Chapter 3: Motion in One Dimension

PHY0101/PHY(PEN)101

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Outline

3.1 Position, Velocity, and Speed
3.2 Instantaneous Velocity and Speed
3.3 Acceleration
3.4 Motion Diagrams
3.5 One Dimensional Motion with Constant Acceleration
3.6 Free Falling Objects
3.7 Kinematic Equations Derived From Calculus

Motion

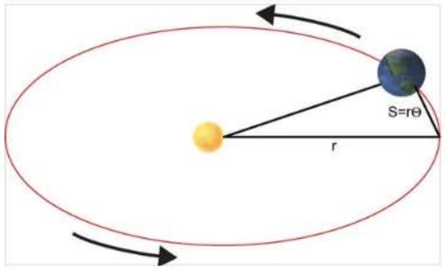
Motion: Change of position over time.

Translational Motion = Straight line motion.

Chapters 2,3,4,5,6,7,8,9

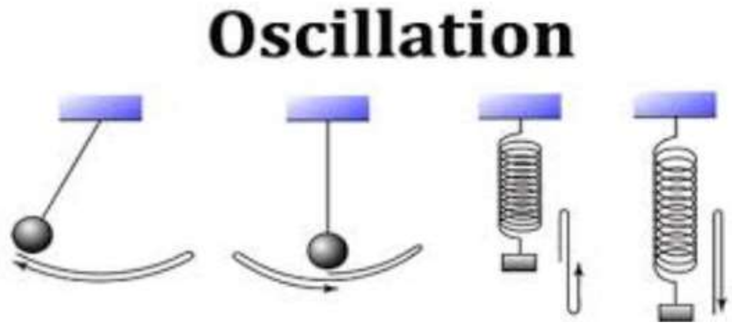
Rotational Motion = Moving (rotating) in a circle.

Chapters 5,6,10,11



Oscillations = Moving (vibrating) back and forth in same path.

Chapter 15



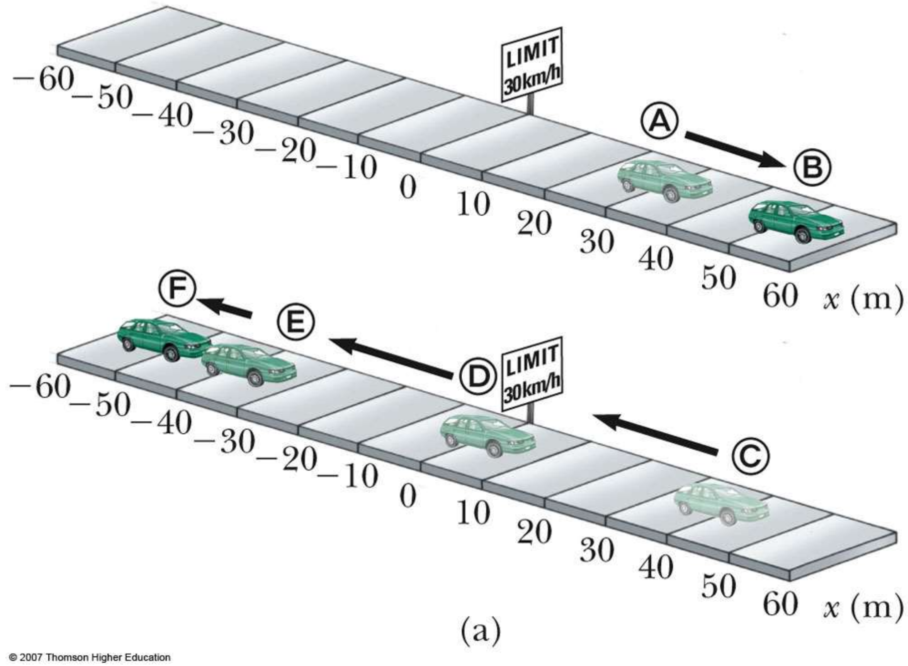
Kinematics

Physics for Scientists and Engineers 6th Edition, Thomson Brooks/Cole © 2004;

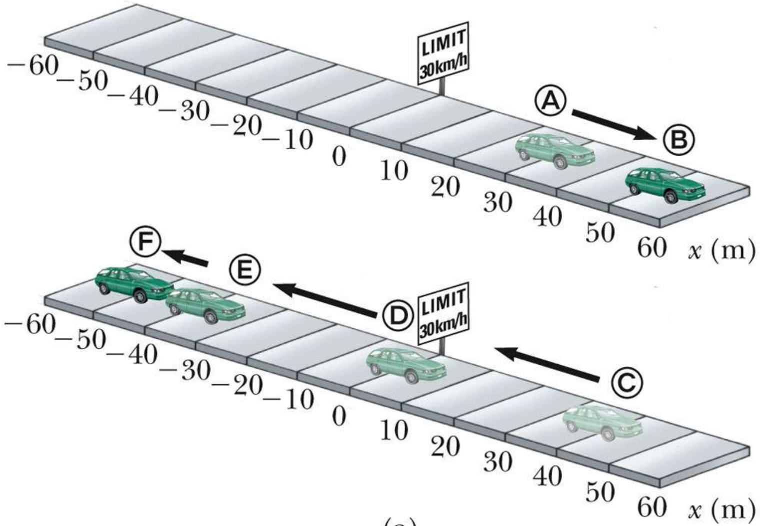
- Describes motion while ignoring the agents that caused the motion
- For now, will consider motion in one dimension
  - Along a straight line
- Will use the particle model
  - A particle is a point-like object, has mass but infinitesimal size

Position

- The object's position is its location with respect to a chosen reference point
  - Consider the point to be the origin of a coordinate system
- In the diagram, allow the road sign to be the reference point

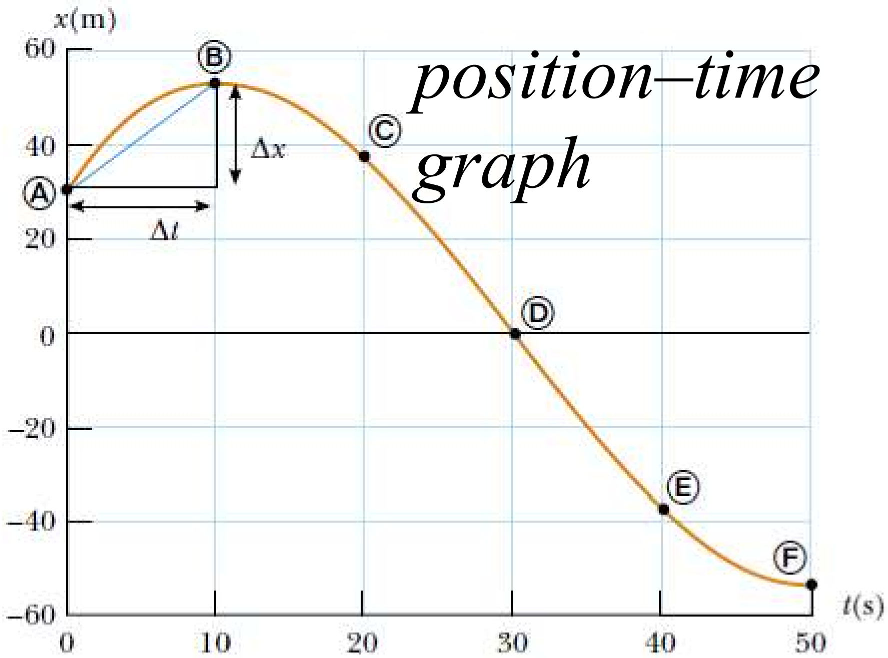


Position-Time Graph



(a)

Position of the Car at Various Times		
Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



Displacement

***The Distance traveled* by an object**

≠ *The Displacement* of the object!

