WEEK 8: PHYSICAL SIGNIFICANCE OF NON-DIMENSIONAL GROUPS

PHYSICAL SIGNIFICANCE OF NON-DIMENSIONAL GROUPS[1]

$$\Pi_Q : T_Q = \frac{Q}{wd^3} \propto \frac{Q}{wrd^2} \propto \frac{Q}{ud^2} \propto \frac{Q}{uA} \propto \frac{V_m A}{uA} \propto \frac{V_m}{u}$$

$$\Pi_h : T_h = \frac{gh}{w^2 d^2} \propto \frac{gh}{w^2 r^2} \propto \frac{gh}{u^2} \propto \frac{uV_Q}{u^2} \propto \frac{V_Q}{u}$$

The equality of the first two Πterms implies that velocity triangles at the two diffirent operating conditions are similar since the ratios $V_m/u$and $V_Q/u$are the same.

$$(V_m/u)m = (V_m/u)P \qquad\qquad (\Pi_Q)m = (\Pi_Q)P$$

$$\Rightarrow$$

$$(V_m/u)m = (V_m/u)P \qquad\qquad (T_n)m = (T_Q)P$$

It is possible to combine the non-dimensional Πterms to obtain new nondimensional parameters as:

$$(\Pi_\eta)_P = \frac{\Pi_Q . \Pi_h}{\Pi_P} = \frac{\left(\frac{Q}{wd^3}\right) \cdot \left(\frac{gh}{w^2 d^2}\right)}{\frac{P}{\rho w^3 d^5}} = \frac{\rho g Q h}{P} = \eta_P$$

and

$$\left(\Pi_\eta\right)_t = \frac{\Pi_P}{\Pi_Q.\Pi_h} = \frac{\frac{P}{\rho w^3 d^5}}{\left(\frac{Q}{wd^3}\right)\cdot\left(\frac{gh}{w^2 d^2}\right)} = \frac{P}{\rho g Q h} = \eta_t$$

When Π_Q, $\Pi_h$and$\Pi_P$are the some of two different operating conditions of geometrically similar machines, the efficiencies of these two machines are the same of these particular operating conditions.

Effects of Reynolds Number on Similarity [1]

When the effect of Reynolds number is neglected the efficiencies of the model and prototype machines are exactly the same if the first three non-dimensional Π terms are the same.

Therefore, when the viscous effects are taken into consideration efficiencies of the model and the prototype are the same at two different operating conditions whenever

1) the Reynolds number are the equality
2) the surface roughness and clearances are geometrically similar
in addition of equality of first three Π terms.

The difference in efficiency at different operating conditions for the model and prototype is usually given by relations that are obtained experimentally such relation is the “Ackeret’s relation” and is given by

$$\frac{1-\eta_P}{1-\eta_m} = 0.5\left[1+\left(R_{em}/R_{ep}\right)^2\right]$$

where η_P and η_m are the efficiencies of the prototype and model, respectively while Re_m and Re_p denotes the Reynolds numbers for the model and prototype,respectively.

REFERENCES

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