Vector vs. Scalar Review

- All physical quantities encountered in this text will be either a scalar or a vector
- A **vector** quantity has both magnitude (value + unit) and direction
- A **scalar** is completely specified by only a magnitude (value + unit)

Vector and Scalar Quantities

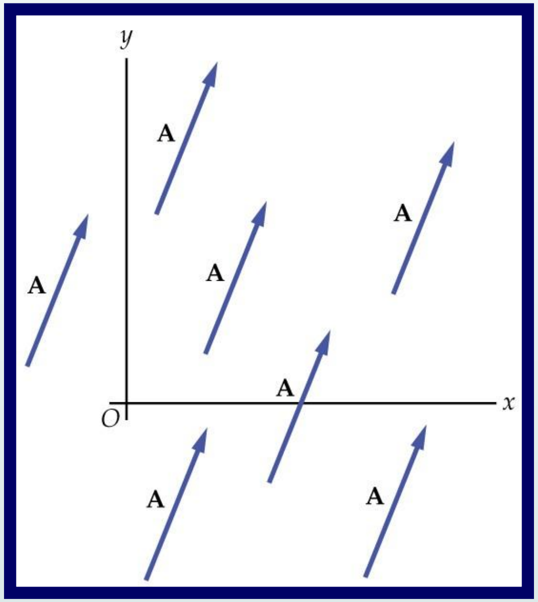
- **Vectors**
  - Displacement
  - Velocity (magnitude and direction!)
  - Acceleration
  - Force
  - Momentum

- **Scalars:**
  - Distance
  - Speed (magnitude of velocity)
  - Temperature
  - Mass
  - Energy
  - Time

To describe a vector we need more information than to describe a scalar! Therefore vectors are more complex!

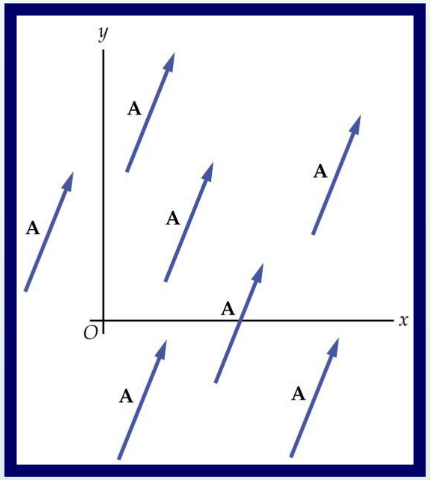
Important Notation

- How to **describe vectors**:
  - The bold font: Vector A is **A**
  - Or an **arrow** above the vector**:** $\vec{A}$
  - In the pictures, we will always show vectors as arrows
  - Arrows point the direction
  - To describe the magnitude of a vector we will use absolute value sign: $\left|\vec{A}\right|$ or just A,
  - Magnitude is always positive, the magnitude of a vector is equal to the length of a vector.



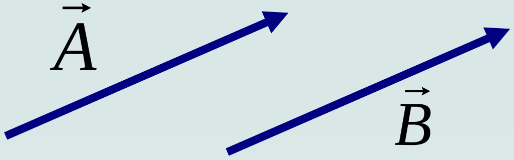
Properties of Vectors

- Equality of Two Vectors
  - Two vectors are **equal** if they have the same magnitude and the same direction



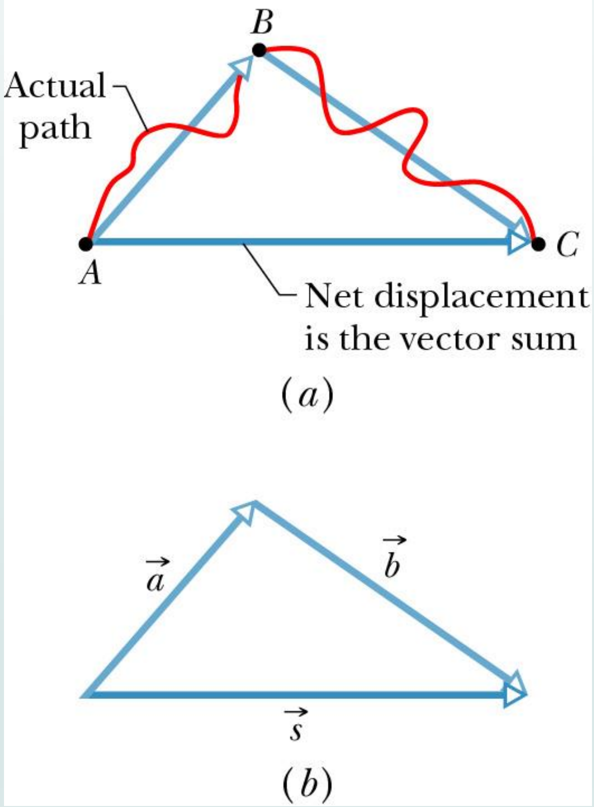
- Movement of vectors in a diagram
  - Any vector can be moved parallel to itself without being affected
- Negative Vectors
  - Two vectors are **negative** if they have the same magnitude but are 180° apart (opposite directions)

