Chapter 4: Motion in Two Dimensions

PHY0101/PHY(PEN)101

Assoc. Prof. Dr. Fulya Bağcı

Outline

- 4.1 The Position, Velocity, and
- Acceleration Vectors
- **Outline**
4.1 The Position, Velocity, and
Acceleration Vectors
4.2 2D Motion with Constant Acceleration
4.3 Projectile Motion
- 4.3 Projectile Motion
- 4.4 Uniform Circular Motion
- 4.5 Tangential and Radial Acceleration
- 1.1 The Festion, Velocity, and
Acceleration Vectors
4.2 2D Motion with Constant Acceleration
4.4 Uniform Circular Motion
4.5 Tangential and Radial Acceleration
4.6 Relative Velocity and Relative Acceleration

Position, Velocity, Acceleration

- **Position, Velocity, Acceleration**
• Just as in 1d, in 2d, an object's motion is completely known if it's position, velocity, **Position, Velocity, Acceleration**
Just as in 1d, in 2d, an object's motion is
completely known if it's position, velocity,
and acceleration are known. **Position, Velocity, Acceleration**
Just as in 1d, in 2d, an object's motion is
completely known if it's position, velocity
and acceleration are known.
Position Vector = **r Position, Velocity, Acceleration**

Fust as in 1d, in 2d, an object's motion is

completely known if it's position, velocity,

and acceleration are known.
 Position Vector = r

- In terms of unit vectors discussed last
- Position Vector \equiv r

for an object at position (x, y) in $x - y$ plane: $r \equiv x i + y j$

For an object moving: **r** depends on time t: $r = r(t) = x(t)$ i + y(t) j

- Suppose that an object moves from point $A(r_i)$ to
point $B(r_f)$ in the x-y plane: $\begin{bmatrix} \overline{y} \\ \overline{y} \end{bmatrix}$ point $B(r_f)$ in the x-y plane: $\begin{bmatrix} y \\ 1 \end{bmatrix}$
- The Displacement Vector is: $\mathbf{\equiv} \Delta \mathbf{r} = \mathbf{r_f} - \mathbf{r_i}$ || \odot If this happens in in a time $||_{\vec{r}}$ $\Delta t = t_f - t_i$
- The Average Velocity is:

Obviously, this in the same $\frac{xy}{y}$ plane is located with the pos direction as the displacement. to the particle. The displacement of the particle as it moves from \circledR to \circledR It is independent of the path in the time interval $\Delta t = t_f - t_i$ is between A and B equal to the vector $\Delta \vec{r} = \vec{r}_f - \vec{r}_f$

 $v_{avg} \equiv (\Delta r / \Delta t)$ Figure 4.1 A particle moving in the xy plane is located with the position

©http://www.phys.ttu.edu/~cmyle s/Prof. Charles W. Myles

• As Δt gets smaller $\&$ smaller, clearly, \bf{A} and \bf{B} get closer $\&$ closer together. Just as in 1d, we As *At* gets smaller & smaller, clearly, **A** and **B** get closer & closer together. Just as in 1d, we define *The Instantaneous Velocity* As Δt gets smaller & smaller, clearly, \bf{A} and \bf{B} get closer & closer together. Just as in 1d, we define **The Instantaneous Velocity**
 \equiv **Velocity at Any Instant of Time.** \equiv *Velocity at Any Instant of Time.* \equiv average velocity over an infinitesimally short time • As Δt gets smaller $\&$ smaller, clearly, \bf{A} and \bf{B} get closer $\&$ closer together. Just as in 1d, we define **The Instantaneous Velocity**
 \equiv **Velocity at Any Instant of Time.**
 \equiv average velocity ove $v \equiv \lim_{\Delta t \to 0} [(\Delta r)/(\Delta t)] \equiv (dr/dt)$ α smaner, elearly, A and **B**

f together. Just as in 1d, we
 ntaneous Velocity

Any **Instant of Time.**
 α is an infinitesimally short time

instantaneous velocity is:
 $[(\Delta r)/(\Delta t)] \equiv (\mathbf{d}r/\mathbf{d}t)$
 $\alpha r)/(\Delta t)$ for $\lim_{\Delta t \to 0}$ = ratio (Δr)/(Δt) for smaller & smaller ∆t. Mathematicians call this a derivative. \Rightarrow The instantaneous velocity $v \equiv$ time derivative of displacement r

-
- **Instantaneous Velocity** $v = (dr/dt)$.
• The magnitude $|v|$ of vector $v \equiv$ **speed**. As motion progresses the speed & direction of y can both • **Instantaneous Velocity** $v = (dr/dt)$.

