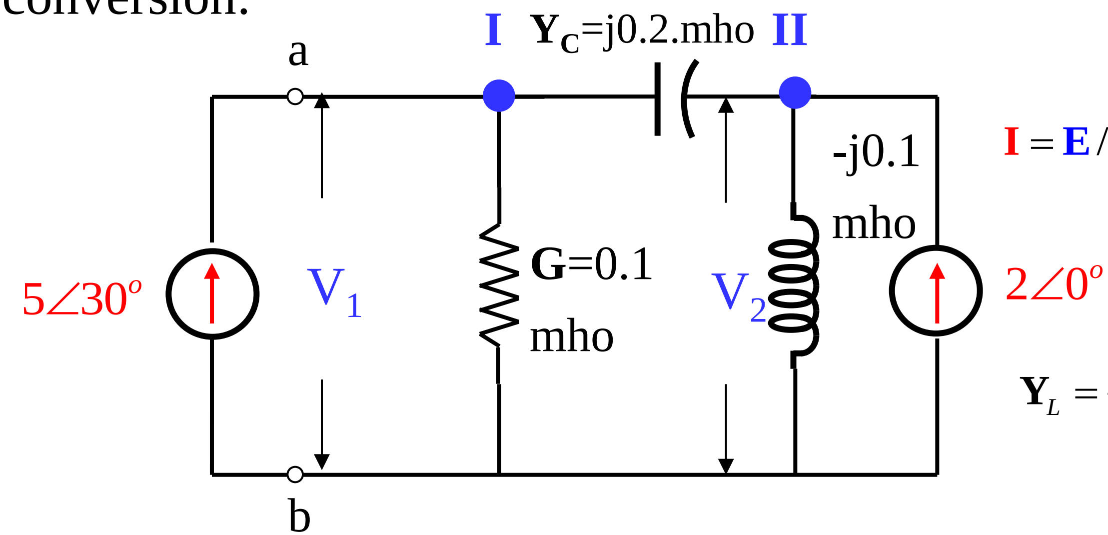
Example-5.7: Find the voltages V_1 and V_2 in the following circuit by applying Node-Voltage Method



Solution-5.7:



After source conversion:



$$\mathbf{I} = \mathbf{E} / \mathbf{Z}_L = \frac{20\angle 90^o}{10\angle 90^o} = 2\angle 0^o \ A$$

$$\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = \frac{1}{j10} = -j0.1 \ mho$$

KCL:

I. Node: $$(0.1 + j0.2)\mathbf{V}_1 - (j0.2)\mathbf{V}_2 = 5\angle 30^o$$

II. Node: $$-(j0.2)\mathbf{V}_1 + (j0.2 - j1.0)\mathbf{V}_2 = 2\angle 0^o$$

$$\mathbf{V}_1 = 38.8\angle 80.1^o \ V \qquad \mathbf{V}_2 = 58.0\angle 76.7^o \ V$$ found.