$$\vec{\mathbf{A}} = -\vec{\mathbf{B}};\ \vec{\mathbf{A}} + \left(-\vec{\mathbf{A}}\right) = 0$$



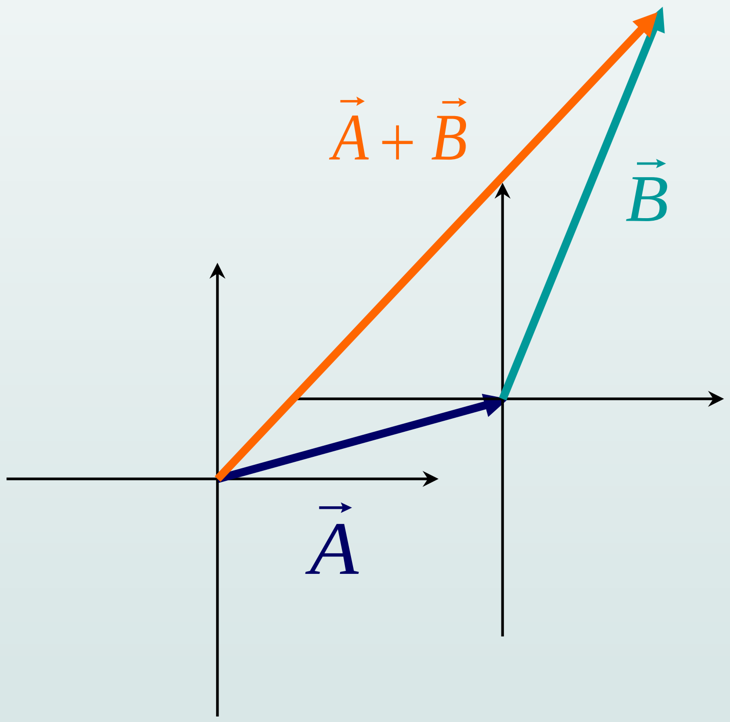
Adding Vectors

- When adding vectors, their directions must be taken into account
- Units must be the same
- Geometric Methods
  - Use scale drawings
- Algebraic Methods
  - More convenient



Adding Vectors Geometrically (Triangle Method)

- Draw a vector with an appropriate length and a coordinate system in the specified direction
- Draw the next vector at the appropriate length and $\vec{B}$ in the specified direction, according to the coordinate system that is parallel to the coordinate system used for "tail to tail" when the vector ends.
- The result is drawn from the original to the end of the last vector

