

# ENE 503 – Computational Fluid Dynamics

## WEEK 12: MATHEMATICAL TERMS

### MATHEMATICAL TERMS:

- Derivatives:

- Full  $\frac{du}{dx}$

- The derivative of  $u$  w.r.t.  $x$ , where  $u = u(x)$

- Partial  $\frac{\partial u}{\partial x}$

- The derivative of  $u$  w.r.t.  $x$ , where  $u = u(x, y, z, t)$

- Substantial  $\frac{DN}{dt} = \frac{\partial N}{\partial x} + \vec{u} \cdot \nabla N$

At this point it is possible to define an extensive property,  $N$  in terms of its corresponding intensive property,  $\eta$  as below:

$$N = \int_{m_s} \eta dm$$

where  $m_s$  is the mass of the system.

Since  $dm = \rho d\forall$ , then

$$N = \int_{\forall_s} \eta \rho d\forall$$

with  $\nabla_s$  being the volume of the system.

- **Vector differential operator**

➤ Del  $\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}$

➤ The gradient: Consider  $\zeta(x, y, z, t)$  a scalar field

The gradient of  $\zeta$  is defined as:

$$\nabla \zeta = \frac{\partial \zeta}{\partial x} i + \frac{\partial \zeta}{\partial y} j + \frac{\partial \zeta}{\partial z} k$$

which gives a vector field.

➤ The divergence: Consider  $\vec{V}(x, y, z, t) = U i + V j + W k$  a vector field

The divergence of  $\vec{V}$  is therefore:

$$\nabla \cdot \vec{V} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}$$

➤ The curl: The curl of  $\vec{V}$  is

$$\nabla \times \vec{V} = \left( \frac{\partial W}{\partial y} - \frac{\partial V}{\partial z} \right) i + \left( \frac{\partial U}{\partial z} - \frac{\partial W}{\partial x} \right) j + \left( \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \right) k$$

- **Definitions - Rectangular coordinates:**

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \mathbf{i} + \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \mathbf{j} + \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \mathbf{k}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \mathbf{A} = \nabla^2 A_x \mathbf{i} + \nabla^2 A_y \mathbf{j} + \nabla^2 A_z \mathbf{k}$$

• **Identities:**

➤ The curl: The curl of  $\vec{V}$  is

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

$$\begin{aligned} (\mathbf{A} \cdot \nabla) \mathbf{B} &= \left[ A_x \frac{\partial B_x}{\partial x} + A_y \frac{\partial B_x}{\partial y} + A_z \frac{\partial B_x}{\partial z} \right] \mathbf{i} \\ &+ \left[ A_x \frac{\partial B_y}{\partial x} + A_y \frac{\partial B_y}{\partial y} + A_z \frac{\partial B_y}{\partial z} \right] \mathbf{j} \\ &+ \left[ A_x \frac{\partial B_z}{\partial x} + A_y \frac{\partial B_z}{\partial y} + A_z \frac{\partial B_z}{\partial z} \right] \mathbf{k} \end{aligned}$$

$$\int_{\tau} \nabla f \, d\tau = \int_S f \, \mathbf{da}$$

$$\int_{\tau} (\nabla \times \mathbf{A}) \, d\tau = - \int_S \mathbf{A} \times \mathbf{da}$$

where  $S$  is the surface which bounds volume  $\tau$