ENE 503 – Computational Fluid Dynamics

WEEK 6: NUMERICAL DISCRETIZATION CONTINUES

NUMERICAL DISCRETIZATION (Continues):

- **FEM:**

  - Discretization of domain
  - Derive element equations
    - Construct the variation formulation of the governing equations over an element
    - Obtain approximation of the variation equation over an element (using Ritz or a Weighted Residual method such as Galerkin, Least Squares etc)
    - Assemble individual element equations for the whole problem
    - Impose the boundary conditions of the problem
    - Solve the assembled equations
    - Post-processing of the results.

- **FEM:**

  - Domain is divided into control volumes
  - Integrate the differential equation over the control volume and apply the divergence theorem.
  - To evaluate derivative terms, values at the control volume faces are needed: have to make an assumption about how the value varies.
  - Result is a set of linear algebraic equations: one for each control volume.
  - Solve iteratively or simultaneously.
  - Using finite volume method, the solution domain is subdivided into a finite number of small control volumes (cells) by a grid.
  - The grid defines the boundaries of the control volumes while the computational node lies at the center of the control volume.

- **FVM Discretization example:**

  - The species transport equation (constant density, incompressible flow) is given by:

$$\frac{\partial \phi}{\partial t}+\frac{\partial}{\partial x_i}(u_i \phi)=\frac{\partial}{\partial x_i}\left(D\frac{\partial \phi}{\partial x_i}\right)+S$$

  Here ϕ is the concentration of the chemical species and *D* is the diffusion coefficient. *S* is a source term.

  - Discretize the above equation for a two-dimensional flow field, given in Figure 1. for a control volume containing the point *P* by using finite volume method (FVM) based **central differencing scheme**

  and

  - obtain a final simple algebraic form of this convection-diffusion equation.

  - determine each coefficient in this final discretization equation

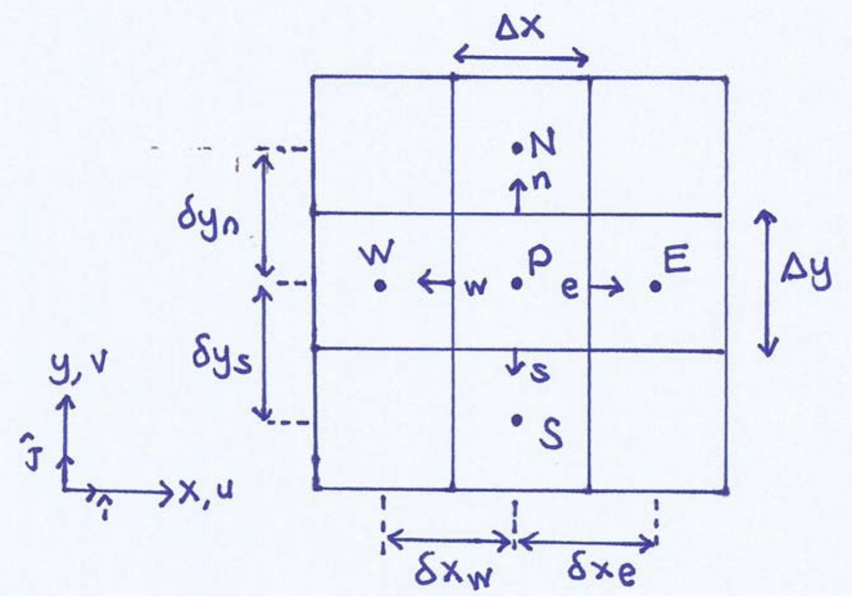

Figure 1

  - The differential equation above is converted into a solvable algebraic equations under steady state assumption

    - Convection term is balanced by the diffusion term

- The balance over the control volume is accomplished as:

$$A_e u_e c_e - A_w u_w c_w + A_n u_n c_n - A_s u_s c_s = DA_e \left.\frac{dc}{dx}\right|_e - DA_w \left.\frac{dc}{dx}\right|_w + DA_n \left.\frac{dc}{dy}\right|_n - DA_s \left.\frac{dc}{dy}\right|_s + S_p$$

- The values at the faces are determined by interpolation from the values at the the cell centers.