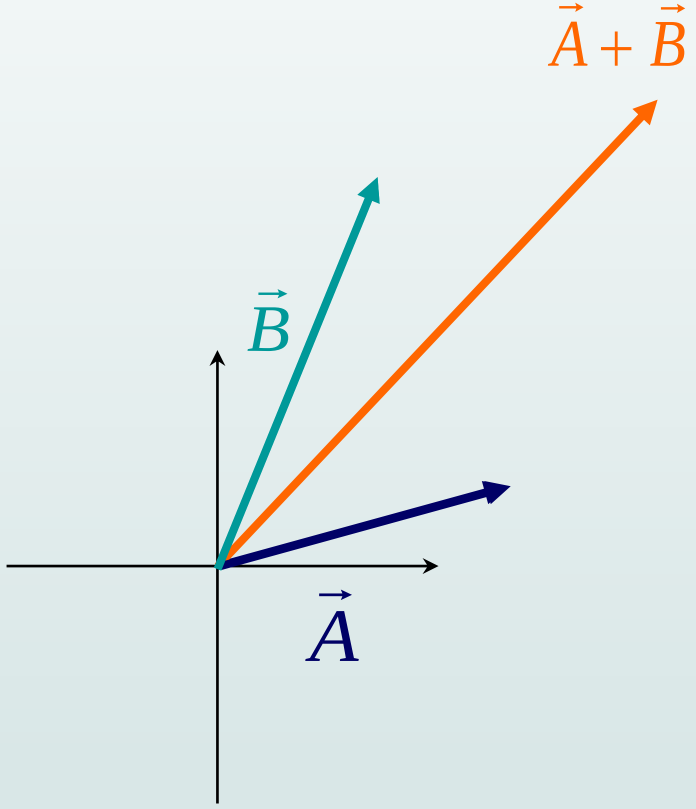


Adding Vectors Graphically

- When you have many vectors, just keep repeating the process until all are included
- The resultant is still drawn from the origin of the first vector to the end of the last vector

Adding Vectors Geometrically (Polygon Method)

- Draw the first vector $\vec{A}$ with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector $\vec{B}$ with the appropriate length and in the direction specified, with respect to the same coordinate system
- Draw a parallelogram
- The resultant is drawn as a diagonal from the origin



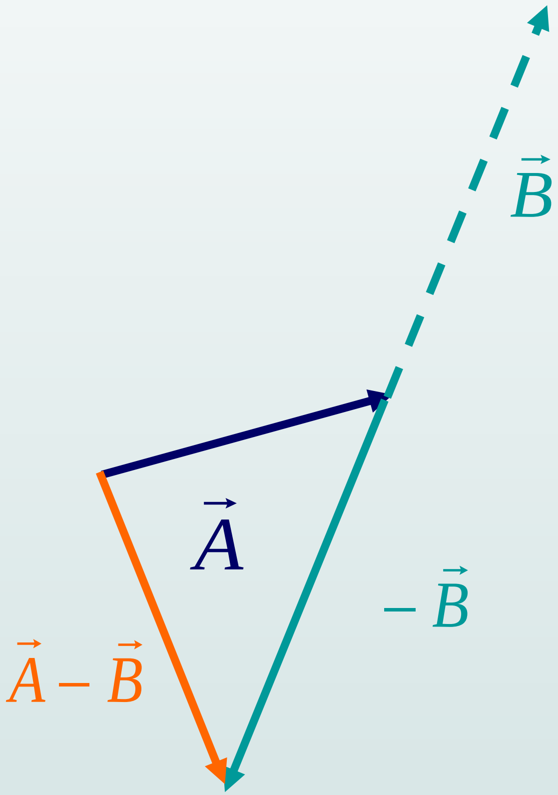
$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

Vector Subtraction

- Special case of vector addition
  - Add the negative of the subtracted vector

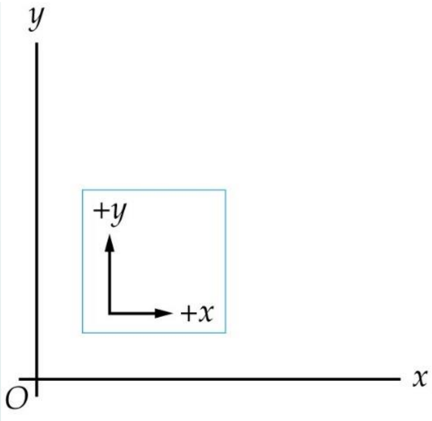
$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$$

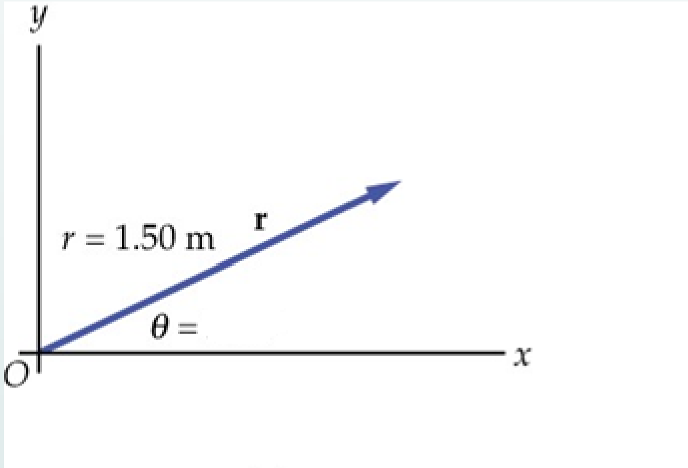
- Continue with standard vector addition procedure



Describing Vectors Algebraically

Vectors: Described by the number, units and direction!