- **Displacement** ≡ ***The change in position*** of an object.

$$\Delta x \equiv x_f - x_i$$

- **Displacement** is a **vector** (magnitude & direction).
- **Distance** is a **scalar** (magnitude).
- **Assume a player moves from one end of the court to the other and back. Distance is twice the length of the court. (Distance is always positive.) Displacement is zero.**
- **Displacement has SI unit of meter.**

Average Velocity

Speed: how far an object travels in a given time interval

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

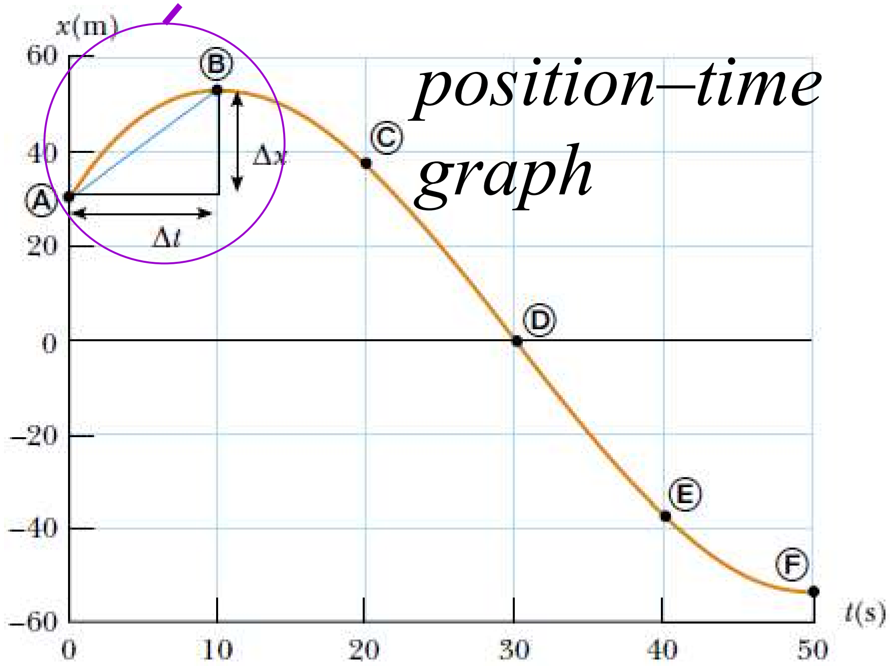
Velocity includes directional information:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}}$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t}$$

Note that the SI unit of speed or velocity is m/s.

- If the coordinate of the particle increases in time, then the average velocity is positive.
- We can interpret average velocity geometrically by drawing a straight line between any two points on the position–time graph



Example 2.1 Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car between points A and F.

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_\mathrm{F} - x_\mathrm{A}}{t_\mathrm{F} - t_\mathrm{A}}$$

$$= \frac{-53\ \mathrm{m} - 30\ \mathrm{m}}{50\ \mathrm{s} - 0\ \mathrm{s}} = \frac{-83\ \mathrm{m}}{50\ \mathrm{s}}$$

$$= -1.7\ \mathrm{m/s}$$

Displacement:

$$\Delta x = x_\mathrm{F} - x_\mathrm{A}$$

$$= -53\ \mathrm{m} - 30\ \mathrm{m}$$

$$= -83\ \mathrm{m}$$

Distance:

From A to B 22 m and from B to F 105 m, totally 127 m

$$\text{Average speed} = \frac{127\ \mathrm{m}}{50\ \mathrm{s}}$$

$$= 2.5\ \mathrm{m/s}$$

Instantaneous Velocity

≡ velocity at any instant of time

≡ average velocity over an infinitesimally short time

- Mathematically, the instantaneous velocity is defined by:

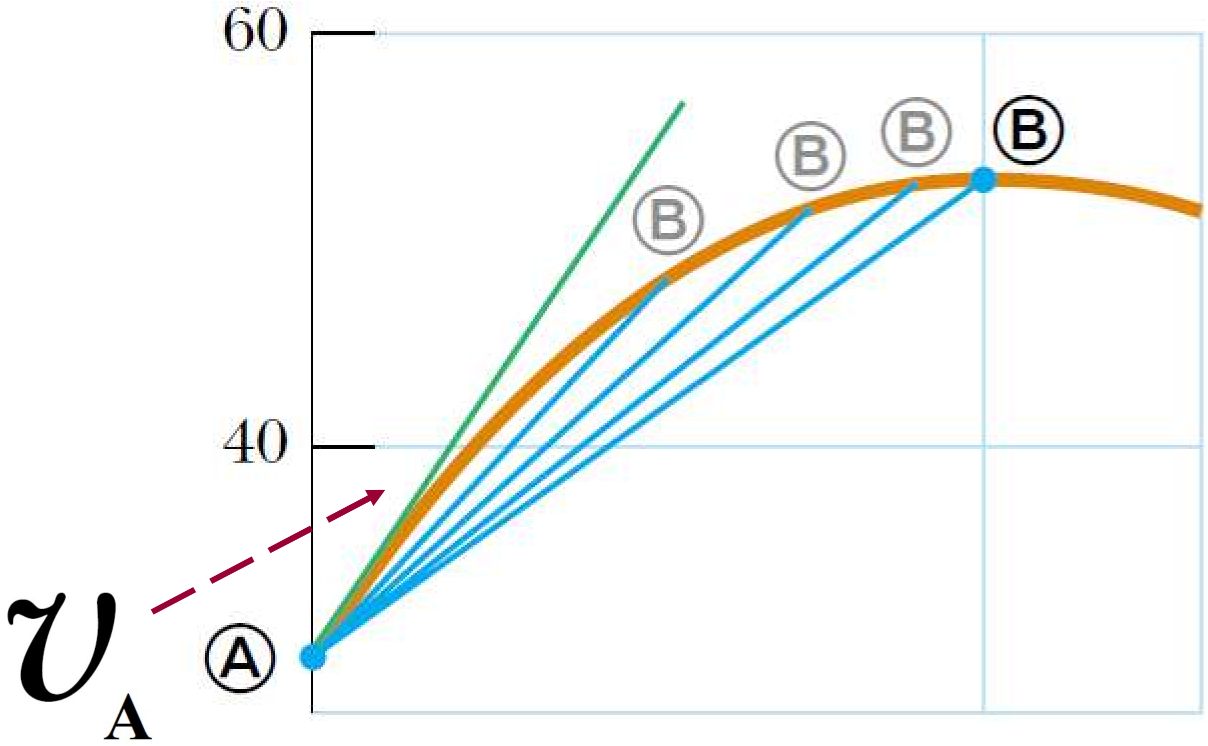
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ ≡ ratio considered as a whole for smaller & smaller Δ**t**. As you should know, mathematicians call this a derivative.

