MATHEMATICAL MODELLING[1-5]

THE REPRESENTATION IN TERMS OF DEVIATION VARIABLES[1-5]

FIRST ORDER SYSTEM RESPONSES [1-5]

Transfer function[1-5]

References:

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A thermometer is immersed in flowing liquid

$$hAT_b - hAT - 0 = mC_p \frac{dT}{dt}$$ Unsteady state Eq1

$$hAT_{bs} - hAT_s = 0$$ steady state Eq 2

Substracting Eq. (2) from Eq. (1);

$$hA(T_b - T_{bs}) - hA(T - T_s) = mC_p \frac{d(T - T_s)}{dt}$$

$T_b - T_{bs} = T_b'$ $T - T_s = T'$ The deviation variables

$$hA\,T_b' - hA\,T' = m\,C_p \frac{dT'}{dt}$$

$$hA\,(T_b' - T') = m\,C_p \frac{dT'}{dt}$$

$$T_b' - T' = \frac{m\,C_p}{hA}\frac{dT'}{dt}$$

$$\tau = \frac{m\,C_p}{hA} \rightarrow \tau\frac{dT'}{dt} + T' = T_b'$$

$$\tau s T'(s) + T'(s) = T_b'(s)$$

$$\frac{T'(s)}{T_b'(s)} = \frac{1}{\tau s + 1}$$

$$\tau = \frac{m\,C_p}{hA} = \frac{(1.2\,g)*(0.25\,cal/g\,^\circ C)}{(0.15\,cal/cm^2\,\min\ ^\circ C)(2\,cm^2)}$$

$$\tau = 1\min \quad \rightarrow \quad K_p = 1$$

$$T_b'(t) = (90-30)\,u(t) + (55-90)\,u(t-2)$$

$$T_b'(t) = 60\,u(t) - 35\,u(t-2)$$

$$T_b'(s) = \frac{60}{s} - \frac{35\,e^{-2s}}{s}$$

$$T'(s) = \frac{1}{\tau s + 1} T_b'(s)$$

$$T'(s) = \frac{1}{s+1}\left[\frac{60}{s} - \frac{35\,e^{-2s}}{s}\right]$$

$$T'(s) = 60\underbrace{\frac{1}{s(s+1)}}_{X(s)} - 35\,e^{-2s}\underbrace{\frac{1}{s(s+1)}}_{X(s)}$$

$$X(s) = \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$A = \left[\frac{s}{s(s+1)}\right]_{s=0} = 1$$

$$B = \left[\frac{(s+1)}{s(s+1)}\right]_{s=-1} = -1$$

$$X(s) = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

$$X(t) = (1 - e^{-t})\,u(t)$$

$$L^{-1}\left\{\frac{e^{-2s}}{s(s+1)}\right\} = (1 - e^{-(t-2)})\,u(t-2)$$

$$T'(t) = 60\left[1 - e^{-t}\right]u(t) - 35\left[1 - e^{-(t-2)}\right]u(t-2)$$

$$T'(t) = T(t) - T_s = T(t) - 30$$

Maximum temperature is obtained at t = 2 min when the thermocouple is removed from the bath. (T_b = 90 °C)

