Chapter 9: Linear Momentum and Collisions PHY0101/PHY101

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Outline (Serway 9th Edition) **Outline (Serway 9th Edition**

9.1 Linear Momentum

9.2 Isolated System (Momentum) **Outline (Serway 9th Edition)**
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9.2 Isolated System (Momentum)
9.3 Nonisolated System (Momentum) **Outline (Serway 9th Edition)**

9.1 Linear Momentum

9.2 Isolated System (Momentum)

9.3 Nonisolated System (Momentum)

9.4 Collisions in One Dimension **Outline (Serway 9th Edition)**

9.1 Linear Momentum

9.2 Isolated System (Momentum)

9.3 Nonisolated System (Momentum)

9.4 Collisions in One Dimension

9.5 Collisions in Two Dimensions

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9.1 Linear Momentum

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9.4 Collisions in One Dimension

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9.6 The Center of Mass 9.1 Linear Momentum
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9.4 Collisions in One Dimension
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9.7 Systems of Many Particles

9.8 Deformable Systems 9.2 Isolated System (Moldentum)

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9.4 Collisions in One Dimension
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9.7 Systems of Many Particles
9.8 Deformable Systems
9.9 Rocket Propulsion
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Collisions and Impulse
consider details of collision

• *Briefly* consider details of collision

- **Collisions and Impulse**
Briefly consider details of collision
– Assume collision lasts a very small time Δt
During collision, net force <u>on one</u> of the objects
Nowton's 2nd Law): $\Sigma \mathbf{F} = \Delta \mathbf{n}/4t$, $(= \frac{d\mathbf{n}}{dt$ **Collisions and Impulse**
• *Briefly* consider details of collision
– Assume collision lasts a very small time Δt
• During collision, net force <u>on one</u> of the objects
(Newton's 2nd Law): $\sum \mathbf{F} = \Delta \mathbf{p}/\Delta t$ (= dp/ (Newton's 2^{nd} Law): $\sum \mathbf{F} = \Delta \mathbf{p}/\Delta \mathbf{t}$ (= dp/dt)
- Or: $\Delta p = (\sum F) \Delta t$ (momentum change of the object)

 $\Delta p \equiv I \equiv$ Impulse that collision gives the object

(change in momentum for the object!)

- Assume collision lasts a very small time Δt

• During collision, net force **on one** of the objects

(Newton's 2nd Law): $\sum \mathbf{F} = \Delta \mathbf{p}/\Delta \mathbf{t}$ (= dp/dt)

Or: $\Delta \mathbf{p} = (\sum \mathbf{F})\Delta \mathbf{t}$ (momentum change of the During collision, net force **on one** of the objec

(Newton's 2nd Law): $\sum \mathbf{F} = \Delta \mathbf{p}/\Delta \mathbf{t}$ (= \mathbf{dp}/\mathbf{dt})
 $\therefore \Delta \mathbf{p} = (\sum \mathbf{F})\Delta \mathbf{t}$ (momentum change of the object)
 $\Delta \mathbf{p} = \mathbf{I} = \underline{\text{Impulse}}$ that coll to $\mathbf{p_f}$, **t** limits: **t**_i to **t**_f)) n's 2nd Law): $\sum \mathbf{F} = \Delta \mathbf{p} / \Delta \mathbf{t}$ (= dp/dt)
 $\sum \mathbf{F} \Delta \mathbf{t}$ (momentum change of the object)

= **Impulse** that collision gives the object

(change in momentum for the object!)

es this as integral over time

• Usual case: Replace time integral of net force by time average force: $[\int (\sum F) dt/(\Delta t)] \approx (\sum F)_{avg}$ Impulse, I = $\Delta p = (\sum F)dt \approx (\sum F)_{avg} \Delta t$ **Usual case:** Replace time integral of net force b
average force: $[\int (\sum \mathbf{F}) dt/(\Delta t)] \approx (\sum \mathbf{F})_{\text{avg}}$
 Impulse, I = $\Delta \mathbf{p} = \int (\sum \mathbf{F}) dt \approx (\sum \mathbf{F})_{\text{avg}} \Delta t$
 $\Delta t = t_f - t_i = \text{average collision time}$
 Math: Time integral = area under ΣF • **Math:** Time integral = area under ΣF the force vs. time curve: \rightarrow

Impulse, $I = \Delta p = area$ under the curve. Δt is usually very small

© Brooks/Cole Thomson 2006 College Physics t_i t_f (a)

