Chapter 8: Conservation of Energy

PHY0101/PHY(PEN)101

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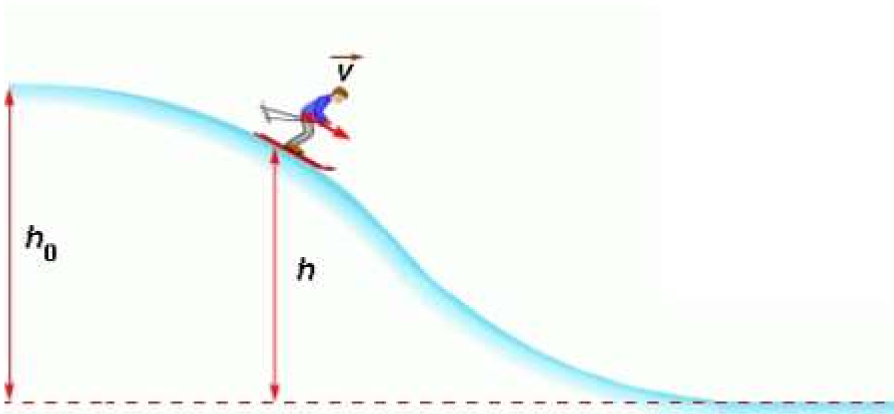


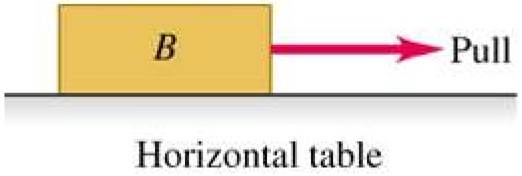
Outline

8.1 Nonisolated System (Energy)
8.2 Isolated System (Energy)
8.3 Situations Involving Kinetic Friction
8.4 Changes in Mechanical Energy for Nonconservative Forces
8.5 Power

8.1 The Nonisolated System (Energy)

- Energy crosses through the *boundary* of the system during some time due to interaction with the environment.
- Work is a method of transferring energy to a system by applying force to the system. The application of force results in displacement
- Mechanical waves are a means of transferring energy by allowing a disturbance to propagate through air or another medium. Exp. sound waves, seismic waves and ocean waves.
- Heat is a mechanism of energy transfer that is driven by a temperature difference between a system and its environment.
- Matter transfer involves situations in which matter physically crosses the boundary of a system, carrying energy with it. Exp. Convection
- Electrical transmission involves energy transfer into or out of a system by means of electric currents.
- Electromagnetic radiation refers to electromagnetic waves such as light, microwaves and radio waves crossing the boundary of a system

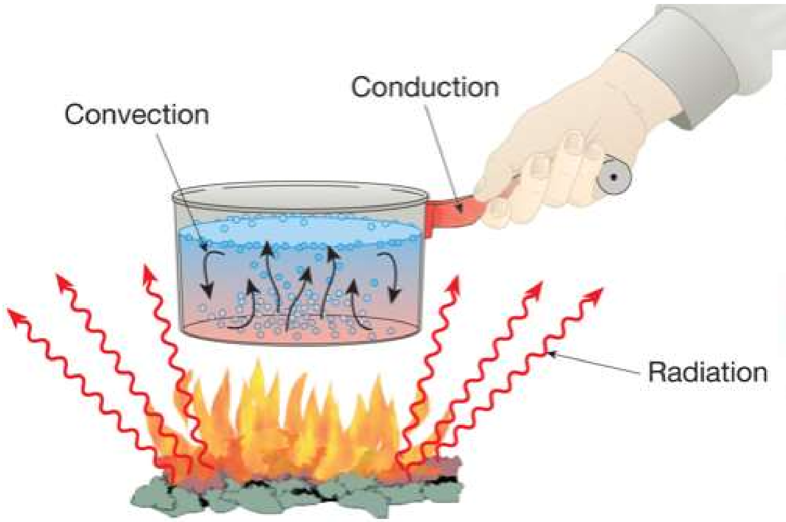
Energy Transfer Mechanisms



Work

Mechanical waves (sound)

Electrical transmission



Heat

Matter transfer

Electromagnetic radiation

We can neither create nor destroy energy—energy is always conserved. Thus, if the total amount of energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by a transfer mechanism. This is a general statement of the principle of conservation of energy.

$$\Delta E_{system} = \sum T$$

E_{system} is the total energy of the system, including all methods of energy storage (kinetic, potential, and internal), and T (for *transfer*) is the amount of energy transferred across the system boundary by some mechanism.