• The magnitude $|v|$ of vector $v =$ **speed.** As motion progresses, the speed & direction of v can both change. For an object moving from $A(v)$ to $B(v)$ in progresses, the speed $&$ direction of v can both change. For an object moving from $\mathbf{A}(\mathbf{v_i})$ to $\mathbf{B}(\mathbf{v_f})$ in the $x-y$ plane:

Velocity Change $\equiv \Delta \nu = \nu_f - \nu_i$

This happens in time $\Delta t = t_f - t_i$

• Average Acceleration

 $a_{\text{avg}} \equiv (\Delta v / \Delta t)$ As both the speed $\&$ direction of v change, over an arbitrary path

i

Example: Motion of a Turtle

A turtle starts at the origin and moves with the speed of v_0 =10 cm/s in the direction of 25 $^{\circ}$ to the horizontal. $cos25^{\circ} = 0.906, sin25^{\circ} = 0.423$

(a) Find the coordinates of a turtle 10 seconds later.

(b) How far did the turtle walk in 10 seconds?

Notice, you can solve the
equations independently for the
horizontal (x) and vertical (y) Notice, you can solve the
equations independently for the
horizontal (x) and vertical (y)
components of motion and then Notice, you can solve the
equations independently for the
horizontal (x) and vertical (y)
components of motion and then
combine them! Notice, you can solve the
equations independently for the
horizontal (x) and vertical (y)
components of motion and then
combine them! Notice, you can solve the
equations independently for the
horizontal (x) and vertical (y)
components of motion and then
combine them!
 $\vec{v}_{0y} = v_0 \sin \theta$

EX components:
 $v_{0x} = v_0 \cos 25^\circ = 9.06 \text{ cm/s}$ **DY** components:

 \overrightarrow{u} \overrightarrow{u} \overrightarrow{u}

$$
v_{0y} = v_0 \sin 25^\circ = 4.23 \text{ cm/s}
$$

$$
\Delta y = v_{0y} t = 42.3 \text{ cm}
$$

ODistance from the origin:

$$
d = \sqrt{\Delta x^2 + \Delta y^2} = 100.0 \text{ cm}
$$

Average and Instantaneous
Acceleration Acceleration

Summary in 2D motion

Position $\overrightarrow{r}(t) = x\hat{i} + y\hat{j}$ **Average velocity** $\left| \vec{v}_{avg} \right| = \frac{\Delta v}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} = v_{avg,x} \hat{i} + v_{avg,y} \hat{j}$ \Box Instantaneous velocity **a** Acceleration $a_x \equiv \frac{a_x}{dt} = \frac{1}{dt^2}$ $\hat{j} = a_x \hat{i} + a_y \hat{j}$ dt $d\nu$ i dt $\overline{d}v$ dt $d\vec{v}$ t $\vec{\nu}$ $\vec{a}(t) = \lim_{\lambda \to 0} \frac{\Delta v}{\lambda t} = \frac{av_x}{\lambda t} \hat{i} + \frac{av_y}{\lambda t} \hat{j} = a_x \hat{i} + a_y$ $x \hat{i} + u v$ t $f(t) = \lim_{h \to 0} \frac{\Delta v}{\Delta t} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} = a_x \hat{i} + a_y \hat{j}$ 0 $=\frac{uv}{v}=\frac{uv_{x}}{v}\hat{i}+\frac{uv_{y}}{v}\hat{j}=a_{x}\hat{i}+$ Δt $\Delta \overline{1}$ $=$ \rightarrow ($\frac{1}{\overrightarrow{u}}$ $\frac{1}{\overrightarrow{u}}$ \vec{a} t \mathcal{V} i t \mathcal{X} t \vec{r} $\vec{v}_{avg} = \frac{\Delta r}{\Delta t} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} = v_{avg,x} \hat{i} + v_{avg,y} \hat{j}$ Δ $+$ Δt Δ $=$ Δt Δi $=$ $\frac{1}{\nu}$ \vec{r} $\hat{j} = v_x \hat{i} + v_y \hat{j}$ dt $\hat{i} + \frac{dy}{dx}$ dt dx dt $d\vec{r}$ t \vec{r} $\vec{v}(t) = \lim_{t \to 0} \frac{\Delta t}{\Delta t} = \frac{dt}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y$ $\hat{v}(t) = \lim_{h \to 0} \frac{\Delta r}{h} = \frac{dr}{h} = \frac{dx}{h} \hat{i} + \frac{dy}{h} \hat{j} = v_x \hat{i} + v_y \hat{j}$ 0 $=\frac{u_i}{i} = \frac{u_i}{i} \hat{i} + \frac{u_j}{i} \hat{j} = v_i \hat{i} +$ Δt Δi $=$ \rightarrow ($\frac{1}{\vec{r}}$ $\frac{1}{\vec{r}}$ \vec{v} dt dx $v_x \equiv$ dt dy $v_y \equiv$ 2 dt d^2x dt $d\nu$ $a_x \equiv \frac{a v_x}{1}$ $x \equiv \frac{u v_x}{\mu} =$ 2 2 dt d^2y dt $\equiv \frac{dv_y}{dt} =$ a_{y}