• The approximation of replacing Σ_F
 $I = \Delta p = \int (\sum F) dt$ $I = \Delta p = \int (\sum F) dt$

with

 $I = \Delta p \approx (\sum F)_{avg} \Delta t$ ^{($\Sigma F)_{avg}$}

is equivalent to replacing the true area under the curve by the rectangle shown. $I = \Delta p = \int (\sum F) dt$
with
 $I = \Delta p \approx (\sum F)_{avg} \Delta t$
is equivalent to replacing the true
area under the curve by the
rectangle shown.
This is known as
the Impulse Approximation

the Impulse Approximation

Example 9.3: Crash Test

- Crash test: Car, $m = 1500$ kg, hits wall. 1 dimensional collision. $+x$ is to the right. Before crash, $v = -15$ m/s.
After crash, $v = 2.6$ m/s. Collision lasts $\Delta t = 0.15$ s. Find: Impulse car receives & average force on car.
	- Assume: Force exerted by wall is large compared to other forces
	- Gravity & normal forces are perpendicular & don't effect the horizontal momentum
	- \Rightarrow Use impulse approximation

 $p_i = mv_i = -2.25$ kg m/s, $p_f = mv_f = 2.64$ kg m/s $I = \Delta p = p_f - p_i = 2.64 \times 10^4$ kg m/s $(\sum F)_{\text{avg}} = (\Delta p / \Delta t) = 1.76 \times 10^5$ N

Collisions

- **Collisions**
• Use term "collision" to represent an event during
which two particles come close to each other and
interact by means of forces which two particles come close to each other and interact by means of forces **Collisions**
Jse term "collision" to represent an event during
which two particles come close to each other and
nteract by means of forces
– May involve physical contact, but is generalized to include
cases with interactio **Collisions**
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interact by means of forces
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cases with interac • Use term "collision" to represent an event during
which two particles come close to each other and
interact by means of forces
– May involve physical contact, but is generalized to include
cases with interaction without
	- cases with interaction without physical contact
- initial to final values is assumed to be short
- than any external forces present

This means the impulse approximation can be used!

- Collisions may be result of direct
contact
• Impulsive forces may vary in time in contact \rightarrow F_{21}
- Collisions may be result of direct

 Impulsive forces may vary in time in

 Impulsive forces may vary in time in

 This force is internal to system complicated ways Collisions may be result of direct

ontact
 \longrightarrow \vec{F}_{21}
 m_1 m_2

omplicated ways

- This force is internal to system

Collision needn't include physical

ontact between the objects \longrightarrow • Collisions may be result of direct

contact

• Impulsive forces may vary in time in

complicated ways

– This force is internal to system

• Collision needn't include physical

contact between the objects \rightarrow

• There • Collisions may be result of direct

contact

• Impulsive forces may vary in time in

complicated ways

– This force is internal to system

• Collision needn't include physical

contact between the objects \rightarrow

• There
	-
- contact between the objects \rightarrow
- particles
- Impulsive forces may vary in time in

complicated ways

 This force is internal to system

 Collision needn't include physical

contact between the objects \rightarrow

 There are still forces between the

particles

 This analyzed in the same way as those that include physical contact - This force is internal to system

• Collision needn't include physical

• There are still forces between the

particles

• This type of collision can be

analyzed in the same way as those

that include physical contact

- conserved for ALL **OLLISIONS!!**

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- **Perfectly Elastic collision: BOTH** momentum and kinetic energy are conserved
Perfectly elastic collisions occur on a microscopic level energy are conserved **Fectly Elastic collision: BOTH** momentum and kingy are conserved
Fectly elastic collisions occur on a microscopic level
nacroscopic collisions, only **approximately elastic** collisi
tually occur
• Generally some energy is
	- Perfectly elastic collisions occur on a microscopic level
	- In macroscopic collisions, only approximately elastic collisions actually occur
		-
- Inelastic collision: *Kinetic energy is not conserved*, but momentum is still conserved Perfectly elastic collisions occur on a microscopic level

In macroscopic collisions, only **approximately elastic** collisions

actually occur

• Generally some energy is lost to deformation, sound, etc.

• **Inclastic colli** • Generally some energy is lost to deformation, sound, etc.

• Inelastic collision: *Kinetic energy is not conserved*, but

<u>momentum is still conserved</u>
 Perfectly inelastic collision: Objects stick together after the

Perfectly inelastic collision: Objects stick together after the collision.

- objects do not stick together
- actual collisions fall in between these two types

Momentum is *always conserved* in all collisions!!!!

9.3: Collisions in One Dimension

- 9.3: Collisions in One Dimension
• Given some information, using conservation
laws, we can determine a LOT about collisions
without linewing the collision forward laws, we can determine a LOT about collisions without knowing the collision forces! 9.3: Collisions in One Di

• Given some information, using co

laws, we can determine a LOT at

without knowing the collision for

• To analyze ALL collisions:

<u>Rule # 1</u>:
-

Rule # 1:

Momentum is ALWAYS conserved in a collision!

 \implies $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ HOLDS for ALL collisions!

Elastic Collisions

• Both momentum AND kinetic energy are conserved!

