

WEEK 3: FUNDAMENTAL RELATIONS FOR THE FLOW THROUGH AN ARBITRARY TURBOMACHINE

FUNDAMENTAL RELATIONS FOR THE FLOW THROUGH AN ARBITRARY TURBOMACHINE (Continues)

The Euler and Bernoulli Equations (Forces and Momentum along a streamline)

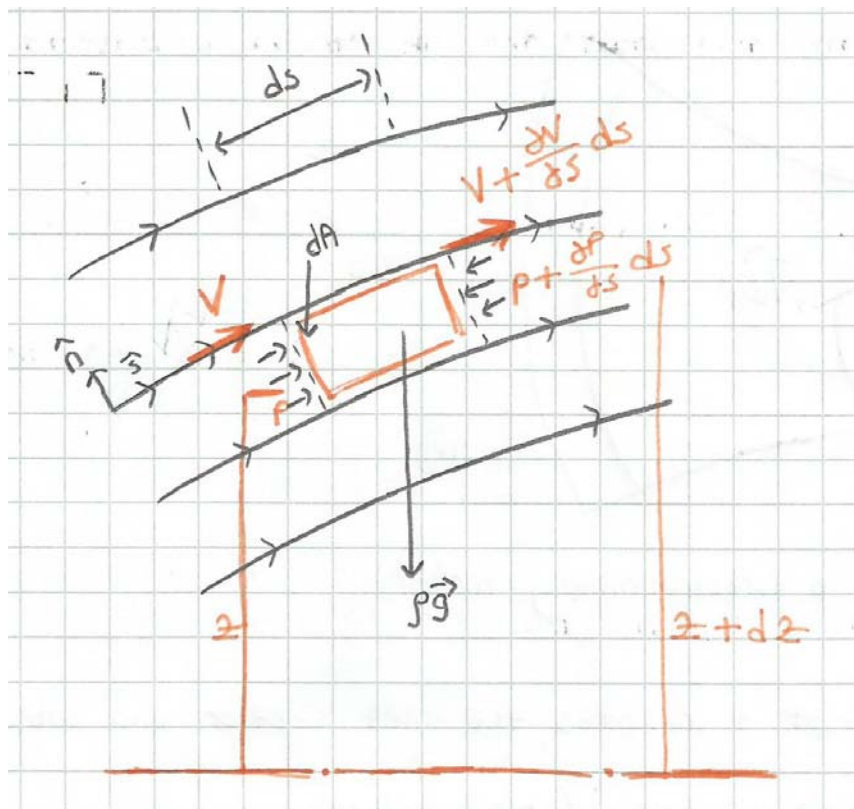


Figure 1. Bernoulli Equation

$$P \cdot dA - \left[P + \frac{\partial P}{\partial s} ds \right] \cdot dA - \rho \cdot g \cdot dA \cdot dz = \rho \cdot V \cdot dA \cdot \left[V + \left(\frac{\partial V}{\partial s} \right) ds \right] - \rho V \cdot V \cdot dA$$

where dz is the change in attitude between two sides of the control volume.

This equation simplified as:

$$\frac{\partial P}{\partial s} + \rho \cdot v \cdot \frac{\partial v}{\partial s} + \rho \cdot g \cdot \frac{\partial z}{\partial s} = 0$$

This is one of the Euler's equations, which is unconditionally true for steady flow in the absence of external forces including friction, work transfer, capillary, magnetic or electrical effects.

It is also very often referred to as "Bernoulli's Equation" although strictly this terms should be reserved for a more restricted form of equation.

The restricted form is obtained if the original equation is integrated along a streamline, i.e. with respect to s, on the assumption that the density is constant. The result of this is:

$$\frac{P}{\rho} + \frac{v^2}{2} + g \cdot z = \text{constant}$$

This equation applies under same condition as the Euler's equation with the added restriction that the fluid density must be constant.

Work is conserved

$$Work = Force \times Distance$$

Momentum is conserved

$$\int Force \cdot dx$$

● **Applications of Bernoulli equations:**

- Bernoulli equation for the steady flow of an incompressible fluid in the absence of gravitational acceleration
- Bernoulli equation for the steady flow of an incompressible fluid

- Bernoulli equation for the flow of an incompressible fluid
- Bernoulli equation for the steady flow of a barotropic fluid

- **Extended Bernoulli Equations:**

- In this equation, mechanical energy converted to thermal energy due to the viscous action between the fluid particles are considered.

REFERENCES

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