Assoc.Prof.Dr. Fulya Bağcı

2D Motion with Constant Acceleration
It can be shown that: **2D Motion with Const**
• It can be shown that:
Motion in the x-

Motion in the x-y plane can be treated as 2 independent **D Motion with Constant Acceleration**

it can be shown that:
 **Motion in the x-y plane can

be treated as 2 independent

motions in the x and y directions.
** \Rightarrow **So, motion in the x direction** \Rightarrow So, motion in the x direction doesn't affect the y motion and motion in the y direction doesn't affect the x motion.

Motion in two dimensions **Motion in two dimensions**
• Motions in each dimension are independent components
• Constant acceleration equations
• $\vec{v} - \vec{v} + \vec{\alpha}t$ $\vec{r} - \vec{r} = \vec{v}t + \frac{1}{2}\vec{\alpha}t^2$ **Motion in two diments**
• Motions in each dimension are independent
• Constant acceleration equations
 $\vec{v} = \vec{v}_0 + \vec{a}t$ $\vec{r} - \vec{r} = \vec{v}_0t + \frac{1}{2}\vec{a}t^2$

-
-

$$
\vec{v} = \vec{v}_0 + \vec{a}t \qquad \vec{r} - \vec{r} = \vec{v}_0t + \frac{1}{2}\vec{a}t^2
$$

Motion in two dimensions
\nNotions in each dimension are independent components
\nConstant acceleration equations
\n
$$
\vec{v} = \vec{v}_0 + \vec{a}t \qquad \vec{r} - \vec{r} = \vec{v}_0t + \frac{1}{2}\vec{a}t^2
$$
\n
$$
\text{Constant acceleration equations hold in each dimension}
$$
\n
$$
v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t
$$
\n
$$
x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \qquad y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2
$$
\n
$$
v_x^2 = v_{0x}^2 + 2a_x (x - x_0) \qquad v_y^2 = v_{0y}^2 + 2a_y (y - y_0)
$$
\n
$$
- t = 0 \text{ beginning of the process;}
$$
\n
$$
- \vec{a} = a_x \hat{i} + a_y \hat{j} \text{ where } a_x \text{ and } a_y \text{ are constant;}
$$
\n
$$
- \text{Initial velocity } \vec{v}_0 = v_{0x} \hat{i} + v_{0y} \hat{j} \text{ initial displacement } \vec{r}_0 = x_0 \hat{i} + y_0 \hat{j}
$$

- $t = 0$ beginning of the process;
- $\vec{a} = a_x \hat{i} + a_y \hat{j}$ where a_x and a_y are constant;
- $\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$ $\vec{v}_0 = v_{0x} \hat{i} + v_{0y} \hat{j}$ initial displacement $\vec{r}_0 = x_0 \hat{i} + y_0 \hat{j}$ \vec{v}

