Stability analysis [1-5]

References:

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3. Seborg D.E., Mellichamp D. A., Edgar T.F, Doyle F.J., 2011, Process Dynamics and Control , John Wiley and Sons ISBN: 978-0-470-64610-6

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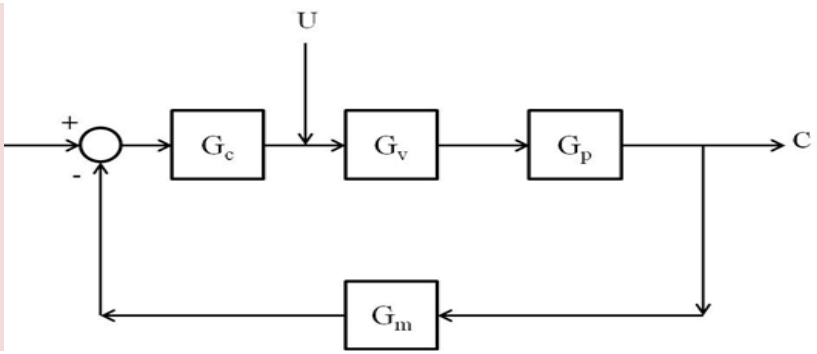
instability by using Routh array.

Open loop transfer function:

$$G = G_p G_C G_m G_v = \frac{K_C}{(s+1)^2(s+2)(s+3)}$$



Characteristic equation:

$$1 + G = 0$$

$$1 + \frac{K_C}{s^4 + 7s^3 + 17s^2 + 17s + 6} = 0$$

$$s^4 + 7s^3 + 17s^2 + 17s + (6 + K_C) = 0$$

Routh array:

Row			
1	1	17	$6 + K_c$
2	7	17	
3	b_1	b_2	
4	c_1		
5	d_1		

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} = \frac{7 * 17 - 1 * 17}{7} = 14.57$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} = \frac{7 * (6 + \mathrm{Kc}) - 0}{7} = (6 + \mathrm{Kc})$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} = \frac{14.57 * 17 - 7(6 + \mathrm{Kc})}{14.57} = (14.12 - 0.48\mathrm{Kc})$$

$$c_2 = \frac{0 - 0}{b_1} = 0$$

$$d_1 = \frac{c_1 b_2 - b_1 c_2}{c_1} = \frac{(14.12 - 0.48\mathrm{Kc}) * (6 + \mathrm{Kc}) - 0}{(14.12 - 0.48\mathrm{Kc})} = (6 + \mathrm{Kc})$$

Row			
1	1	17	6+ K_c
2	7	17	
3	14.57	6+ K_c	
4	14.12-0.48 K_c		
5	6+ K_c		

If the element in the n^{th} (fourth) row of the array is zero, the system is on the verge of instability; hence;

$14.12 - 0.48Kc = 0$

$Kc = 29.42$

Open loop transfer function:

$$G = G_p G_C G_m G_v = \frac{K_C}{s^3 + 8s^2 + 20s + 15}$$

Characteristic equation:

$$1 + G = 0$$

$$1 + \frac{K_C}{s^3 + 8s^2 + 20s + 15} = 0$$

$$s^3 + 8s^2 + 20s + (15 + K_C) = 0$$

Routh array:

Row		
1	1	20
2	8	$15 + K_c$
3	b_1	
4	c_1	

$$b_1 = \frac{8 * 20 - 1 * (15 + K_C)}{8} = 20 - \frac{15 + K_C}{8}$$

$$c_1 = \frac{\left(20 - \frac{15 + K_C}{8}\right)(15 + K_C)}{\left(20 - \frac{15 + K_C}{8}\right)} = 15 + K_C$$