⇒ Instantaneous Velocity

v ≡ Time Derivative of Displacement x

- Instantenous velocity: $v_x = \dfrac{dx}{dt}$



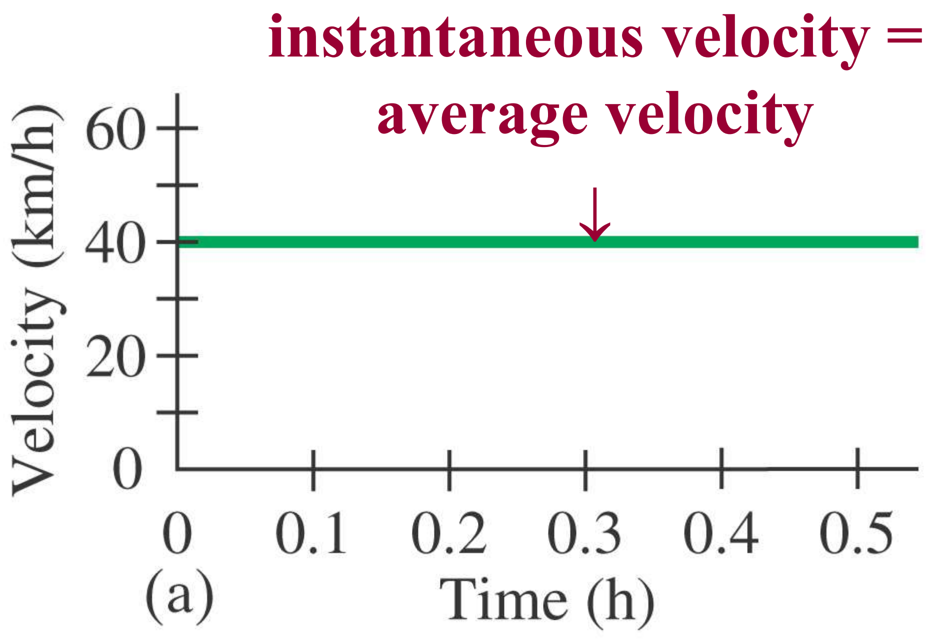
- It is the slope of the tangent line to x(t).
- From here on, we use the word velocity to designate instantaneous velocity.

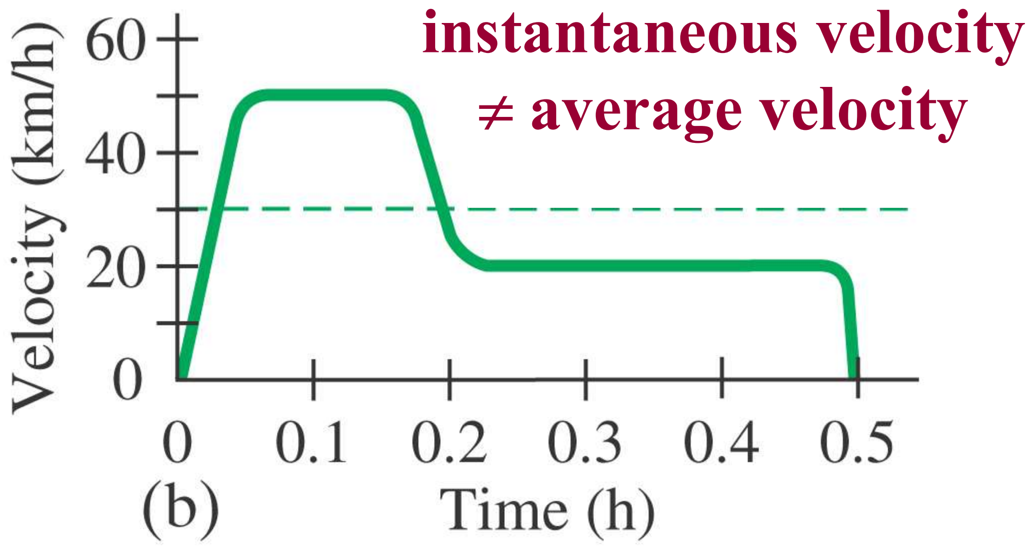
These graphs show

(a) constant velocity →

and

(b) varying velocity →



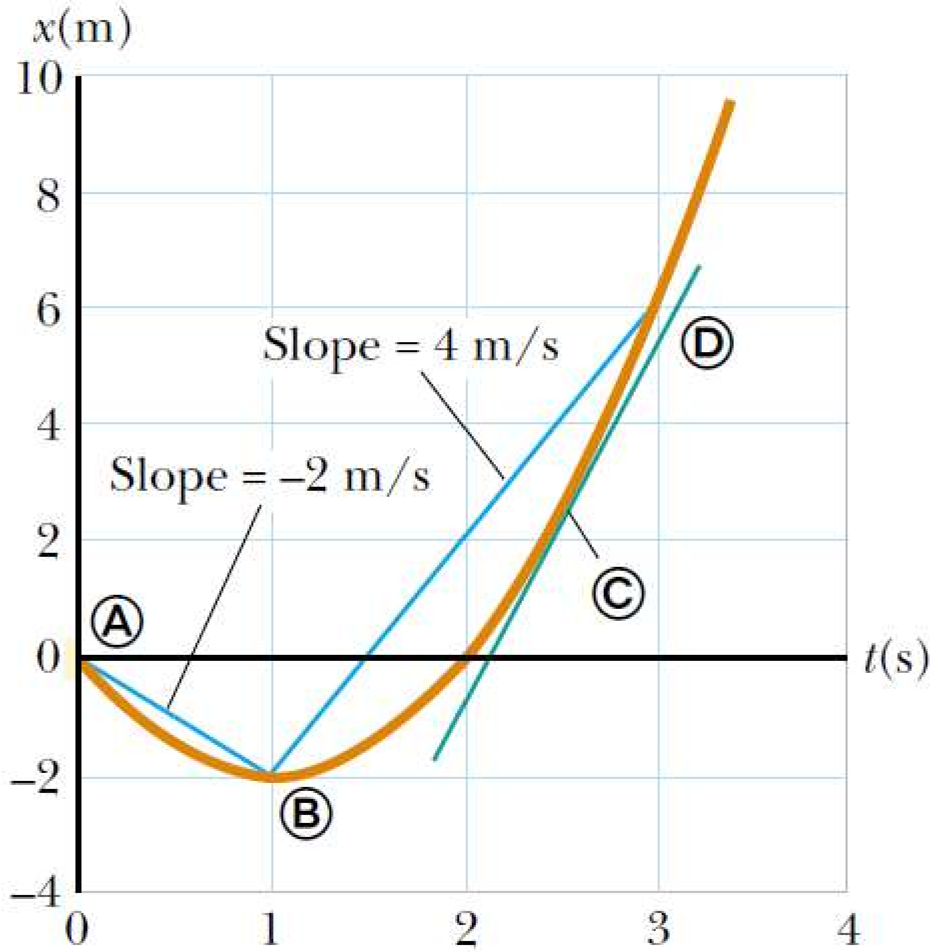


Example 2.3 Average and Instantaneous Velocity

$$x = -4t + 2t^2$$

Note that the particle moves in the negative *x* direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive *x* direction at times $t > 1$ s.