Example 4.1: A particle starts from the origin at $t=0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of Example 4.1: A particle starts from the origin at $t=0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of acceleration only, g Example 4.1: A particle starts from the origin at $t=0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of acceleration only, g Example 4.1: A particle starts from the origin at $t=0$ with are velocity having an x component of 20 m/s and a y component m/s. The particle moves in the xy plane with an x component acceleration only, given by $a_x=4.0$ in the origin at $t=0$ with an initial
 \therefore 20 m/s and a y component of -15

y plane with an x component of

=4.0 m/s². (a) Determine the

at any time and the total velocity at $t=0$ with an initial
d a y component of -15
th an x component of
. (a) Determine the
e and the total velocity Example 4.1: A particle starts from the origin at $t=0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of acceleration only, g Example 4.1: A particle starts from the origin
velocity having an x component of 20 m/s and
m/s. The particle moves in the xy plane wit
acceleration only, given by $a_x=4.0 \text{ m/s}^2$.
components of the velocity vector at an

$$
v_{xf} = v_{xi} + a_x t = (20 + 4.0t) \text{ m/s}
$$

\n
$$
v_{yf} = v_{yi} + a_y t = -15 \text{ m/s} + 0 = -15 \text{ m/s}
$$

\n
$$
\mathbf{v}_f = v_{xi} \mathbf{\hat{i}} + v_{yi} \mathbf{\hat{j}} = \frac{[(20 + 4.0t)\mathbf{\hat{i}} - 15\mathbf{\hat{j}}] \text{ m/s}}{[(20 + 4.0t)\mathbf{\hat{i}} - 15\mathbf{\hat{j}}] \text{ m/s}}
$$

\n
$$
\mathbf{a} = 4.0\mathbf{\hat{i}} \text{ m/s}^2 \qquad \mathbf{v}_i = [20\mathbf{\hat{i}} - 15\mathbf{\hat{j}}] \text{ m/s}
$$

(b) Calculate the velocity and speed of the particle at t=5.0 s. $v_f = [(20 + 4.0(5.0))]$ = $15 \hat{j}$ m/s = $(40 \hat{i} - 15 \hat{j})$ m/s $\theta = \tan^{-1}\left(\frac{v_{\text{yf}}}{v_{\text{xf}}}\right) = \tan^{-1}\left(\frac{-15 \text{ m/s}}{40 \text{ m/s}}\right) = -21^{\circ}$

4.3 Projectile Motion

2

2

 $\overline{0}$

 $x^2 + 2a_x(x - x_0) v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

 $2a_y(y-y_0)$

□2-D problem and define a coordinate system: 4.3 Projectile Motic

a 2-D problem and define a coordinate system:

x- horizontal, y- vertical (up +)

a Try to pick $x_0 = 0$, $y_0 = 0$ at $t = 0$

a Horizontal motion + Vertical motion **Try to pick** $x_0 = 0$, $y_0 = 0$ at $t = 0$ \Box Horizontal motion + Vertical motion **u**Horizontal: $a_x = 0$, constant velocity motion **Overtical:** $a_y = -g = -9.8 \text{ m/s}^2$, $v_{0y} = 0$ **Equations:** $v_y = v_{0y} + a_y t$ 2 ² $y-y_0 = v_{0y}t + \frac{1}{2}a_yt$ $v_x = v_{0x} + a_x t$ 4.3 **FTOJECUTE IVIOUDII**

D problem and define a coordinate system:

prizontal, y- vertical (up +)

v to pick $x_0 = 0$, $y_0 = 0$ at $t = 0$

rizontal motion + Vertical motion

rizontal: $a_x = 0$, constant velocity motion

r

2

 $x - x_0 = v_{0x}t + \frac{1}{2} a_x t$

 $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

 $\overline{0}$

□X and Y motions happen independently, so we can treat them separately

Try to pick $x_0 = 0$, $y_0 = 0$ at $t = 0$ Horizontal motion + Vertical motion **u**Horizontal: $a_x = 0$, constant velocity motion **OVertical:** $a_y = -g = -9.8 \text{ m/s}^2$ \Box x and y are connected by time t $\Box y(x)$ is a parabola

Maximum Height of a Projectile

Horizontal Range of a Projectile

$$
t=2t_A=\frac{2\vartheta_i sin\theta_i}{g}
$$

$$
R = \vartheta_{xi} t = \vartheta_i cos \theta_i \frac{2\vartheta_i sin \theta_i}{g} = \frac{2\vartheta_i^2 sin \theta_i cos \theta_i}{g}
$$

$$
R = \frac{2\vartheta_{xi}\vartheta_{yi}}{g}
$$

Example 4.3: The Long Jump

A long-jumper leaves the ground at an angle $\theta_i = 20^\circ$ above the horizontal at a speed of $v_i = 11.0 \text{ m/s}$. (cos20°=0.94, sin20°=0.34)