(a)

Vectors: Can be described by their magnitude and direction. For example: Your displacement is 1.5 m at an angle of 30^0.

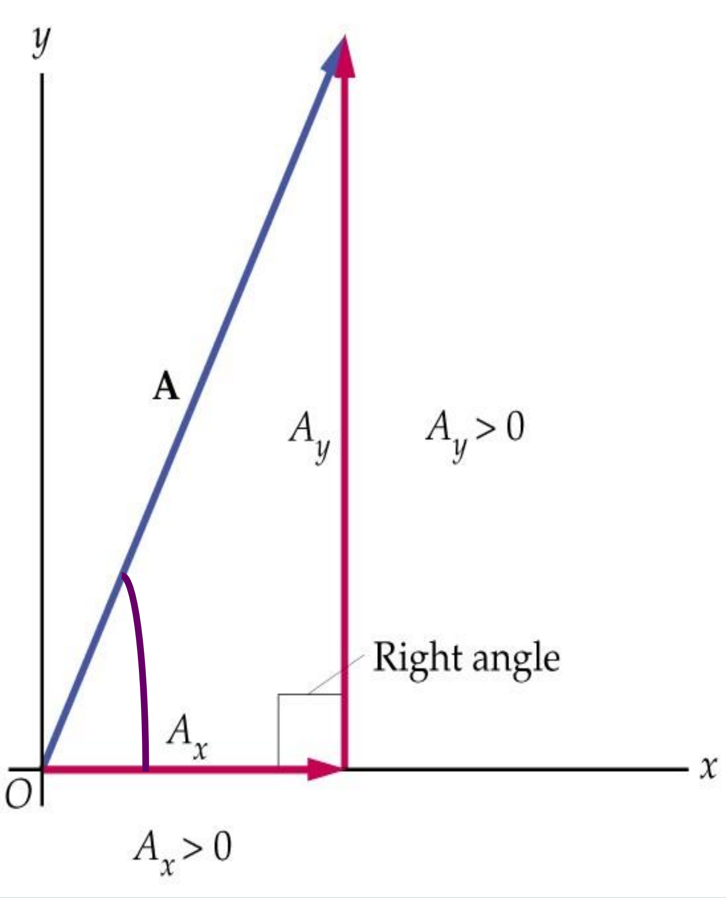
Can be described by components? For example: your displacement is *1.28 m* in the positive x direction and *0.75m* in the positive y direction.

Components of a Vector

- A **component** is a part
- It is useful to use **rectangular components**
  - These are the projections of the vector along the x- and y-axes

Figure 3.13
Physics for Scientists and Engineers 6th Edition, Thomson Brooks/Cole © 2004; Chapter 3