 \Rightarrow

Before collision $\overrightarrow{\mathbf{v}}_{1i}$ \vec{v}_{2i}

 $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

 $(\frac{1}{2})m_1(v_{1i})^2 + (\frac{1}{2})m_2(v_{2i})^2$ $=$ $(\frac{1}{2})m_1(v_{1f})^2 + (\frac{1}{2})m_2(v_{2f})^2$ Th

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 m_1 $m₂$ (a)

Note!!!

- *Special case:* 2 hard objects (like billiard balls) collide $($ = "Elastic Collision")
- ↓ Note!!!
• Special case: 2 hard objects (like b
balls) collide (= "Elastic Collision"
• To analyze Elastic collisions:
Rule # 1: Still holds! Momentum is Rule # 1: Still holds! Momentum is conserved!!
- \implies $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ Rule # 2: For Elastic Collisions ONLY (!!) Total Kinetic energy is conserved!! \implies $(\frac{1}{2})m_1(v_{1i})^2 + (\frac{1}{2})m_2(v_{2i})^2$ e Elastic collisions:
 Still holds! **Momentum is conserved!!**
 $\mathbf{n}_1 \mathbf{v}_{1i} + \mathbf{m}_2 \mathbf{v}_{2i} = \mathbf{m}_1 \mathbf{v}_{1f} + \mathbf{m}_2 \mathbf{v}_{2f}$

For Elastic Collisions **ONLY** (!!)
 etic energy is conserved!!
 $(\mathbf{v}_{1i})^2 + (\frac{1}{2})$ $=$ $(\frac{1}{2})m_1(v_{1f})^2 + (\frac{1}{2})m_2(v_{2f})^2$

collisions: (Figure):

 $FIGURE 7-13$ Two equal mass objects (a) approach each other with equal speeds, (b) collide, and then (c) bounce off with equal speeds in the opposite directions if the collision is elastic, or (d) bounce back much less or not at all, if the collision is inelastic.

- Special case: Head-on Elastic Collisions
	-

• Special case: Head-on Elastic Collisions.

Special case: Head-on Elastic Collisions.
– Momentum is conserved (*ALWAYS*!)
 $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ V_{1i} , V_{2i} , V_{1f} , V_{2f} are one dimensional vectors! Special case: Head-on Elastic Collisions.

– Momentum is conserved (ALWAYS!)
 $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$
 v_{1i} , v_{2i} , v_{1f} , v_{2f} are one dimensional vectors!

– Kinetic Energy is conserved (ELASTIC! $(KE)_{before} = (KE)_{after}$ $(\frac{1}{2})m_1(v_{1i})^2 + (\frac{1}{2})m_2(v_{2i})^2 = (\frac{1}{2})m_1(v_{1f})^2 + (\frac{1}{2})m_2(v_{2i})^2$ (*XAYS!*)

(*M*₂V_{2f}

onal vectors!

(*LASTIC!*)

fter

(v_{1f})² + (¹/₂)</sub> m_2 (v_{2f})²

i, v{1f} , v_{2f} , m_1 , m_2 $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$
 v_{1i} , v_{2i} , v_{1f} , v_{2f} are one dimensional vectors!

- Kinetic Energy is conserved (*ELASTIC*!)

(**KE**)_{before} = (**KE**)_{after}
 $\sum_{i=1}^{n_1} (v_{1i})^2 + (\sum_{i=1}^{n_2} m_2(v_{2i$ -2 equations, 6 quantities: v_{1i} , v_{2i} , v_{1f} , v_{2f} , m_1 , m_2 \Rightarrow Clearly, must be given 4 out of 6 to solve problems! Solve with CAREFUL algebra!!

$$
m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}
$$
 (1)

 $(\frac{1}{2})m_1(v_{1i})^2 + (\frac{1}{2})m_2(v_{2i})^2 = (\frac{1}{2})m_1(v_{1f})^2 + (\frac{1}{2})m_2(v_{2f})^2$ (2)

- $(\mathbf{w}_{1f})^2 + (\frac{1}{2}) \mathbf{m}_2 (\mathbf{v}_{2f})^2$ (1)

(v_{1f})² + (½) $\mathbf{m}_2 (\mathbf{v}_{2f})^2$ (2)

(2), the results of $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ (1)
 $(\frac{1}{2})m_1(v_{1i})^2 + (\frac{1}{2})m_2(v_{2i})^2 = (\frac{1}{2})m_1(v_{1f})^2 + (\frac{1}{2})m_2(v_{2f})^2$ (2)

• Now, some algebra with (1) & (2), the results of

which will help to simplify problem solving: which will help to simplify problem solving: $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2i}$
 $m_1(v_{1i})^2 + (\frac{1}{2})m_2(v_{2i})^2 = (\frac{1}{2})m_1(v_{1f})^2$