- a) How far does he jump in the horizontal direction?
	- (Assume his motion is equivalent to that of a particle.)
- b) What is the maximum height reached?

$$
v_{xi} = v_i cos \theta_i, v_{yi} = v_i sin \theta_i, v_{xf} = v_{xi}
$$

\n
$$
x_f = v_{xi} t, v_{yf} = v_{yi} - gt
$$

\n
$$
y_f = v_{yi} t - \frac{1}{2}gt^2
$$

\n
$$
(v_{yf})^2 = (v_{yi})^2 - 2gy_f
$$

The Long Jump: Solutions

A long-jumper leaves the ground at an angle $\theta_i = 20^\circ$ above the horizontal at a speed of $v_i = 8.0$ m/s. (cos20°=0.94, sin25°=0.34)

- a) How far does he jump in the horizontal direction? (Assume his motion is equivalent to that of a particle.)
- b) What is the maximum height reached?

$$
v_{xi} = v_i \cos(\theta_i) = 7.5 \text{ m/s}
$$

\n
$$
v_{yi} = v_i \sin(\theta_i) = 4.0 \text{ m/s}
$$

\n**a)** How far does he jump in the
\nhorizontal direction? Range =
\n
$$
\mathbf{R} = (2v_{xi}v_{yi}/g) = 2(7.5)(4)/(9.8)
$$

\n
$$
\mathbf{R} = 7.94 \text{ m}
$$

\nKinematic Equations
\n
$$
v_{xi} = v_i \cos\theta_i, \quad v_{yi} = v_i \sin\theta_i, v_{xf} = v_{xi}
$$

\n
$$
x_f = v_{xi}t, \quad v_{yf} = v_{yi} - gt
$$

\n
$$
y_f = v_{yi}t - (\frac{1}{2})gt^2
$$

\n
$$
(v_{yf})^2 = (v_{yi})^2 - 2gy_f
$$

Kinematic Equations $v_{xi} = v_i \cos \theta_i$, $v_{yi} = v_i \sin \theta_i$, $v_{xf} = v_{xi}$ $(v_{\text{vf}})^2 = (v_{\text{vi}})^2 - 2gy_f$ jump in the

on? Range =

7.5)(4)/(9.8)
 m

uations

in θ_i , $v_{xf} = v_{xi}$
 $v_{yi} - gt$

2)gt²

2 2gy_f

The Long Jump: Solutions

A long-jumper leaves the ground at an angle $\theta_i = 20^\circ$ above the horizontal at a speed of $v_i = 11.0 \text{ m/s}$. (cos20°=0.94, sin20°=0.34)

- a) How far does he jump in the horizontal direction?
	- (Assume his motion is equivalent to that of a particle.)
- b) What is the maximum height reached?

©Serways Physics 9th Ed. (Serway, Jewett)

 $v_{\rm xi} = v_i \cos(\theta_i) = 7.5$ m/s $v_{vi} = v_i \sin(\theta_i) = 4.0$ m/s $R = 7.94$ m

b) What is the maximum height? $h = [(v_{yi})^2/(2g)]$ $h = 0.72$ m

Kinematic Equations $v_{xi} = v_i \cos \theta_i$, $v_{yi} = v_i \sin \theta_i$, $v_{xf} = v_{xi}$ $x_i = v_i \cos(\theta_i) = 7.5 \text{ m/s}$
 $y_i = v_i \sin(\theta_i) = 4.0 \text{ m/s}$
 $\mathbf{R} = 7.94 \text{ m}$
 $\mathbf{R} = [(\mathbf{v}_{yi})^2/(2g)]$
 $\mathbf{h} = [(\mathbf{v}_{yi})^2/(2g)]$
 $\mathbf{h} = 0.72 \text{ m}$
 Sumeratic Equations
 $\mathbf{r}_i \cos \theta_i$, $\mathbf{v}_{yi} = \mathbf{v}_{i} \sin \theta_i$, $\mathbf{v}_{xf} =$ $v_i \sin(\theta_i) = 4.0 \text{ m/s}$
 $R = 7.94 \text{ m}$

the maximum height?
 $n = [(v_{yi})^2/(2g)]$
 $h = 0.72 \text{ m}$

<u>ematic Equations</u>
 $s\theta_i, v_{yi} = v_i \sin \theta_i, v_{xf} = v_{xi}$
 $= v_{xi} t, v_{yf} = v_{yi} - gt$
 $y_f = v_{yi} t - (\frac{1}{2})gt^2$
 $v_{yf} = (v_{yi})^2 - 2gy_f$ $(v_{\rm vf})^2 = (v_{\rm vi})^2 - 2gy_f$ m

un height?