(a) Determine the displacement of the particle in the time intervals t=0 to t=1 s and t=1 to t=3 s.



$$x = -4t + 2t^2$$

$$\Delta x_{A \to B} = x_f - x_i = x_B - x_A$$
$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2]$$
$$= -2\ \text{m}$$

$$\Delta x_{B \to D} = x_f - x_i = x_D - x_B$$
$$= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2]$$
$$= +8\ \text{m}$$

(B) Calculate the average velocity during these two time intervals.

$$\bar{v}_{x(A \to B)} = \frac{\Delta x_{A \to B}}{\Delta t} = \frac{-2\ \text{m}}{1\ \text{s}} = -2\ \text{m/s}$$

$$\bar{v}_{x(B \to D)} = \frac{\Delta x_{B \to D}}{\Delta t} = \frac{8\ \text{m}}{2\ \text{s}} = +4\ \text{m/s}$$

(C) Find the instantaneous velocity of the particle at t = 2.5 s.

$$v_x = +6\ \text{m/s}$$

Acceleration

- Velocity can change with time. An object with velocity that is changing with time is said to be ***accelerating.***
- **Average acceleration** = ratio of change in velocity to elapsed time.

$$\bar{a} \equiv \frac{\Delta v}{\Delta t} = (v_2 - v_1)/(t_2 - t_1)$$

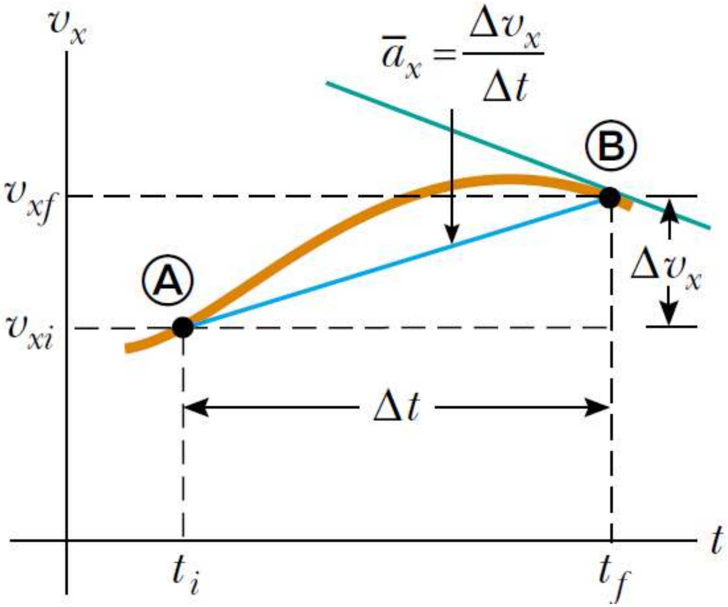
Acceleration is a **vector**.

- **Instantaneous acceleration**

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}.$$

- **Units: velocity/time = distance/(time)^2 = m/s^2**

- Just as the velocity of a moving particle is the slope at a point on the particle's *x*-*t* graph, the acceleration of a particle is the slope at a point on the particle's v_x-t graph.



- Velocity as a function of time

$$v_f(t) = v_i + a_{avg}\Delta t$$

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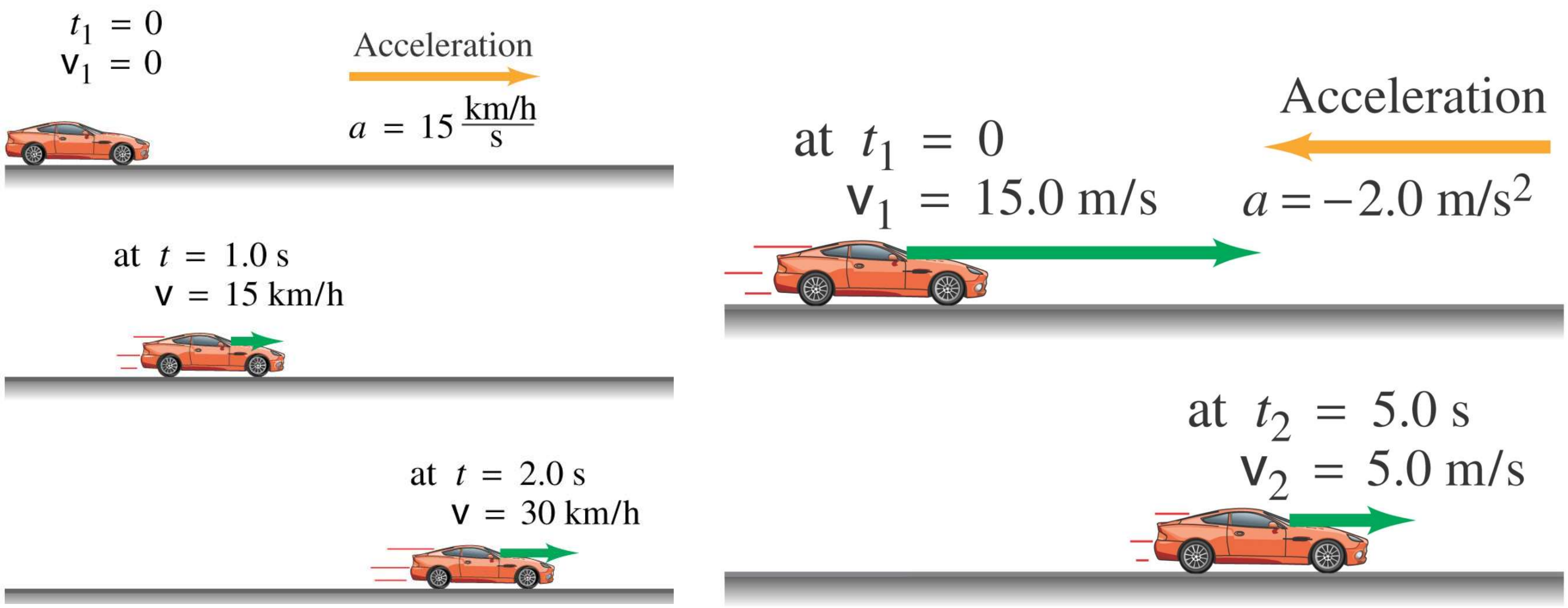
- For 1D motion, when the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down. Acceleration is caused by force.



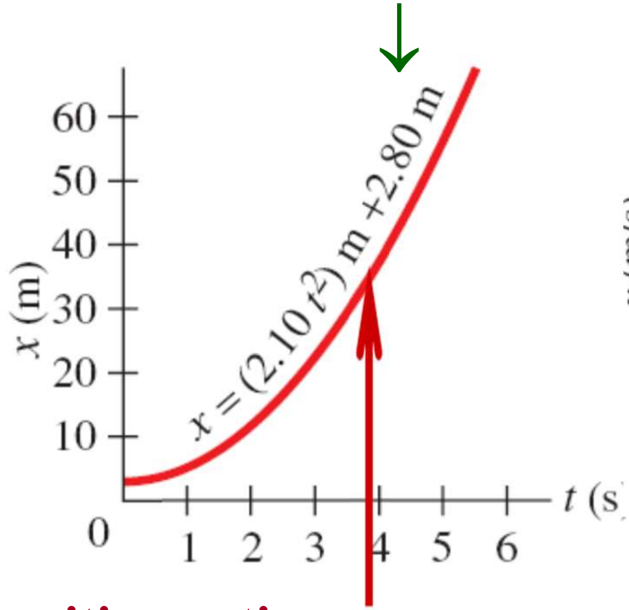
positive a negative a

Example : Acceleration given x(t). A particle moves in a straight line so that its position is given by **$x = (2.10\ m/s^2)t^2 + (2.80\ m)$**. Calculate: **(a)** its average acceleration during the time interval from **$t_1 = 3$ s** to **$t_2 = 5$ s**, & **(b)** its instantaneous acceleration as a function of time.