Now, some algebra with (1) & (2),

which **will help to simplify proble**

- Rewrite (1) as: $m_1(v_{1i} - v_{1f}) = m$

- Rewrite (2) as: = $m_1v_{1f} + m_2v_{2f}$ (1)
 $v = (1/2)m_1(v_{1f})^2 + (1/2)m_2(v_{2f})^2$ (2)

vith (1) & (2), the results of

mplify problem solving:

(v_{1i} - v_{1f}) = m₂(v_{2f} - v_{2i}) (a) 2f (1)

+ (1/2) m₂ (v_{2f})² (2)

the results of

m solving:

(v_{2f} - v_{2i}) (a) $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2i} = (1/2)m_1(v_1)$

Now, some algebra with (1) & (

which will help to simplify pro

- Rewrite (1) as: $m_1(v_{1i} - v_{1f}) =$

- Rewrite (2) as:
 $m_1[(v_{1i})^2 - (v_{1f})^2] = m_2$
	-
	-

 $m_1[(v_{1i})^2 - (v_{1f})^2] = m_2[(v_{2f})^2 - (v_{2i})^2]$ (b) $w_{2i} = m_1v_{1f} + m_2v_{2f}$
 w_{2i})² = (1/2)m₁(v_{1f})² + (1/2) m₂ (v_{2f})²

ca with (1) & (2), the results of
 p simplify problem solving:

m₁(v_{1i} - v_{1f}) = m₂(v_{2f} - v_{2i}) (

- (v_{1f})²] = m₂[(v (1)
 $\frac{1}{2}$) m₂ (v_{2f})² (2)

results of
 solving:
 $\frac{1}{2}$ r – v_{2i}) (a)

- (v_{2i})²] (b) $m_1(v_{1i})^2 + (\frac{1}{2})m_2(v_{2i})^2 = (\frac{1}{2})m_1(v_{1i})^2$

Now, some algebra with (1) & (

which **will help to simplify pro**

- Rewrite (1) as: $m_1(v_{1i} - v_{1f}) =$

- Rewrite (2) as:
 $m_1[(v_{1i})^2 - (v_{1f})^2] = m_2$

- Divide (b) by (a)

which will help to simplify problem solving:
\n– Rewrite (1) as:
$$
m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})
$$
 (a)
\n– Rewrite (2) as:
\n $m_1[(v_{1i})^2 - (v_{1f})^2] = m_2[(v_{2f})^2 - (v_{2i})^2]$ (b)
\n– Divide (b) by (a):
\n $\Rightarrow v_1 + v_{1f} = v_2 + v_{2f}$ or
\n $v_1 - v_2 = v_{2f} - v_{1f} = -(v_{1f} - v_{2f})$ (3)
\nRelative velocity before=- Relative velocity after
\nElastic head-on (1d) collisions only!!

Elastic head-on (1d) collisions only!!

• Summary: 1d Elastic collisions: Rather than directly use momentum conservation + KE conservation, often convenient to use: v use momentum conservation + KE
vation, often convenient to use:
 ntum conservation:
 $\mathbf{v}_{1i} + \mathbf{m}_2 \mathbf{v}_{2i} = \mathbf{m}_1 \mathbf{v}_{1f} + \mathbf{m}_2 \mathbf{v}_{2f}$ (1)

with:
 $-\mathbf{v}_2 = \mathbf{v}_{2f} - \mathbf{v}_{1f} = -(\mathbf{v}_{1f} - \mathbf{v}_{2f})$ (3)

3

Momentum conservation:

$$
\rightarrow \quad m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad (1)
$$

use
these! along with:

$$
\rightarrow \quad \mathbf{v}_1 - \mathbf{v}_2 = \mathbf{v}_{2f} - \mathbf{v}_{1f} = -(\mathbf{v}_{1f} - \mathbf{v}_{2f}) \tag{3}
$$

• (1) & (3) are equivalent to momentum conservation $+$ KE conservation, since (3) was derived from these conservation laws!

Example : Pool (Billiards)
\nBefore:
$$
(m) \rightarrow v
$$

\n $m_1 = m_2 = m$, $v_{1i} = v$, $v_{2i} = 0$, $v_{1f} = ?$, $v_{2f} = ?$
\n**Monentum Conservation:** $mv + m(0) = mv_{1f} + mv_{2f}$
\nMasses cancel $\Rightarrow v = v_{1f} + v_{2f}$ (I)
\n**Relative velocity results for elastic head on collision:**
\n $v - 0 = v_{2f} - v_{1f}$ (II)
\nSolve (I) & (II) simultaneously for $v_{1f} \& v_{2f}$:
\n $\Rightarrow v_{1f} = 0$, $v_{2f} = v$
\nBall 1: to rest. Ball 2 moves with original velocity of ball 1

Before: Ball 1 $(m) \rightarrow v$ $\mathbf{v} = \mathbf{0}$