Components of a Vector



- The x-component of a vector is the projection along the x-axis

$$\cos\theta = \frac{A_x}{A} \qquad A_x = A\cos\theta$$

- The y-component of a vector is the projection along the y-axis

$$\sin\theta = \frac{A_y}{A} \qquad A_y = A\sin\theta$$

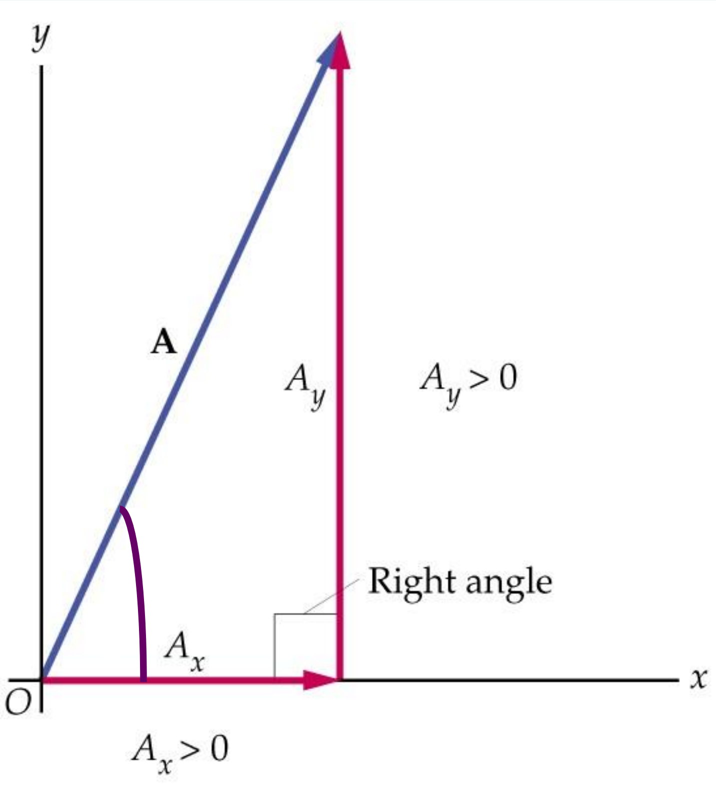
- Then,

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

- The components are the legs of the right triangle whose hypotenuse is *A*

$$\begin{cases} A_x = A\cos(\theta) \\ A_y = A\sin(\theta) \end{cases}$$

$$\begin{cases} |\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2} \\ \tan(\theta) = \dfrac{A_y}{A_x} \quad \text{or} \quad \theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right) \end{cases}$$



Unit Vectors

- Components of a vector are vectors

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

- Unit vectors *i*-hat, *j*-hat, *k*-hat

$$\hat{i} \rightarrow x \quad \hat{j} \rightarrow y \quad \hat{k} \rightarrow z$$

- Unit vectors used to specify direction
- Unit vectors have a magnitude of 1
- Then

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

Magnitude + Sign   Unit vector

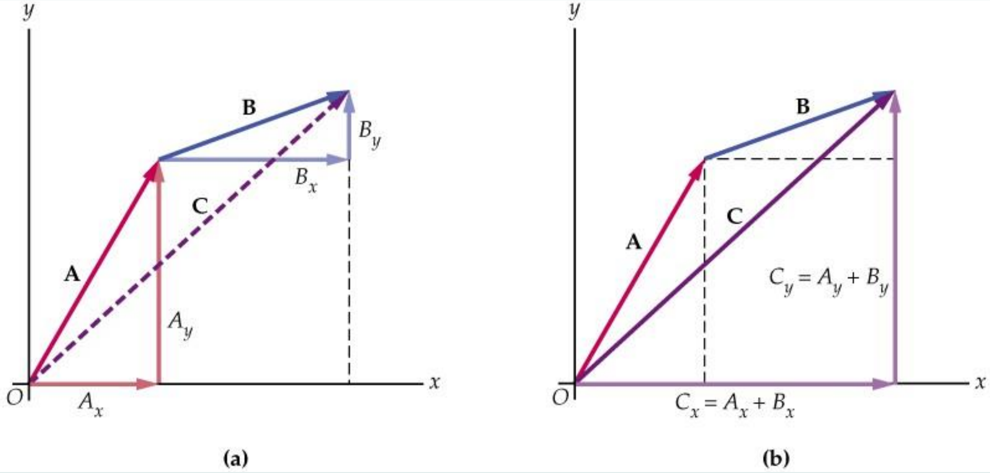
Figure 3.16
Physics for Scientists and Engineers 6th Edition, Thomson Brooks/Cole © 2004; Chapter 3

Adding Vectors Algebraically

- Consider two vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j}$$



- Then

$$\vec{A} + \vec{B} = (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j})$$

$$= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

- If $\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$
- so $C_x = A_x + B_x \qquad C_y = A_y + B_y$

Scalar Product of Two Vectors (dot product)

- The scalar product of two vectors is written as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$

Figure 7.16
Physics for Scientists and Engineers 6th Edition, Thomson Brooks/Cole © 2004; Chapter 7

- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv A\,B\,cos\;\theta$
  - θ is the angle *between A* and *B*

$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$

Dot Product

- The dot product says something about how parallel two vectors are.
- The dot product (scalar product) of two vectors can be thought of as the projection of one onto the direction of the other.