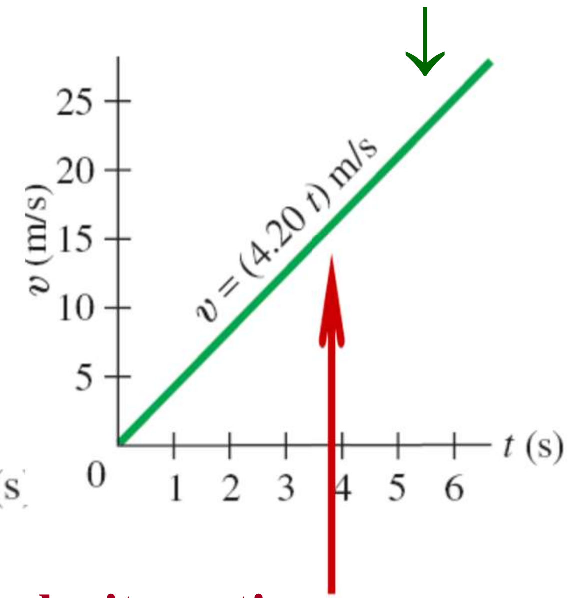
In this case, position *vs* time curve is a parabola

Velocity *vs* time curve is a straight line

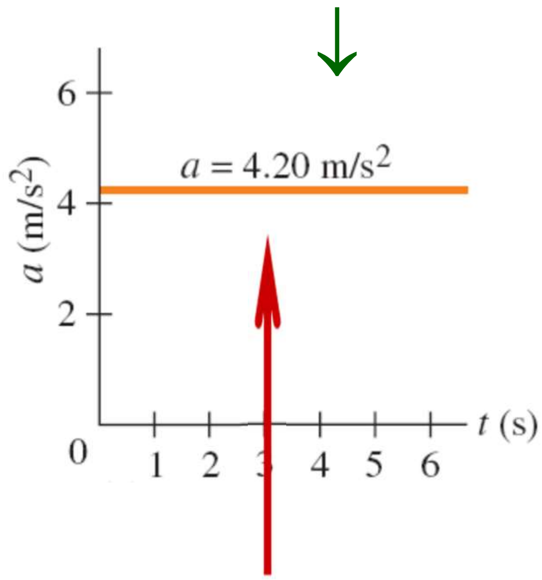
Acceleration is constant here!



position *vs* time curve



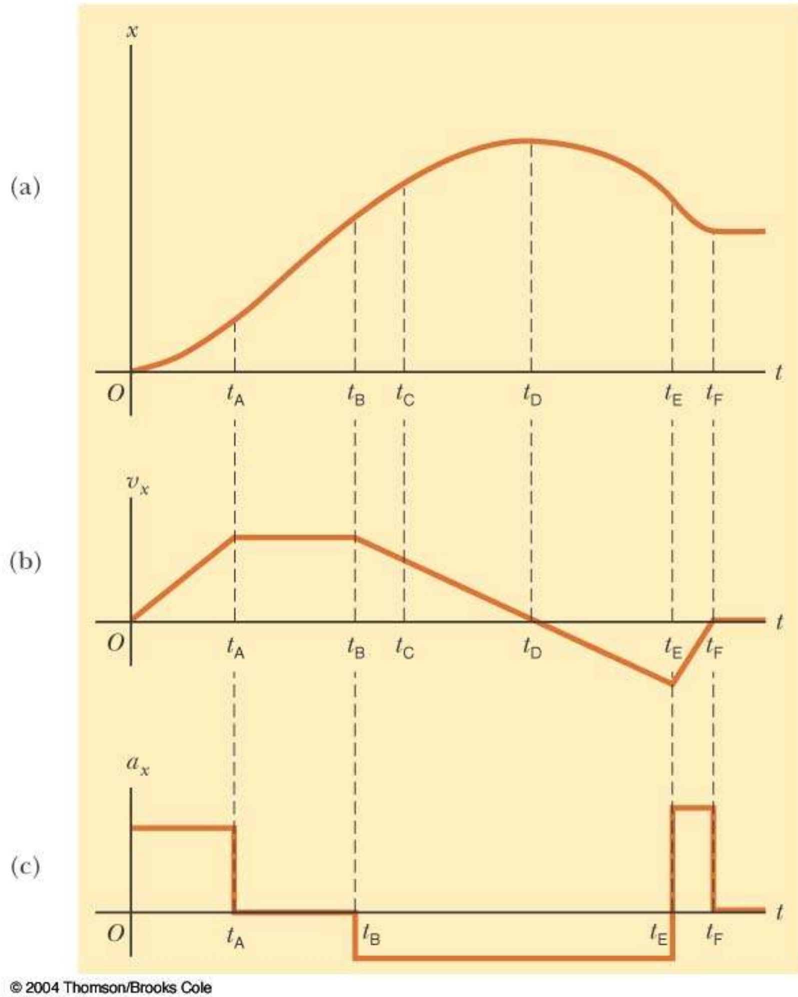
velocity *vs* time curve



acceleration *vs* time curve

Example 2.4: Graphical Relations between x, v, & a

Problem: The position of an object moving along the ***x*** axis varies with time as in the figure. Graph the velocity versus time and acceleration versus time curves for the object.



Example 2.5: Average & Instantaneous Acceleration

Problem: The velocity of a particle moving along the x axis varies in time according to the expression **v = (44 - 10*t*²)**, where ***t*** is in seconds.

a) Calculate the average acceleration in the time interval ***t* = 0 to *t* = 2.0 s.**

b) Calculate the acceleration at ***t* = 2.0 s.**

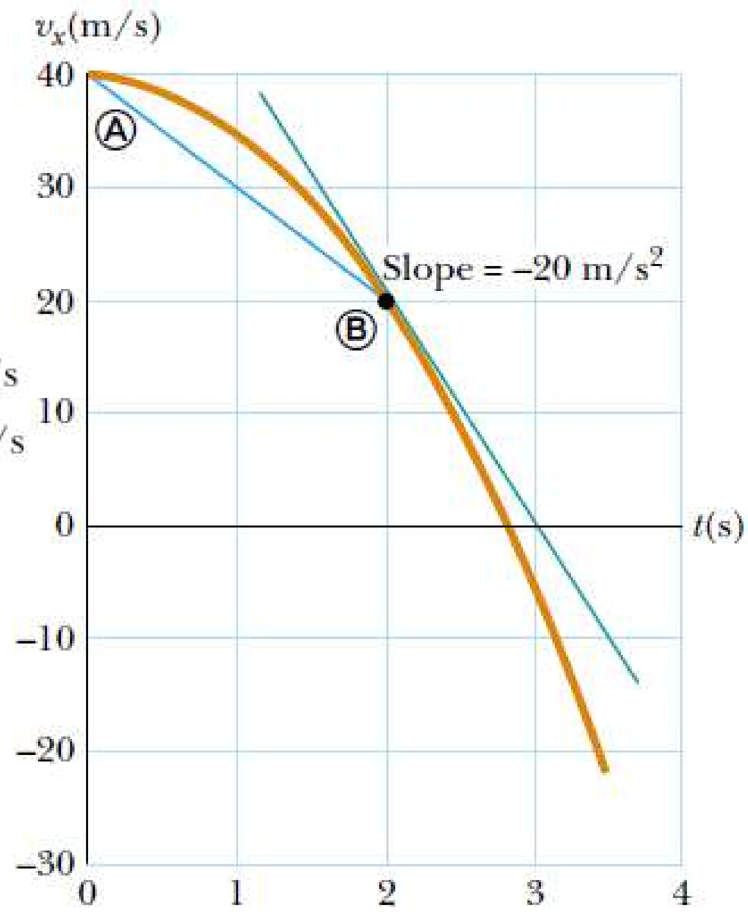
$$v_{xA} = (40 - 5t_A{}^2)\ \text{m/s} = [40 - 5(0)^2]\ \text{m/s} = +40\ \text{m/s}$$

