ENE 503 – Computational Fluid Dynamics

WEEK 5: NUMERICAL DISCRETIZATION

NUMERICAL DISCRETIZATION:

- **Contents:**

  - Introduction to numerical discretization
  - Finite difference method (FDM)
  - Finite element method (FEM)
  - Finite volume method (FVM)

- **Introduction:**

  - Given the governing equations describing fluid flow motion, one can reproduce the information about the flow
  - The governing equations of fluid motion are represented ina series of partial differential equations which contain the raw flow variables
  - The computer solve these partial differential equation by dealing with numbers.
  - Therefore, the computer can transform the flow problem into a numerical one.
  - The process through which this transformation occurs is known as "discretization" – making things discrete in a finite space
  - Therefore, all partial differential equation eventually become algebraic in nature and can be solved by computer directly.
  - The most well-known discretization techniques are:
    - FDM
    - FEM
    - FVM

  also used

    - Control volume methods (CVM)
    - Spectral methods (SM)
    - Filter scheme methods (FSM)
    - Boundary integral equation methods (BIEM)

- **Simplification of Navier-Stokes equations:**

The Navier-Stokes equations are defined as:

- The continuity equation:

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{V}) = 0$$

- The momentum equation:

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \nabla . \vec{V} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 \vec{V} + \frac{1}{\rho} \vec{F}$$

- Energy equation

$$\rho \frac{DE}{Dt} = -\, div(p\vec{V}) + \left[\frac{\partial(u\tau_{xx})}{\partial x} + \frac{\partial(u\tau_{yx})}{\partial y} + \frac{\partial(u\tau_{zx})}{\partial z} + \frac{\partial(v\tau_{xy})}{\partial x} + \frac{\partial(v\tau_{yy})}{\partial y} + \frac{\partial(v\tau_{zy})}{\partial z} + \frac{\partial(w\tau_{xz})}{\partial x} + \frac{\partial(w\tau_{yz})}{\partial y} + \frac{\partial(u\tau_{zz})}{\partial z} \right] + div(k \; grad \; T) + S_E$$

- **The FDM:**

- Taylor Series expansion is used to build up a library of equations that describe the derivatives of a particular variable
- This mathematical process allows the value of a variable at a particular point in space to be calculated from either the value of that variable at the previous point, or the value of the variable at the next point.

$$U\left(x+h\right) = U(x) + h\frac{dU}{dx} + \frac{1}{2}h^2\frac{d^2U}{dx^2} + \frac{1}{6}h^3\frac{d^3U}{dx^3} + \ldots\ldots \quad (1)$$

$$U\left(x+h\right) = U(x) + h f^1 + \frac{1}{2}h^2 f^2 + \frac{1}{6}h^3 f^3 + \ldots\ldots \quad (2)$$

where *U* is the velocity component in the *x*-direction, *h* is the infinitesimal integral distance in the *x*-direction and derivatives are taken with respect to *x*.

- Equation (1) can be rearranged to calculate d*U*/d*x* as in Equation (3). This process is called “forward differencing”
- Equation (2) can also be used to calculate dU/dx as in Equation (4). This process is called “backward differencing”
- And Equation (1) and (2) can be combined to calculate d*U*/d*x* as in Equation (5). This process is called “central differencing”

$$\frac{dU}{dx} = \frac{1}{h} U\left(x+h\right) - U(x) + O\left(h\right) \tag{3}$$

$$\frac{dU}{dx} = \frac{1}{h} U(x) - U\left(x-h\right) + O\left(h\right) \tag{4}$$

$$\frac{dU}{dx} = \frac{1}{2h} U\left(x+h\right) - U\left(x-h\right) + O\left(h^2\right) \tag{5}$$

- The Taylor series is an infinite series and therefore the O (h) is introduced to represent the “rest of the terms” here.

References:

1. Versteeg H.K., and W. Malalasekera V., 1995, “Computational Fluid Dynamics: The Finite Volume Method", Longman Scientific & Technical, ISBN 0-582-21884-5