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$$\Delta K + \Delta U + \Delta E_{internal} = W + Q + T_{MW} + T_{MT} + T_{ET} + T_{ER}$$

- Work-kinetic energy theorem is a special case of the more general case principle of conservation of energy.

8.2 Isolated System (Energy)

- If for a system all terms at the right side of the conversation of energy equation are zero, the system is an isolated system.
- No energy crosses the boundary by any method.
- For example, after we have lifted the book, there is gravitational potential energy stored in the system , which can be calculated from the work done by external agent on the system, using W= ΔU_g

$$W_{on\ book} = mg.\Delta r = -mg\hat{j}.(y_f - y_i)\hat{j} = mgy_i - mgy_f$$

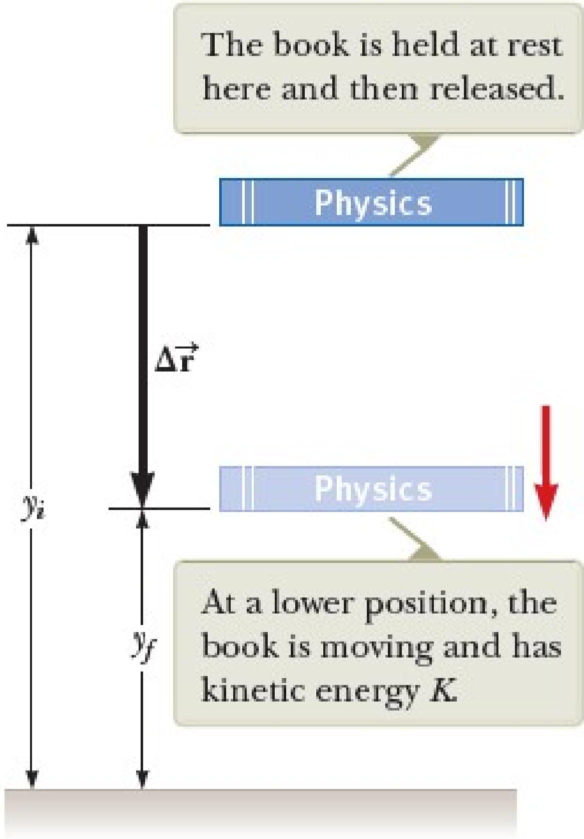
$$W_{on\ book} = \Delta K_{book}$$

$$\Delta K_{book} = mgy_i - mgy_f$$

$$mgy_i - mgy_f = -(mgy_f - mgy_i) = -\Delta U_g$$

$$\Delta K = -\Delta U_g$$

$$\Delta K + \Delta U_g = 0$$



The left side represents a sum of changes of the energy stored in the system. The right-hand side is zero because there are no transfers of energy across the boundary of the system; the book–Earth system is *isolated* from the environment. We developed this equation for a gravitational system, but it can be shown to be valid for a system with any type of potential energy. Therefore, for an isolated system,

$$\Delta K + \Delta U_g = 0$$

$$\Delta E_{mechanical} = 0$$

This equation states for conservation of mechanical system for an isolated system with no nonconservative forces acting.

$$K_f - K_i + U_f - U_i = 0$$

$$K_f - K_i + U_f - U_i = 0$$

For the gravitational energy of falling book,

$$K_f - K_i + U_f - U_i = 0$$

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

$$E_{total,i} = E_{total,final}$$

If nonconservative forces act on a system, mechanical energy is not conserved but total energy of the system is always conserved.

$$\Delta E_{\text{system}} = 0$$

Example 8.1 Ball in Free Fall

A ball of mass m is dropped from a height h above the ground.
(A) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground. Choose the system as the ball and the Earth.
(B) Find the speed of the ball again at height y by choosing the ball as the system.