2g)]

m

uations

in θ_i , $v_{xf} = v_{xi}$
 $v_{yi} - gt$

2gy_f

Example 4.4: Non-Symmetric Projectile Motion

A stone is thrown. $x_i = y_i = 0$
 $\begin{array}{ccc} y & v_i = 20.0 \text{ m/s} \\ 0.00 & 0.00 \end{array}$ $y_f = -45.0$ m, $v_i = 20$ m/s, $\theta_i = 30^\circ$ a) Time to hit the ground? b) Speed just before it hits? c) Distance from the base of the building where it lands? $(cos30^{\circ} = 0.87, sin30^{\circ} = 0.5)$

Kinematic Equations $v_{xi} = v_i \cos \theta_i$, $v_{yi} = v_i \sin \theta_i$ $v_{xf} = v_{xi}$, $x_f = v_{xi}$ © Brooks/Cole Thoms

2006 College Physics

matic Equations

cos θ_i , $v_{yi} = v_i sin \theta_i$

= v_{xi} , $x_f = v_{xi}t$
 $v_{yf} = v_{yi} - gt$

= $v_{yi}t - (1/2)gt^2$
 $v_{zi}^2 - 2gy_f$ **z**
 g c Brooks/Cole Thomson
 nematic Equations
 r v_icos θ _i, **v**_{yi} = **v**_isin θ _i
 f = **v**_{xi}, **x**_{**f** = **v**_{xi}**t**
 v_{yf} = **v**_{yi} - **gt**
 y_f = **v**_{yi}**t** - (¹/₂)**gt**²
 y_f = **}** $(v_{\rm vf})^2 = (v_{\rm vi})^2 - 2gy_f$ Cole Thomson
ge Physics

ations

= $v_i sin \theta_i$

= $v_{xi} t$

gt

2)gt²

- 2gy_f

Example 4.4: Solution

A stone is thrown! $x_i = y_i = 0$
 $\begin{array}{ccc} y & y_i = 20.0 \text{ m/s} \\ 0.0 & 0.0 \end{array}$ $y_f = -45.0$ m, $v_i = 20$ m/s, $\theta_i = 30^\circ$ a) Time to hit the ground? b) Speed just before it hits? c) Distance from the base of the building where it lands? First, calculate $v_{\text{vi}} = v_i \cos(\theta_i) = 17.3 \text{ m/s}$ $v_{\text{vi}} = v_i \sin(\theta_i) = 10.0 \text{ m/s}$ a) Time to hit the ground? (Time when $y_f = -45.0$ m) be to interest in hits?

the dip is before it hits?

img where it lands?

calculate
 $v_{xi} = v_i \cos(\theta_i) = 17.3 \text{ m/s}$
 $v_{yi} = v_i \sin(\theta_i) = 10.0 \text{ m/s}$

to hit the ground?

e when $y_f = -45.0 \text{ m}$
 $y_f = -45 \text{ m} = v_{yi} t - {1/2}gt^2$
 v_{xi A general quadratic must be solved using the quadratic equation. This gives:

$$
t=4.22~s
$$

Kinematic Equations $v_{xi} = v_i \cos \theta_i$, $v_{yi} = v_i \sin \theta_i$ $v_{xf} = v_{xi}$, $x_f = v_{xi}$ © Brooks/Cole Thomson

