ENE 505 – Applied Computational Fluid Dynamics in Renewable Energy Technologies

WEEK 2: MATHEMATICAL TERMS

MATHEMATICAL TERMS:

- **Derivatives:**

  ➤ Full $\quad \dfrac{du}{dx}$

  - The derivative of *u* w.r.t. *x*, where $u = u(x)$

  ➤ Partial $\quad \dfrac{\partial u}{\partial x}$

  - The derivative of *u* w.r.t. *x*, where $u = u(x, y, z, t)$

  ➤ Substantial $\quad \dfrac{DN}{dt} = \dfrac{\partial N}{\partial x} + \vec{u}.\nabla N$

At this point it is possible to define an extensive property, *N* in terms of its corresponding intensive property, η as below:

$$N = \int_{m_s} \eta \, dm$$

where m_s is the mass of the system.

Since $dm = \rho \, d\forall$, then

$$N = \int_{\forall_s} \eta \rho \, d\forall$$

with $\forall_s$ being the volume of the system.

- **Vector differential operator**

  ➢ Del $\nabla = i \dfrac{\partial}{\partial x} + j \dfrac{\partial}{\partial y} + k \dfrac{\partial}{\partial z}$

  ➢ The gradient: Consider $\xi(x,y,z,t)$ a scalar field

  The gradient of ξ is defined as:

  $$\nabla \xi = \dfrac{\partial \xi}{\partial x} i + \dfrac{\partial \xi}{\partial y} j + \dfrac{\partial \xi}{\partial z} k$$

  which gives a vector field.

  ➢ The divergence: Consider $\vec{V}(x,y,z,t) = Ui + Vj + Wk$ a vector field

  The divergence of $\vec{V}$ is therefore:

  $$\nabla . \vec{V} = \dfrac{\partial U}{\partial x} + \dfrac{\partial V}{\partial y} + \dfrac{\partial W}{\partial z}$$

  ➢ The curl: The curl of $\vec{V}$ is

  $$\nabla \times \vec{V} = \left(\dfrac{\partial W}{\partial y} - \dfrac{\partial V}{\partial z} \right) i + \left(\dfrac{\partial U}{\partial z} - \dfrac{\partial W}{\partial x} \right) j + \left(\dfrac{\partial V}{\partial x} - \dfrac{\partial U}{\partial y} \right) k$$

- **Definitions - Rectangular coordinates:**

  $$\nabla f = \dfrac{\partial f}{\partial x} i + \dfrac{\partial f}{\partial y} j + \dfrac{\partial f}{\partial z} k$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)i + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)j + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)k$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 \mathbf{A} = \nabla^2 A_x i + \nabla^2 A_y j + \nabla^2 A_z k$$

- **Identities:**

> The curl: The curl of $\vec{V}$ is

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

$$(\mathbf{A} \cdot \nabla)\mathbf{B} = \left[A_x \frac{\partial B_x}{\partial x} + A_y \frac{\partial B_x}{\partial y} + A_z \frac{\partial B_x}{\partial z}\right]i$$
$$+ \left[A_x \frac{\partial B_y}{\partial x} + A_y \frac{\partial B_y}{\partial y} + A_z \frac{\partial B_y}{\partial z}\right]j$$
$$+ \left[A_x \frac{\partial B_z}{\partial x} + A_y \frac{\partial B_z}{\partial y} + A_z \frac{\partial B_z}{\partial z}\right]k$$

$$\int_\tau \nabla f \, d\tau = \int_S f \, \mathbf{da}$$
$$\int_\tau (\nabla \times \mathbf{A}) \, d\tau = -\int_S \mathbf{A} \times \mathbf{da}$$

where S is the surface which bounds volume τ