(A) $\Delta K + \Delta U_g = 0$

$$\left(\tfrac{1}{2}mv_f^2 - 0\right) + (mgy - mgh) = 0$$

$$v_f^2 = 2g(h - y) \quad \rightarrow \quad v_f = \sqrt{2g(h - y)}$$

$$\Delta K = W$$

(B) $\left(\tfrac{1}{2}mv_f^2 - 0\right) = \vec{\mathbf{F}}_g \cdot \Delta\vec{\mathbf{r}} = -mg\hat{\mathbf{j}} \cdot \Delta y\hat{\mathbf{j}}$

$$= -mg\Delta y = -mg(y - h) = mg(h - y)$$

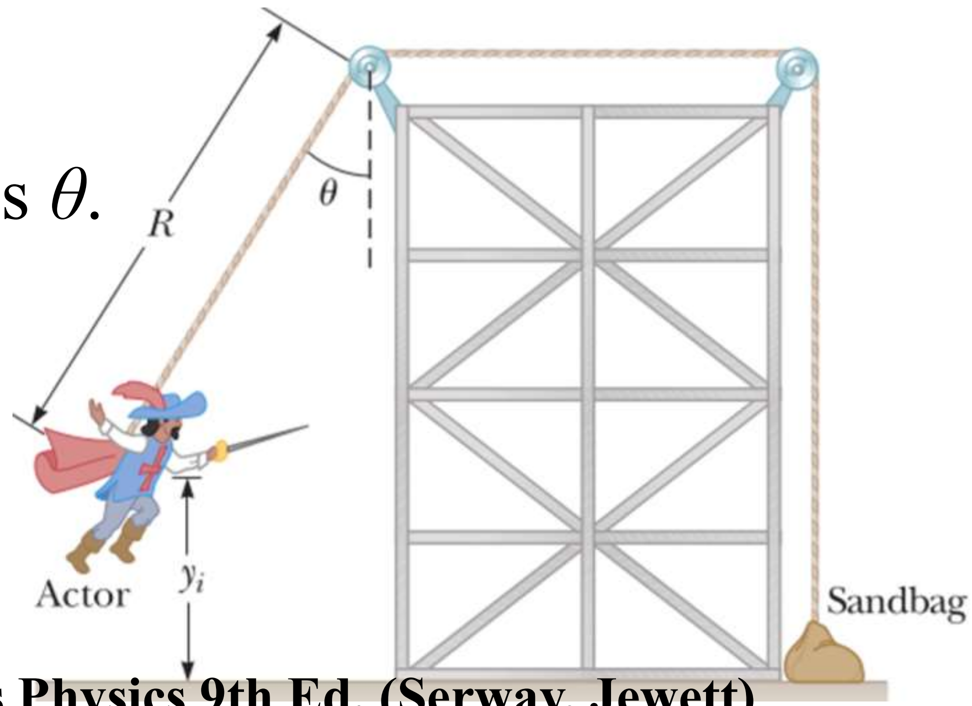
$$v_f^2 = 2g(h - y) \quad \rightarrow \quad v_f = \sqrt{2g(h - y)}$$

Example 8.2 A Grand Entrance

Mass of an actor is 65 kg. Sandbag is 130 kg. Two pulleys are frictionless. Cable between the harness and the nearest pulley is 3.0 m so that the pulley can be hidden behind a curtain.

For the apparatus to work successfully, the sandbag must never lift above the floor as the actor swings from above the stage to the floor. Initial angle that the actor's cable makes with the vertical is θ.

What is the maximum value θ can have before the sandbag lifts off the floor?



$$\Delta K + \Delta U_g = 0$$

$$(1)\quad \left(\tfrac{1}{2}m_{\text{actor}}v_f^2 - 0\right) + (0 - m_{\text{actor}}gy_i) = 0$$

$$(2)\quad v_f^2 = 2gR(1 - \cos\theta)$$

$$\sum F_y = T - m_{\text{actor}}g = m_{\text{actor}}\frac{v_f^2}{R}$$

$$(3)\quad T = m_{\text{actor}}g + m_{\text{actor}}\frac{v_f^2}{R}$$

$$m_{\text{bag}}g = m_{\text{actor}}g + m_{\text{actor}}\frac{2gR(1 - \cos\theta)}{R}$$