2006 College Physics

matic Equations

cos θ_i , $v_{yi} = v_i sin \theta_i$

= v_{xi} , $x_f = v_{xi}t$
 $v_{yf} = v_{yi} - gt$

= $v_{yi}t - (\frac{1}{2})gt^2$
 $y_i^2 = (v_{yi})^2 - 2gy_f$ y

y © Brooks/Cole Thomson

y 2006 College Physics

nematic Equations

y_icos θ_i , $v_{yi} = v_i sin \theta_i$
 $f = v_{xi}$, $x_f = v_{xi}t$
 $v_{yf} = v_{yi} - gt$
 $y_f = v_{yi}t - \frac{1}{2}gt^2$
 $y_{f} = (v_{yi})^2 - 2gy_f$ $(v_{\rm vf})^2 = (v_{\rm vi})^2 - 2gy_f$ ole Thomson

e Physics

<u>ations</u>

= v_isin θ_i

= v_{xi} t

gt

2)gt²

- 2gy_f

Example 4.4: Solution

A stone is thrown! $x_i = y_i = 0$
 $\begin{array}{ccc} y & v_i = 20.0 \text{ m/s} \\ 0.0 & 0.00 \end{array}$ $y_f = -45.0$ m, $y_i = 20$ m/s, $\theta_i = 30^\circ$ a) Time to hit the ground? b) Speed just before it hits? c) Distance from the base of the building where it lands? First, calculate $v_{\text{vi}} = v_i \cos(\theta_i) = 17.3 \text{ m/s}$ $v_{\text{vi}} = v_i \sin(\theta_i) = 10.0 \text{ m/s}$ $t_{\text{hit}} = 4.22$ s b) Velocity just before it hits? Time to hit the ground?

Speed just before it hits?

Distance from the base of the

building where it lands?

First, calculate
 $v_{xi} = v_i \cos(\theta_i) = 17.3 \text{ m/s}$
 $v_{yi} = v_i \sin(\theta_i) = 10.0 \text{ m/s}$
 $t_{hit} = 4.22 \text{ s}$

Speed $v_{xf} = v_{yi}$ eed just before it hits?

tance from the base of the

lding where it lands?
 $v_{xi} = v_i \cos(\theta_i) = 17.3 \text{ m/s}$
 $v_{yi} = v_i \sin(\theta_i) = 10.0 \text{ m/s}$
 $t_{hit} = 4.22 \text{ s}$

locity just before it hits?
 v_{xi} , v_{xi} , $v_{yi} = v_{yi} - gt$ so v_{xf} Speed $(v_f)^2 = (v_{xf})^2 + (v_{yf})^2$ v_{xf} $v_f = 35.8$ m/s Angle: $\tan(\theta_f) = (v_{yf}/v_{xf}) = -(31.3/17.3) = -1.8$ $V_f = v_{yi} t$ $\theta_f = -60.9^\circ$

Kinematic Equations $v_{xi} = v_i \cos \theta_i$, $v_{yi} = v_i \sin \theta_i$ $v_{xf} = v_{xi}$, $x_f = v_{xi}$ © Brooks/Cole Thomson

2006 College Physics

matic Equations

cos θ_i , $v_{yi} = v_i sin \theta_i$

= v_{xi} , $x_f = v_{xi}t$
 $v_{yf} = v_{yi} - gt$

= $v_{yi}t - (\frac{1}{2})gt^2$
 $v_{zi}^2 - 2gy_f$ **z**
 z © Brooks/Cole Thomson
 z 2006 College Physics
 nematic Equations
 $\cdot v_i \cos\theta_i$, $v_{yi} = v_i \sin\theta_i$
 $f = v_{xi}$, $x_f = v_{xi}t$
 $v_{yf} = v_{yi} - gt$
 $y_f = v_{yi}t - \frac{1}{2}gt^2$
 $y_{f} = (v_{yi})^2 - 2gy_f$ $(v_{\rm vf})^2 = (v_{\rm vi})^2 - 2gy_f$ Cole Thomson

ge Physics

<u>ations</u>

= $v_i sin\theta_i$

= $v_{xi} t$

gt

2)gt²

- 2gy_f

Example 4.4: Solution

A stone is thrown! $x_i = y_i = 0$
 $y_i = 20.0 \text{ m/s}$ $y_f = -45.0$ m, $y_i = 20$ m/s, $\theta_i = 30^\circ$ a) Time to hit the ground? b) Speed just before it hits? c) Distance from the base of the building where it lands? First, calculate

 $v_{\text{vi}} = v_i \cos(\theta_i) = 17.3 \text{ m/s}$ $v_{\rm vi} = v_i \sin(\theta_i) = 10.0 \text{ m/s}$ $t_{\text{hit}} = 4.22$ s $v_f = 35.8 \text{ m/s}, \theta_f = -60.9^{\circ}$ c) Distance from the base of the building where it lands? $x_f = v_{xi} t_{hit} = (17.3)(4.22) = 73.0$ m $v_{vf} = v_{vi} - gt$

