

ENE 505 – Applied Computational Fluid Dynamics in Renewable Energy Technologies

WEEK 5: NUMERICAL DISCRETIZATION

NUMERICAL DISCRETIZATION:

- **Contents:**

- Introduction to numerical discretization
- Finite difference method (FDM)
- Finite element method (FEM)
- Finite volume method (FVM)

- **Introduction:**

- Given the governing equations describing fluid flow motion, one can reproduce the information about the flow
- The governing equations of fluid motion are represented in a series of partial differential equations which contain the raw flow variables
- The computer solve these partial differential equation by dealing with numbers.
- Therefore, the computer can transform the flow problem into a numerical one.
- The process through which this transformation occurs is known as “discretization” – making things discrete in a finite space
- Therefore, all partial differential equation eventually become algebraic in nature and can be solved by computer directly.
- The most well-known discretization techniques are:
 - FDM
 - FEM
 - FVM

also used

- Control volume methods (CVM)
- Spectral methods (SM)
- Filter scheme methods (FSM)

- Boundary integral equation methods (BIEM)

- **Simplification of Navier-Stokes equations:**

The Navier-Stokes equations are defined as:

- The continuity equation:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \vec{V}) = 0$$

- The momentum equation:

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 \vec{V} + \frac{1}{\rho} \vec{F}$$

- Energy equation

$$\begin{aligned} \rho \frac{DE}{Dt} = & -\text{div}(p \vec{V}) + \left[\frac{\partial(u\tau_{xx})}{\partial x} + \frac{\partial(u\tau_{yx})}{\partial y} + \frac{\partial(u\tau_{zx})}{\partial z} + \frac{\partial(v\tau_{xy})}{\partial x} \right. \\ & \left. + \frac{\partial(v\tau_{yy})}{\partial y} + \frac{\partial(v\tau_{zy})}{\partial z} + \frac{\partial(w\tau_{xz})}{\partial x} + \frac{\partial(w\tau_{yz})}{\partial y} + \frac{\partial(u\tau_{zz})}{\partial z} \right] \\ & + \text{div}(k \text{ grad } T) + S_E \end{aligned}$$

- **The FDM:**

- Taylor Series expansion is used to build up a library of equations that describe the derivatives of a particular variable
- This mathematical process allows the value of a variable at a particular point in space to be calculated from either the value of that variable at the previous point, or the value of the variable at the next point.

$$U(x+h) = U(x) + h \frac{dU}{dx} + \frac{1}{2} h^2 \frac{d^2U}{dx^2} + \frac{1}{6} h^3 \frac{d^3U}{dx^3} + \dots \quad (1)$$

$$U(x+h) = U(x) + h f^1 + \frac{1}{2} h^2 f^2 + \frac{1}{6} h^3 f^3 + \dots \quad (2)$$

where U is the velocity component in the x -direction, h is the infinitesimal integral distance in the x -direction and derivatives are taken with respect to x .

- Equation (1) can be rearranged to calculate dU/dx as in Equation (3). This process is called “forward differencing”

- Equation (2) can also be used to calculate dU/dx as in Equation (4). This process is called “backward differencing”

- And Equation (1) and (2) can be combined to calculate dU/dx as in Equation (5). This process is called “central differencing”

$$\frac{dU}{dx} = \frac{1}{h}U(x+h) - U(x) + O(h) \quad (3)$$

$$\frac{dU}{dx} = \frac{1}{h}U(x) - U(x-h) + O(h) \quad (4)$$

$$\frac{dU}{dx} = \frac{1}{2h}U(x+h) - U(x-h) + O(h^2) \quad (5)$$

- The Taylor series is an infinite series and therefore the $O(h)$ is introduced to represent the “rest of the terms” here.

References:

1. Versteeg H.K., and W. Malalasekera V., 1995, “Computational Fluid Dynamics: The Finite Volume Method”, Longman Scientific & Technical, ISBN 0-582-21884-5