$$\cos\theta = \frac{3m_{\text{actor}} - m_{\text{bag}}}{2m_{\text{actor}}} = \frac{3(65.0\text{ kg}) - 130\text{ kg}}{2(65.0\text{ kg})} = 0.500$$

$$\theta = 60.0^\circ$$

Example 8.3 Spring Loaded Cork Gun

- Ball, mass **m = 35 g = 0.035 kg** in popgun is shot straight up with spring of unknown constant **k**. Spring is compressed $y_A = -0.12$ m, below relaxed level, $y_B = 0$. Ball gets to a max height $y_C = 20.0$ m above relaxed end of spring.

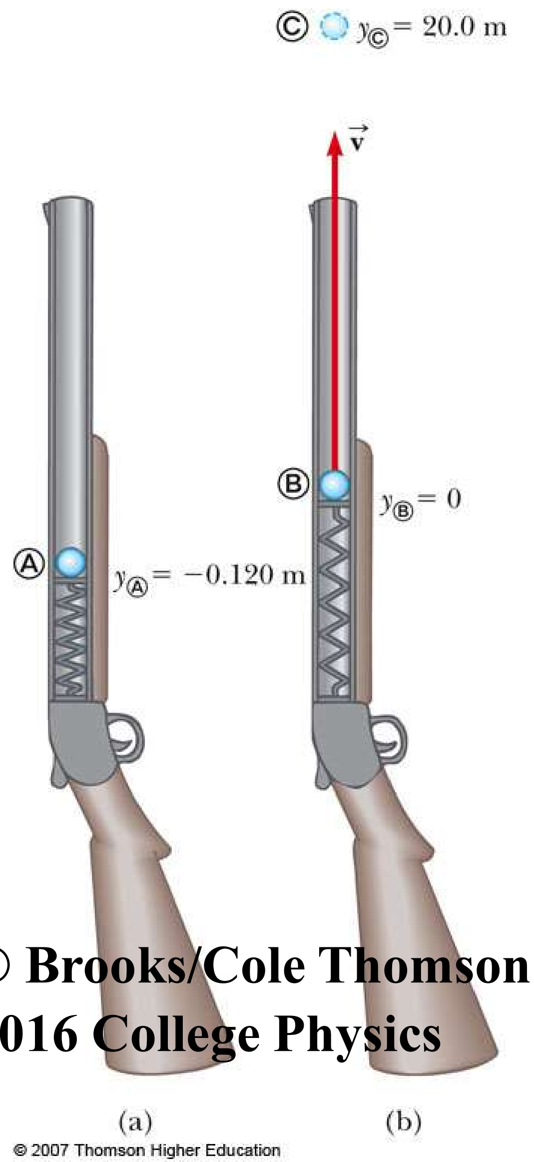
  (A) If no friction, find spring constant **k**.

  (B) Find speed of ball at point **B**.
- Ball starts from rest. Speeds up as spring pushes against it. As it leaves gun, gravity slows it down. **System = ball, gun, Earth**.
- Conservative forces are acting, so use

Conservation of Mechanical Energy

Initial kinetic energy **K = 0.** Choose gravitational potential energy $U_g = 0$ where ball leaves gun. Also elastic potential energy $U_e = 0$ there. At max height, again have **K = 0.** Choose **Note:**

Need two types of potential energy!



- For entire trip of ball,

Mechanical Energy is Conserved!!

or: $K_A + U_A = K_B + U_B = K_C + U_C$.

At each point, $U = U_g + U_e$ so,

$$K_A + U_{gA} + U_{eA} = K_B + U_{gB} + U_{eB} = K_C + U_{gC} + U_{eC}$$

(A) To find spring constant **k**, use:

