ENE 505 – Applied Computational Fluid Dynamics in Renewable Energy Technologies

WEEK 8: TURBULENCE MODELS

<u>TURBULENCE MODELS:</u>

- **RANS based Turbulence Models**

➢ For turbulent, incompressible and Newtonian viscous fluid, the Reynolds-averaged Navier–Stokes equation is given by:

$$\rho \frac{D\overline{u}_i}{Dt} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial \overline{u}_i}{\partial x_i}\right)\right] + \frac{\partial}{\partial x_j}\left(-\rho\overline{u_i^{'}u_j^{'}}\right)$$

Where ρ is the fluid density, $\overline{u_i}$ is the time averaged velocity, $u_j^{'}$ is the deviation from the time averaged velocity, $\overline{p}$ is the time averaged pressure, μ is the dynamic viscosity of the fluid, $-\rho\overline{u_i^{'}u_j^{'}}$ is the Reynold's stress tensor, which appears on the right-hand side of the time-averaged Navier–Stokes equations as a result of time averaging to Navier–stokes equations.

➢ The temporal and spatial co-ordinates correspond to *t* and x_j, respectively. In an eddy-viscosity model, the Reynolds stress is assumed to be proportional to the mean velocity gradients, with the constant of proportionality being the turbulent viscosity, μ_t. This assumption is known as the Boussinesq eddy-viscosity hypothesis, and provides the following expression for the Reynolds stresses [1]

$$-\rho\overline{u_i^{'}u_j^{'}} = \frac{2}{3}\rho k\delta_{ij} + \mu_t\left(\frac{\partial \overline{u_i}}{\partial \overline{x_j}} + \frac{\partial \overline{u_j}}{\partial x_j}\right) + \frac{2}{3}\mu_t\frac{\partial \overline{u_j}}{\partial x_j}\delta_{ij}$$

where k is the turbulent kinetic energy expressed as:

$$k = \frac{1}{2}\overline{u_i^{'} u_j^{'}}$$

- **Standard k-epsilon Model**

➢ The standard k-ε turbulence model is based on transport equations for turbulent kinetic energy k and its dissipation rate,ε , and it is firstly developed by Launder and Spalding [2].

➢ The turbulence kinetic energy, k and its rate of dissipation, ε are obtained from the following transport equations due to [2]

$$\rho \frac{Dk}{Dt} = \frac{\partial}{\partial x_i}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_i}\right] + G_k - \rho\varepsilon$$

$$\rho \frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_i}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_i}\right] + C_{1\varepsilon}\frac{\varepsilon}{k}G_k - C_{2\varepsilon}\rho\frac{\varepsilon^2}{k}$$

where, μ_t is the eddy viscosity, $C_{1\varepsilon}$ and $C_{2\varepsilon}$ are constants in the sense that they are not changed between calculations. σ_k and σ_ε are the turbulent Prandtl numbers for k and ε, respectively. The default values for these coefficients are given as follows: $C_{1\varepsilon} = 1.44$, $C_{2\varepsilon} = 1.92$, C_μ = 0.09, $\sigma_k = 1.0$, and $\sigma_\varepsilon = 1.0$. G_k represents the generation of turbulent kinetic energy due to the mean velocity gradients, calculated in a manner consistent with the Boussinesq hypothesis.

- **The RNG k-epsilon Model**

➢ Further development and Improvement from the standard model has been done by Yakhot et al. [3] and is based on the renormalized (RNG) group theory. An additional sink term is suggested; over the standard k-ε turbulence model, in the turbulence dissipation equation to account for non-equilibrium strain rates and employs different values for various model coefficients.

- The equation of k remains the same and the dissipation equation ε is modified to include the additional sink term:

$$\frac{C_\mu \eta^3 \left(1-\eta/\eta_0\right)}{1+\beta\eta^3}\frac{\varepsilon^3}{k}$$

- **The k-omega Model**

- The *k-ω* model is based on Wilcox [4]. It compromises modifications for low Reynolds number effects, compressibility and shear flow spreading. It is characterized by the turbulent kinetic energy and the frequency, $\omega = k/\varepsilon$, where ε is the rate of dissipation of *k*. The turbulence viscosity can be expressed as:

$$\mu_t = C_\mu \frac{\rho k}{\omega}$$

where the turbulent kinetic energy, *k*, and the specific dissipation rate, ω , are obtained from the following transport equations:

$$\frac{\partial \rho k}{\partial t} + \frac{\partial\left(\rho \overline{u_i} k\right)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P - \rho\omega k$$

$$\frac{\partial \rho \omega}{\partial t} + \frac{\partial\left(\rho \overline{u_i} k\right)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_w}\right)\frac{\partial \omega}{\partial x_j}\right] + \gamma\frac{\omega}{k} - \beta\rho\frac{\omega^2}{C_\mu}$$

References:

1. Hinze, O. (1975) Turbulence. New York, McGraw-Hill Publishing Co.

2. Launder BE, Spalding DB. Mathematical Models of Turbulence. Academic Press: London, 1972.

3. Yakhot V, Orszag SA, Thangham S, Gatski TB, Speziale CG. Development of turbulence models for shear flows

by a double expansion technique. Physics of Fluids 1992; 4(7): 1510–1520.

4. Wilcox, D. (1998), Turbulence modeling for CFD, California: DCW Industries, Inc., La Canada.