$$v_{xB} = (40 - 5t_B{}^2)\ \text{m/s} = [40 - 5(2.0)^2]\ \text{m/s} = +20\ \text{m/s}$$

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20 - 40)\ \text{m/s}}{(2.0 - 0)\ \text{s}}$$

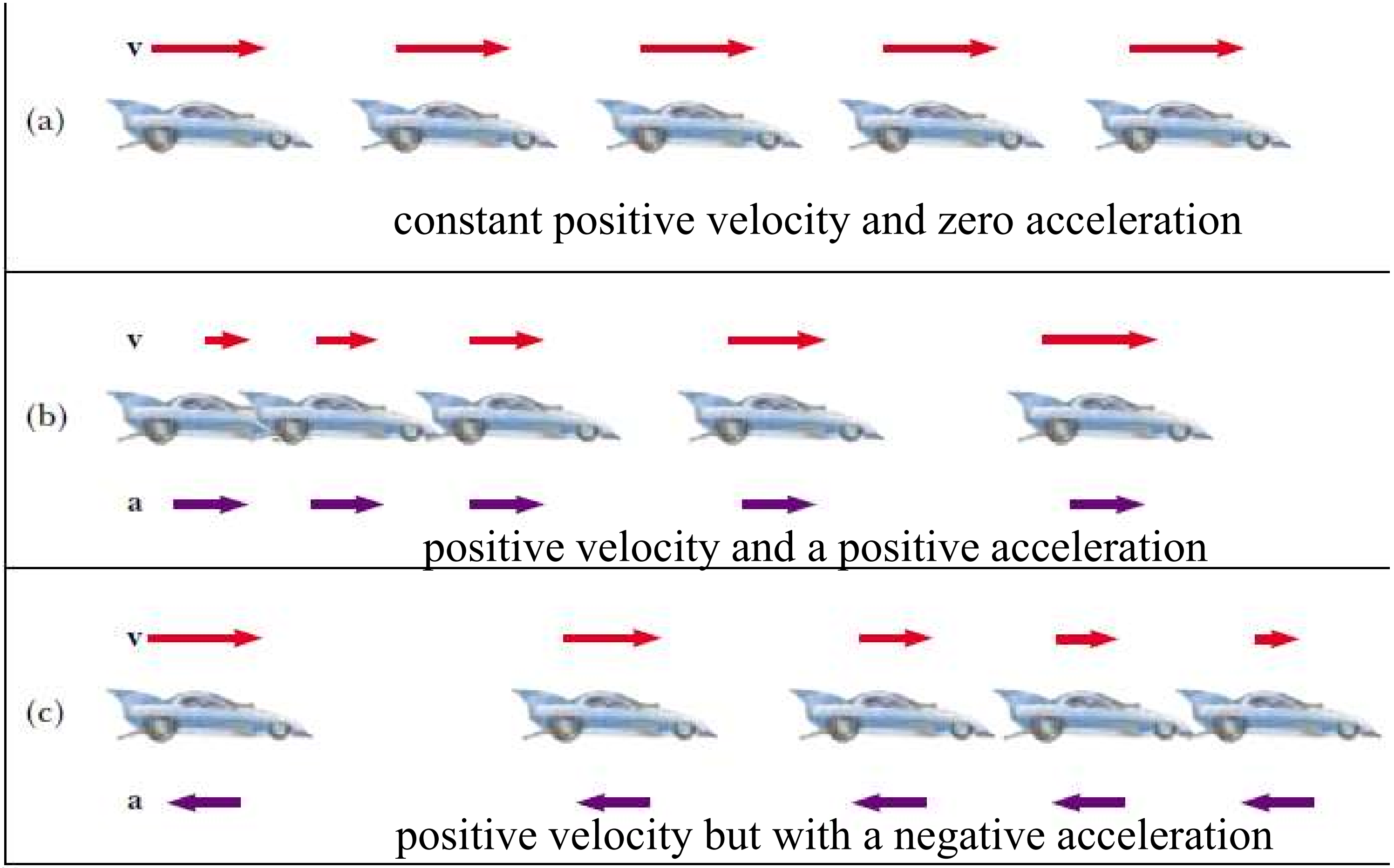
$$= -10\ \text{m/s}^2$$

$$a_x = (-10)(2.0)\ \text{m/s}^2 = -20\ \text{m/s}^2$$



Because the velocity of the particle is positive and the acceleration is negative, the particle is slowing down.

2.4 Motion Diagrams



Active Figure 2.9 (a) Motion diagram for a car moving at constant velocity (zero acceleration). (b) Motion diagram for a car whose constant acceleration is in the direction of its velocity. The velocity vector at each instant is indicated by a red arrow, and the constant acceleration by a violet arrow. (c) Motion diagram for a car whose constant acceleration is in the direction *opposite* the velocity at each instant.

2.5 One-Dimensional Motion with Constant Acceleration

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

$$v_{xf} = v_{xi} + a_x t \qquad \text{(for constant } a_x\text{)}$$

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} \qquad \text{(for constant } a_x\text{)}$$

$$x_f - x_i = \bar{v}t = \tfrac{1}{2}(v_{xi} + v_{xf})\,t$$

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})\,t \qquad \text{(for constant } a_x\text{)}$$