$$T(t) = 30 + T^{'}(t)$$

$$T(2) = 30 + 60\,(1 - e^{-t})$$

$$\boxed{T(2) = 30 + 60\,(1 - e^{-2})} \rightarrow \boxed{T(2) = 81.88\ ^{o}C}$$

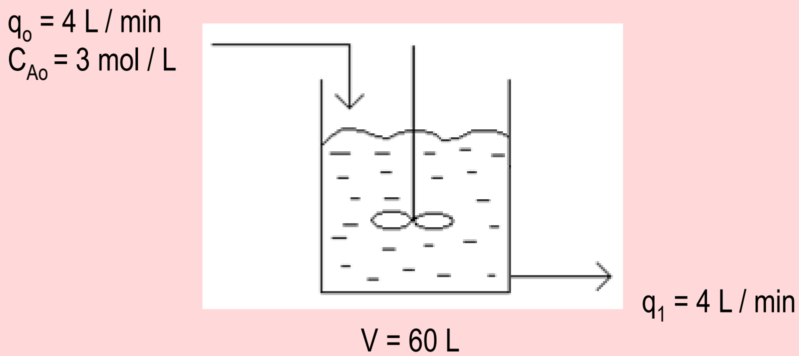
$$T(t) = 30 + T^{'}(t)$$

$$T(15) = 30 + 60\,(1 - e^{-t}) - 35\,(1 - e^{-(t-2)})$$

$$T(15) = 30 + 60\,(1 - e^{-25}) - 35\,(1 - e^{-(25-2)})$$

$$\rightarrow \quad T(15) = 55\ ^{o}C$$

The mixing tank shown in figure is initially at steady state with the inlet salt concentration at 3 mol / L.



Assumptions:

Constant volumetric flow rate

Constant density

Unsteady state : $$qC_{Ao} - qC_A = V\frac{dC_A}{dt} \quad (1)$$

Steady state : $$qC_{Aos} - qC_{As} = 0 \quad (2)$$

Substracting Eq. (2) from Eq. (1); $q\,(C_{Ao}-C_{Aos})-q\,(C_A-C_{As})=V\dfrac{d(C_A-C_{As})}{dt}$

Deviation Variables

$$C_{Ao}-C_{Aos}=C'_{Ao}$$

$$C_A-C_{As}=C'_A$$

The changes observed in the time dependent variable from **the first steady state** to **the second steady state** are defined by **deviation variables**.

$$q\,C'_{Ao}=q\,C'_A+V\frac{dC'_A}{dt}$$

$$C'_{Ao}=C'_A+\left(\frac{V}{q}\right)\frac{dC'_A}{dt}$$

Steady state:

$$q C_{Aos} - q C_{As} = 0$$

$$(4L/\min)*(3mol/L) - (4L/\min)*C_{As} = 0$$

$$C_{As} = 3\,mol/L$$

$$\tau = \frac{V}{q} = \frac{60\,L}{4\,L/\min} = 15\min$$

$$K_p = 1$$

$$C'_{Ao} = C'_A + \left(\frac{V}{q}\right)\frac{dC'_A}{dt}$$

$$C'_{Ao} = C'_A + \tau \frac{dC'_A}{dt}$$

$$C'_{Ao}(s) = C'_A(s) + \tau\, s\, C'_A(s)$$

Transfer function

$$\frac{C'_A(s)}{C'_{Ao}(s)} = \frac{1}{\tau s + 1} = \frac{1}{15s+1}$$

$$\frac{C'_A(s)}{C'_{Ao}(s)} = \frac{1}{15s+1}$$

$$C'_{Ao}(s) = \frac{4}{s}$$

$$C'_A(s) = \left(\frac{4}{s}\right)\left(\frac{1}{15s+1}\right) = \frac{4}{s(15s+1)} = \frac{A}{s} + \frac{B}{15s+1}$$

$$A = \left[\frac{4s}{s(15s+1)}\right]_{s=0} = 4$$

$$B = \left[\frac{4(15s+1)}{s(15s+1)}\right]_{s=-0.067} = -60$$

$$C'_A(s) = \frac{4}{s} - \frac{60}{15s+1} = \frac{4}{s} - \frac{(60/15)}{s+(1/15)} = \frac{4}{s} - \frac{4}{s+0.067}$$

$$C'_A(t) = 4 - 4 * e^{-0.067t} = 4 * (1 - e^{-0.067t})$$

$$C'_A(t) = C_A(t) - C_{A_s}$$

$$C_A(t) = C_{A_s} + C'_A(t)$$

$$C_A(\infty) = 3 mol / L + 4$$

$$C_A(\infty) = 7 \ mol / L$$