- The values at the faces are determined by using **central differencing scheme**.

$$A_e \frac{u_P + u_E}{2}\frac{c_P + c_E}{2} - A_w \frac{u_W + u_P}{2}\frac{c_P + c_W}{2} + A_n \frac{v_N + v_P}{2}\frac{c_N + c_P}{2} - A_s \frac{v_S + v_P}{2}\frac{c_S + c_P}{2} =$$

$$DA_e \frac{c_E - c_P}{\delta X_e} - DA_w \frac{c_P - c_W}{\delta X_w} + DA_n \frac{c_N - c_P}{\delta Y_n} - DA_s \frac{c_P - c_S}{\delta Y_s} + S_p$$

$$\frac{A_e u_P c_P + A_e u_P c_E + A_e u_E c_P + A_e u_E c_E - A_w u_W c_P - A_w u_W c_W - A_w u_P c_P - A_w u_P c_W + A_n v_N c_N}{4}$$

$$+ \frac{A_n v_N c_P + A_n v_P c_N + A_n v_P c_P - A_s v_S c_S - A_s v_S c_P - A_s v_P c_S - A_s v_P c_P}{4} =$$

$$DA_e \frac{c_E}{\delta X_e} - DA_e \frac{c_P}{\delta X_e} - DA_w \frac{c_P}{\delta X_w} + DA_w \frac{c_W}{\delta X_w} + DA_n \frac{c_N}{\delta Y_n} - DA_n \frac{c_P}{\delta Y_n} - DA_s \frac{c_P}{\delta Y_s} + DA_s \frac{c_S}{\delta Y_s} + S_p$$

$$c_P \left[\frac{A_e u_P + A_e u_E - A_w u_W - A_w u_P + A_n v_N + A_n v_P - A_s v_S - A_s v_P}{4} + \frac{DA_e}{\delta X_e} + \frac{DA_w}{\delta X_w} + \frac{DA_n}{\delta Y_n} + \frac{DA_s}{\delta Y_s} \right] =$$

$$c_W \left[\frac{A_w u_W + A_w u_P}{4} + \frac{DA_w}{\delta X_w} \right] + c_N \left[\frac{-A_n v_N - A_n v_P}{4} + \frac{DA_n}{\delta Y_n} \right] + c_E \left[\frac{-A_e u_P - A_e u_E}{4} + \frac{DA_e}{\delta X_e} \right] +$$

$$c_S \left[\frac{A_s v_S + A_s v_P}{4} + \frac{DA_s}{\delta Y_s} \right] + S_p$$

$$\alpha_P = \left[\frac{A_e u_P + A_e u_E - A_w u_W - A_w u_P + A_n v_N + A_n v_P - A_s v_S - A_s v_P}{4} + \frac{DA_e}{\delta X_e} + \frac{DA_w}{\delta X_w} + \frac{DA_n}{\delta Y_n} + \frac{DA_s}{\delta Y_s} \right],$$

$$\alpha_W = \left[\frac{A_w u_W + A_w u_P}{4} + \frac{DA_w}{\delta X_w} \right], \alpha_N = \left[\frac{-A_n v_N - A_n v_P}{4} + \frac{DA_n}{\delta Y_n} \right],$$

$$\alpha_E = \left[\frac{-A_e u_P - A_e u_E}{4} + \frac{DA_e}{\delta X_e} \right], \alpha_S = \left[\frac{A_s v_S + A_s v_P}{4} + \frac{DA_s}{\delta Y_s} \right], S_p = b$$

$$c_P \alpha_P = c_W \alpha_W + c_N \alpha_N + c_E \alpha_E + c_S \alpha_S + b = \sum_{nb} c_{nb} \alpha_{nb} + b$$

References:

1. Versteeg H.K., and W. Malalasekera V., 1995, “Computational Fluid Dynamics: The Finite Volume Method", Longman Scientific & Technical, ISBN 0-582-21884-5.