$$K_A + U_{gA} + U_{eA} = K_C + U_{gC} + U_{eC}$$

or, $0 + mgy_A + (½)k(y_A)^2 = 0 + mgy_C + 0$, giving

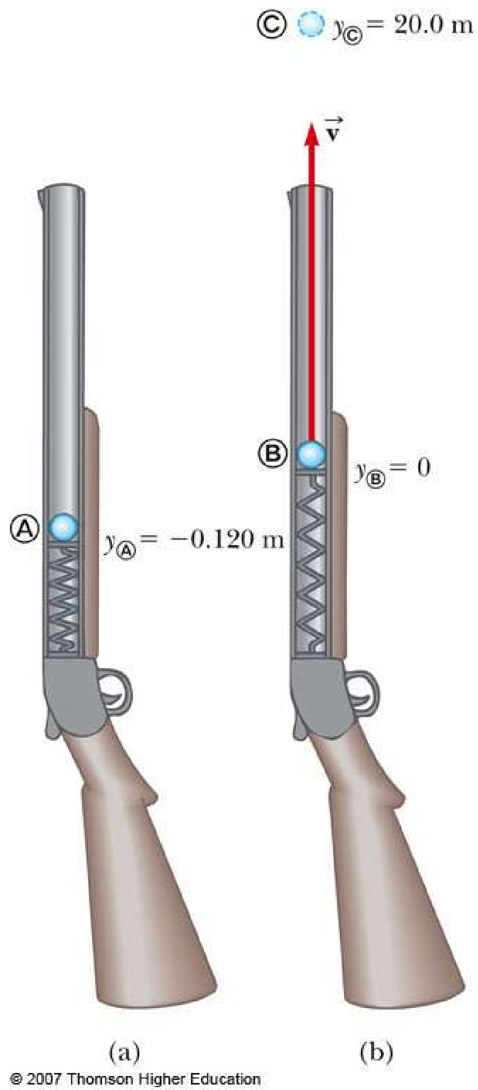
$$k = [2mg(y_C - y_A)/(y_A)^2] = 958\ N/m$$

(B) To find ball's speed at point **B**, use:

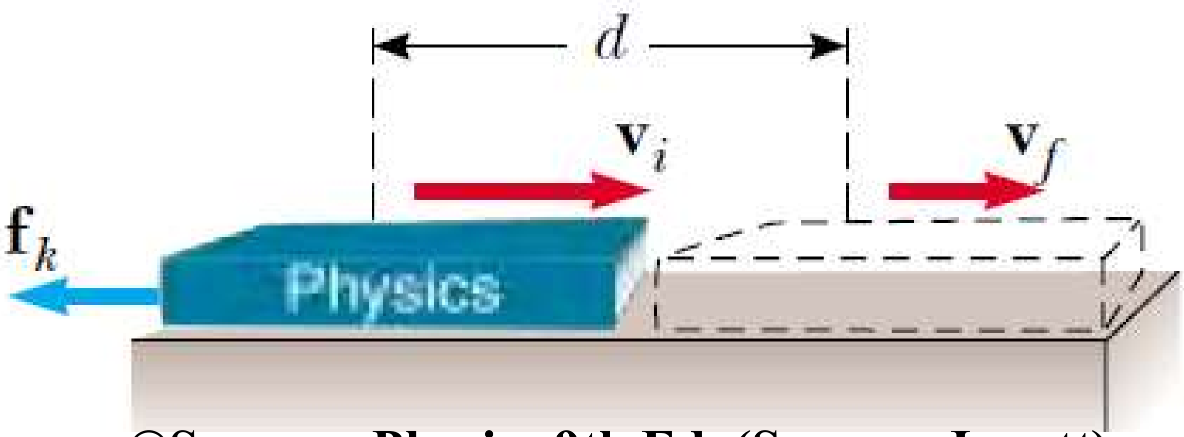
$$K_A + U_{gA} + U_{eA} = K_B + U_{gB} + U_{eB}$$