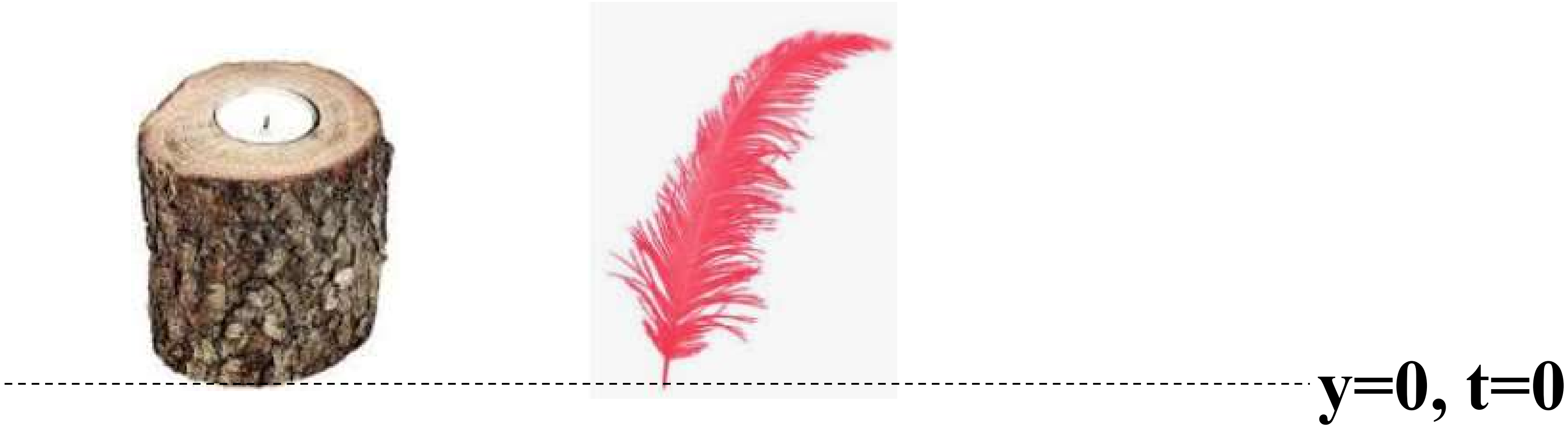
$$x_f = x_i + \tfrac{1}{2}\,[v_{xi} + (v_{xi} + a_x t)]\,t$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}\,a_x t^2 \qquad \text{(for constant } a_x\text{)}$$

$$v_{xf}^{\;2} = v_{xi}^{\;2} + 2a_x\,(x_f - x_i) \qquad \text{(for constant } a_x\text{)}$$

2.6 Freely Falling Objects

Do heavier objects fall faster than lighter ones ???



In the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the **same constant acceleration** under the influence of the **Earth's gravity**.

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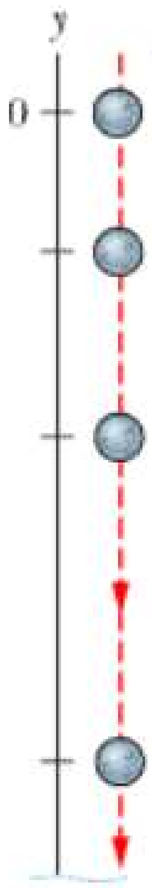
Freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion.

Objects thrown upward or downward and **those released from rest** are all falling freely once they are released. Any freely falling object experiences an acceleration directed downward, regardless of its initial motion.

At the Earth's surface, the value of g is approximately 9.80 m/s^2.

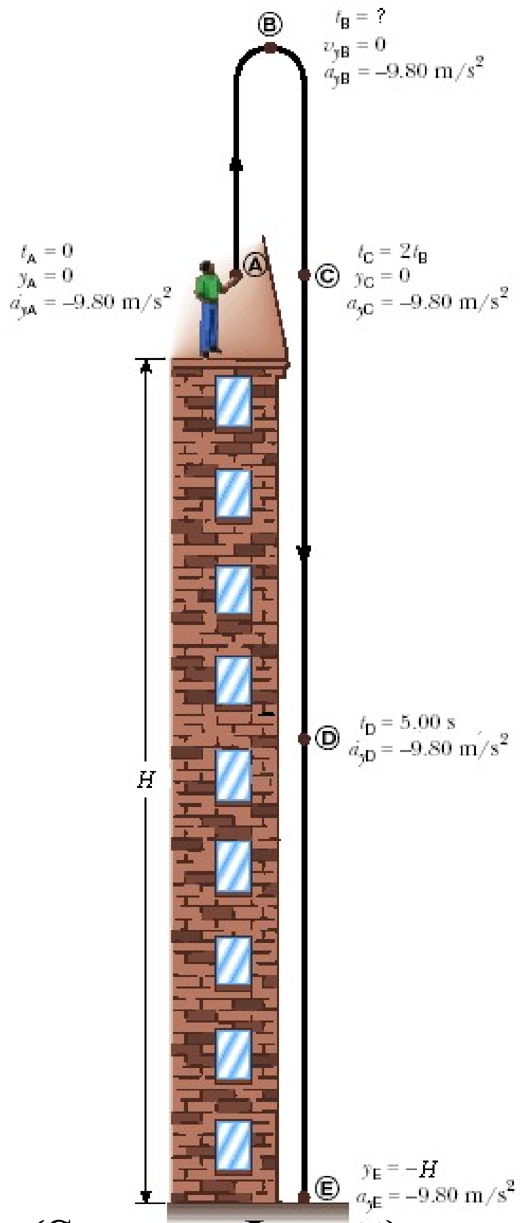


t (s)	y (m)	v (m/s)	a (m/s^2)
0	0	0	–9.8
1	–4.9	–9.8	–9.8
2	–19.6	–19.6	–9.8
3	–44.1	–29.4	–9.8
	–48.0		–9.8

$$a_y = -g = -9.80 \text{ m/s}^2$$

Example 2.12: Not a bad throw for a rookie!

A stone is thrown at point (**A**) from the top of a building with initial velocity $\mathbf{v_0}$ **= 20 m/s** straight up. The building is **H = 50 m** high, and the stone just misses the edge of the roof on its way down, as in the figure.



Calculate:

a) The time at which it reaches its maximum height.

b) It's maximum height above the rooftop.

c) The time at which it returns to the thrower's hand.

d) It's velocity when it returns to the thrower's hand.

e) It's velocity & position at time **t = 5 s.**

Figure©Serways Physics 9th Ed. (Serway, Jewett)

a)

$$v_{yB} = v_{yA} + a_y t$$

$$0 = 20.0\ \text{m/s} + (-9.80\ \text{m/s}^2)\,t$$

$$t = t_B = \frac{20.0\ \text{m/s}}{9.80\ \text{m/s}^2} = \boxed{2.04\ \text{s}}$$

b)

$$y_{max} = y_B = y_A + v_{xA}t + \tfrac{1}{2}a_y t^2$$

$$y_B = 0 + (20.0\ \text{m/s})(2.04\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(2.04\ \text{s})^2$$

$$= \boxed{20.4\ \text{m}}$$

c) t=4.08 s

d)

$$v_{yC} = v_{yA} + a_y t = 20.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(4.08\ \text{s})$$

$$= \boxed{-20.0\ \text{m/s}}$$

e) $v_{y\mathrm{D}} = v_{y\mathrm{A}} + a_y t = 20.0\ \mathrm{m/s} + (-9.80\ \mathrm{m/s^2})(5.00\ \mathrm{s})$