all the elements of the first column must be positive for stability

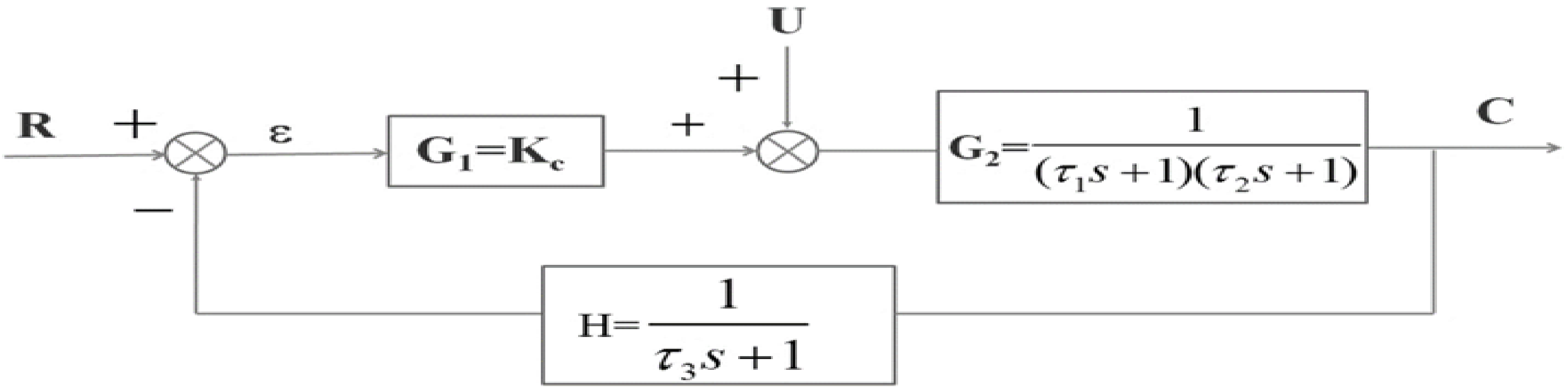
$$20 - \frac{15 + K_C}{8} > 0$$

$$\frac{15 + K_C}{8} < 20$$

$$15 + K_C < 160$$

$$K_C < 145$$

stable only if $K_C < 145$



The characteristic equation 1+G(s)=0

$G=G_1\times G_2\times H$

$$G = K_c \times \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \times \frac{1}{(\tau_3 s+1)}$$

$$G = K_c \times \frac{1}{(6s+1)(3s+1)} \times \frac{1}{(2s+1)}$$

$$1+G(s)=0$$

$$1+K_c \times \frac{1}{(6s+1)(3s+1)} \times \frac{1}{(2s+1)} = 0$$

$$\frac{(6s+1)(3s+1)(2s+1)+K_c}{(6s+1)(3s+1)(2s+1)} = 0,$$

$$(6s+1)(3s+1)(2s+1)+K_c=0$$

Routh Test:

$$36s^3 + 36s^2 + 11s + (1 + K_c) = 0$$

$n=3$, $a_o=36$, $a_1=36$, $a_2=11$, $a_3=(1+K_c)$.

	Row		
	1	$a_o=36$	$a_2=11$
(n-1)	2	$a_1=36$	$a_3=(1+K_c)$
(n)	3	b_1	
(n+1)	4	c_1	

	Row		
	1	36	11
(n-1)	2	36	$(1+K_c)$
(n)	3	b_1	
(n+1)	4	c_1	

$$b_1 = \frac{36 \times 11 - 36 \times (1 + K_c)}{36}$$

$$b_1 = 11 - (1 + K_c)$$

$$b_1 = 10 - K_c$$

$$c_1 = \frac{b_1 \times (1 + K_c) - 0}{b_1}$$

$$c_1 = (1 + K_c)$$

	Row		
	1	36	11
(n-1)	**2**	36	$1+K_c$
(n)	**3**	$10 - K_c$	
(n+1)	**4**	$1 + K_c$	

K_c is positive, $(10- K_c) >0$.

K_c should be smaller than 10

$K_c<10$.

in the (n-1)st row $Cs^2 + D = 0$

C=36,

D=(1+ K_c)=1+10=11

$$36s^2 + 11 = 0$$

$$s_{1,2} = \pm j\frac{\sqrt{11}}{6}$$

$$(s - s_1)(s - s_2)(s - s_3) = 0$$

$$(s - j\frac{\sqrt{11}}{6})(s + j\frac{\sqrt{11}}{6})(s - s_3) = 0$$

$$(36s^2+11)(s-s_3)=36s^3+11s-(s_3)36s^2-(s_3)11=36s^3-(s_3)36s^2+11s-(s_3)11=36s^3-(-1)36s^2+11s-(-1)11$$

$$36s^3 + 36s^2 + 11s + (1 + K_c) = 0$$

The roots of the characteristic equation

$s_1 = j\frac{\sqrt{11}}{6}$, $s_2 = -j\frac{\sqrt{11}}{6}$, $s_3 = -1$