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\vec{A}\cdot\hat{i} = A\cos\theta = A_x$$

- Components

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Projection of a Vector: Dot Product

- The dot product talks about how parallel two vectors are.
- The dot product of two vectors (scalar multiplication) can be thought of as the reflection of one towards the other.

$$\hat{i}\cdot\hat{j}=0;\ \hat{i}\cdot\hat{k}=0;\ \hat{j}\cdot\hat{k}=0$$

$$\hat{i}\cdot\hat{i}=1;\ \hat{j}\cdot\hat{j}=1;\ \hat{k}\cdot\hat{k}=1$$

- Components

$$\vec{A}\cdot\vec{B}=AB\cos\theta$$

$$\vec{A}\cdot\hat{i}=A\cos\theta=A_x$$

$$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$$

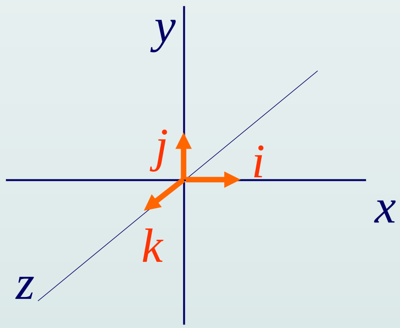
Vector Product (Cross Product)

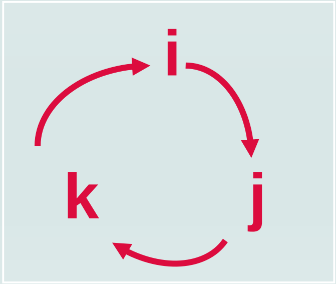
$$\vec{C} = \vec{A} \times \vec{B}$$

- The cross product of two vectors says something about how perpendicular they are.
- Magnitude: $\left|\vec{C}\right| = \left|\vec{A} \times \vec{B}\right| = AB\sin\theta$
  - θ is smaller angle between the vectors
  - Cross product of any parallel vectors = zero
  - Cross product is maximum for perpendicular vectors
  - Cross products of Cartesian unit vectors:

$$\hat{i} \times \hat{j} = \hat{k};\ \hat{i} \times \hat{k} = -\hat{j};\ \hat{j} \times \hat{k} = \hat{i}$$

$$\hat{i} \times \hat{i} = 0;\ \hat{j} \times \hat{j} = 0;\ \hat{k} \times \hat{k} = 0$$





Cross Product

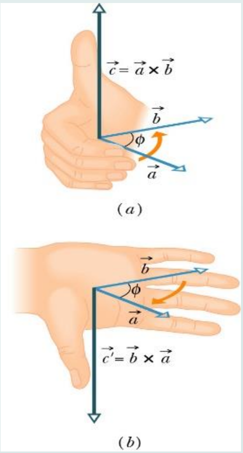
- Direction: C perpendicular to both A and B (right-hand rule)
  - Place A and B tail to tail
  - Right hand, not left hand
  - Four fingers are pointed along the first vector A
  - "sweep" from first vector A into second vector B through the smaller angle between them
  - Your outstretched thumb points the direction of C
- First practice

$$\vec{A}\times\vec{B}=\vec{B}\times\vec{A}?$$

Figure 11.2
Physics for Scientists and Engineers 6th Edition, Thomson Brooks/Cole © 2004; Chapter 11

$$\vec{A}\times\vec{B}=\vec{B}\times\vec{A}?$$

$$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$



Summary

- Polar coordinates of vector A (*A*, *q*)
- Cartesian coordinates (A_x, A_y)
- Relations between them:
- Beware of tan 180-degree ambiguity

$$\begin{cases} A_x = A\cos(\theta) \\ A_y = A\sin(\theta) \end{cases}$$

$$\begin{cases} A = \sqrt{(A_x)^2 + (A_y)^2} \\ \tan(\theta) = \frac{A_y}{A_x} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \end{cases}$$

- Unit vectors: $\mathbf{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
- Addition of vectors: $\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$

  $C_x = A_x + B_x \qquad C_y = A_y + B_y$
- Scalar multiplication of a vector: $a\mathbf{A} = aA_x\hat{i} + aA_y\hat{j}$
- Product of two vectors: scalar product and cross product
  - Dot product is a scalar: $\vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$
  - Cross product is a vector ($\perp \vec{A}\, and\, \vec{B}$): $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$