$= -29.0\ \mathrm{m/s}$

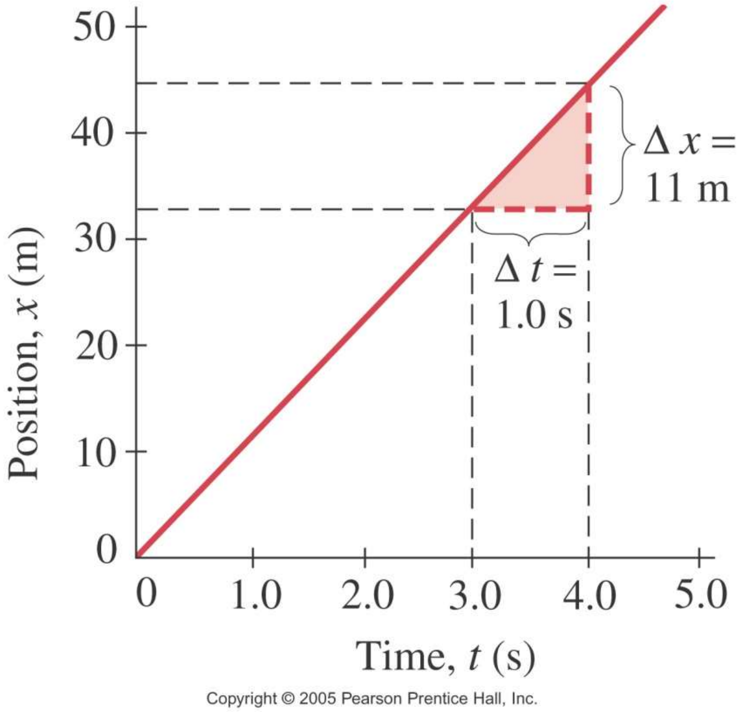
$$y_\mathrm{D} = y_\mathrm{C} + v_{y\mathrm{C}}\, t + \frac{1}{2} a_y t^2$$

$$= 0 + (-20.0\ \mathrm{m/s})(5.00\ \mathrm{s} - 4.08\ \mathrm{s})$$

$$+ \frac{1}{2}(-9.80\ \mathrm{m/s^2})(5.00\ \mathrm{s} - 4.08\ \mathrm{s})^2$$

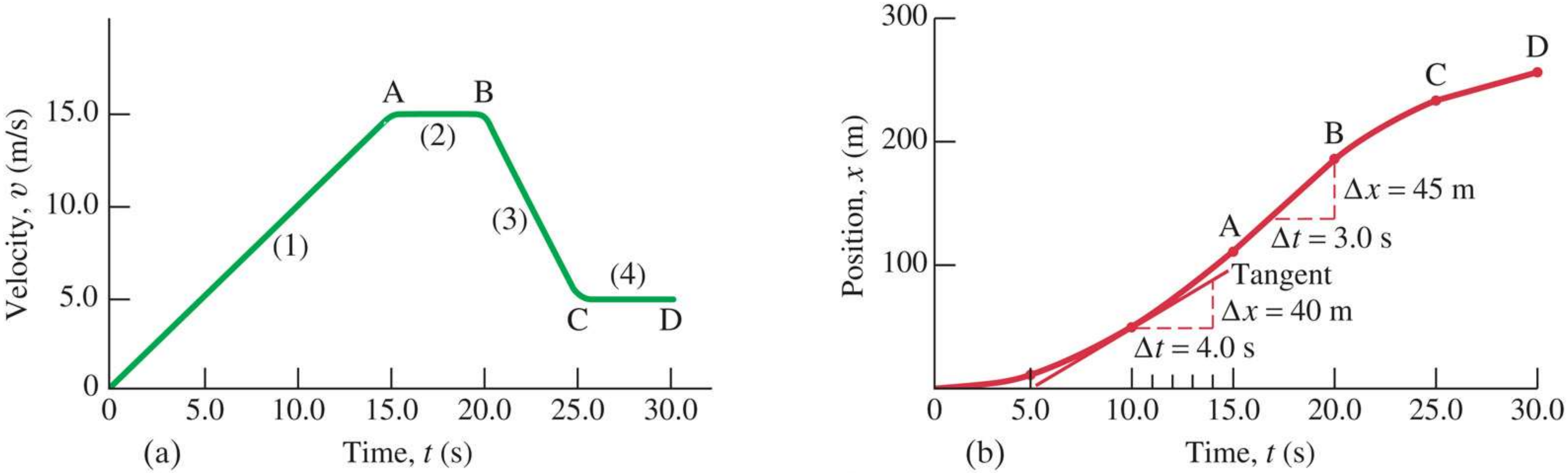
$$= -22.5\ \mathrm{m}$$

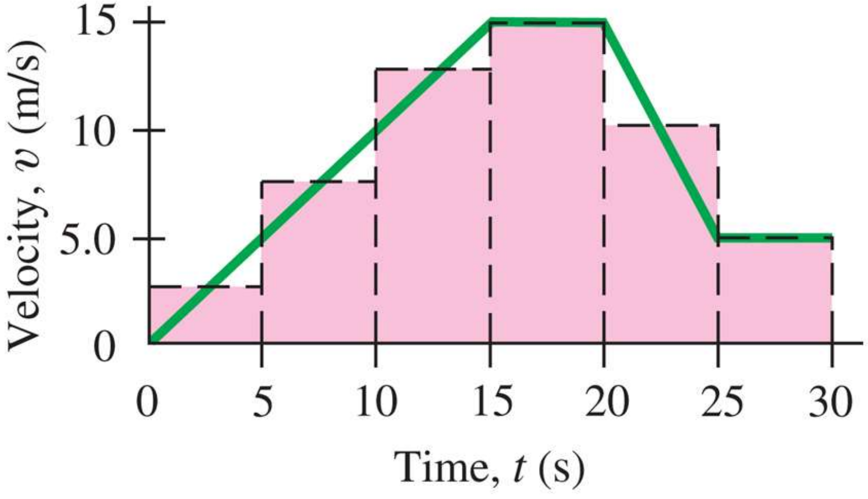
2.7 Kinematic Equations Derived from Calculus



This is a graph of x vs. t for an object moving with constant velocity. The velocity is the slope of the x-t curve.

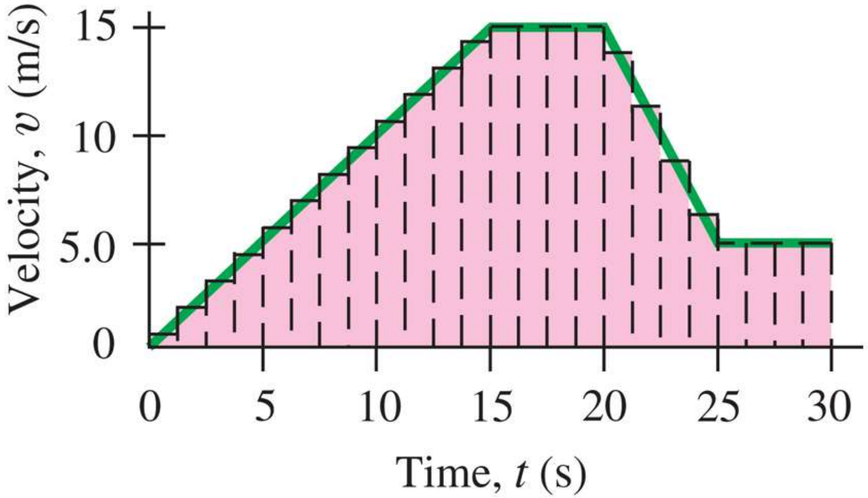
On the left we have a graph of velocity vs. time for an object with varying velocity; on the right we have the resulting *x* vs. *t* curve. The instantaneous velocity is tangent to the curve at each point.





(a)

The displacement, x, is the area beneath the v vs. t curve.



(b)

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn}\, \Delta t_n = \int_{t_i}^{t_f} v_x(t)\, dt$$

- Displacement, distance
- Average velocity, average speed, instantaneous velocity, instantaneous speed
- Average acceleration, instantaneous acceleration
- Equations of kinematics
- Freely falling objects

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