Kinematic Equations $v_{xi} = v_i \cos \theta_i$, $v_{yi} = v_i \sin \theta_i$ $v_{xf} = v_{xi}, \quad x_f = v_{xi}$ **D** Brooks/Cole Thomson
 matic Equations
 matic Equations
 cos θ_i , $v_{yi} = v_i \sin \theta_i$
 $= v_{xi}$, $x_f = v_{xi}t$
 $v_{yf} = v_{yi} - gt$
 $= v_{yi}t - (\frac{1}{2})gt^2$
 $= (v_{yi})^2 - 2gy_f$ y

y C Brooks/Cole Thomson

y 2006 College Physics

nematic Equations

y_i cos θ_i , $v_{yi} = v_i sin \theta_i$
 $f = v_{xi}$, $x_f = v_{xi}t$
 $v_{yf} = v_{yi} - gt$
 $y_f = v_{yi}t - \frac{1}{2}gt^2$
 $y_{f} = (v_{yi})^2 - 2gy_f$ $(v_{\rm vf})^2 = (v_{\rm vi})^2 - 2gy_f$ ble Thomson

Physics

<u>ations</u>

= $v_i sin\theta_i$

= $v_{xi}t$

gt

2)gt²

- 2gy_f

Example: Driving Off a Cliff!!

A movie stunt driver on a motorcycle speeds horizontally off a 50.0-m-high cliff. How fast must the motorcycle leave the cliff top to land on level ground below, 90.0 m from the base of the cliff where the cameras are? **ple: Driving Off a Cliff!!**
on a motorcycle speeds horizontally off a 50.0-m-high
ne motorcycle leave the cliff top to land on level
from the base of the cliff where the cameras are?
<u>ons</u>: $v_{xi} = v_i cos\theta_i$, $v_{yi} = v_i sin\theta_i$, n -high
 e ?
 $= v_{xi}t$
 $- 2gy_f$

Kinematic Equations: $v_{xi} = v_i \cos \theta_i$, $v_{yi} = v_i \sin \theta_i$, $v_{xf} = v_{xi}$ $x_f = v_{xi}$ t $v_{\text{vf}} = v_{\text{vi}} - gt$, $y_f = v_{\text{vi}} t - \frac{1}{2}gt^2$, $(v_{\text{vf}})^2 = (v_{\text{vi}})^2 - 2gy_f$

©http://www.phys.ttu.edu/~cmyle s/ Prof. Charles W. Myles

Solutions: Driving Off a Cliff!!

A movie stunt driver on a motorcycle speeds horizontally off a 50.0-m-high cliff. How fast must the motorcycle leave the cliff top to land on level ground below, 90.0 m from the base of the cliff where the cameras are?

Driving Off a Cliff!!

on a motorcycle speeds horizontally off a 50.0-m-high

ne motorcycle leave the cliff top to land on level

from the base of the cliff where the cameras are?

<u>ons</u>: $v_{xi} = v_i cos\theta_i$, $v_{yi} = v_i sin\theta_i$, 1-high
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$ if **a Cliff!!**
rizontally off a 50.0-m-high
iff top to land on level
where the cameras are?
 $v_i \sin\theta_i$, $v_{xf} = v_{xi}$ $x_f = v_{xi}t$
 $(\frac{1}{2})gt^2$, $(v_{yf})^2 = (v_{yi})^2 - 2gy_f$
 $v_x = v_{xi} = ?$, $v_{yf} = -gt$
 $x_f = v_{xi}t$, $y_f = -(\frac{1}{2})gt^2$
Time **Kinematic Equations:** $v_{xi} = v_i \cos \theta_i$, $v_{yi} = v_i \sin \theta_i$, $v_{xf} = v_{xi}$ $x_f = v_{xi}$ t $v_{\text{vf}} = v_{\text{vi}} - gt$, $y_f = v_{\text{vi}}t - \frac{1}{2}gt^2$, $(v_{\text{vf}})^2 = (v_{\text{vi}})^2 - 2gy_f$ $x+y$ $v_x = v_{xi} = ?$, $v_{vf} = -gt$

©http://www.phys.ttu.edu/~cmyle s/Prof. Charles W. Myles