or, $(½)m(v_B)^2 + 0 + 0 = 0 + mgy_A + (½)k(y_A)^2$, giving

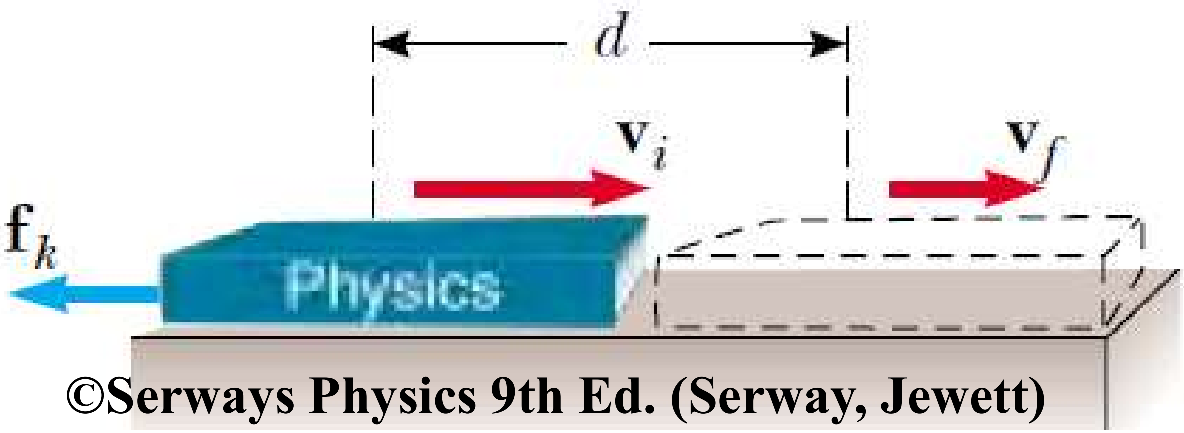
$(v_B)^2 = [k(y_A)^2/m] + 2gy_A$; or, $(v_B) = 19.8\ m/s$



8.3 Situations Involving Kinetic Friction



From your everyday experience with sliding over surfaces with friction, you can probably guess that the surface will be *warmer* after the book slides over it. Thus, the work that was done on the surface has gone into warming the surface rather than increasing its speed. We call the energy associated with an object's temperature its internal energy, symbolized E_{int}.



$$\sum W_{\text{other forces}} + \int \vec{\mathbf{f}}_k \cdot d\vec{\mathbf{r}} = \int \left(\sum \vec{\mathbf{F}}_{\text{other forces}}\right) \cdot d\vec{\mathbf{r}} + \int \vec{\mathbf{f}}_k \cdot d\vec{\mathbf{r}}$$

$$= \int \left(\sum \vec{\mathbf{F}}_{\text{other forces}} + \vec{\mathbf{f}}_k\right) \cdot d\vec{\mathbf{r}}$$

$$\sum W_{\text{other forces}} + \int \vec{\mathbf{f}}_k \cdot d\vec{\mathbf{r}} = \int \sum \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

$$\sum W_{\text{other forces}} + \int \vec{\mathbf{f}}_{\text{k}} \cdot d\vec{\mathbf{r}} = \int m\vec{\mathbf{a}} \cdot d\vec{\mathbf{r}} = \int m \frac{d\vec{\mathbf{v}}}{dt} \cdot d\vec{\mathbf{r}} = \int_{t_i}^{t_f} m \frac{d\vec{\mathbf{v}}}{dt} \cdot \vec{\mathbf{v}}\, dt$$

$$\frac{d}{dt}(\vec{\mathbf{v}}\cdot\vec{\mathbf{v}}) = \frac{d\vec{\mathbf{v}}}{dt}\cdot\vec{\mathbf{v}} + \vec{\mathbf{v}}\cdot\frac{d\vec{\mathbf{v}}}{dt} = 2\,\frac{d\vec{\mathbf{v}}}{dt}\cdot\vec{\mathbf{v}}$$

$$\frac{d\vec{\mathbf{v}}}{dt}\cdot\vec{\mathbf{v}} = \tfrac{1}{2}\,\frac{d}{dt}(\vec{\mathbf{v}}\cdot\vec{\mathbf{v}}) = \tfrac{1}{2}\,\frac{dv^2}{dt}$$

$$\sum W_{\text{other forces}} + \int \vec{\mathbf{f}}_k \cdot d\vec{\mathbf{r}} = \int_{t_i}^{t_f} m\,\frac{1}{2}\,\frac{dv^2}{dt}\,dt = \tfrac{1}{2}m\int_{v_i}^{v_f} d(v^2) = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \Delta K$$

$$\sum W_{\text{other forces}} - f_k d = \Delta K$$

The change in kinetic energy is equal to the work done by all forces other than friction minus a term $f_k d$ associated with the friction force.

$$\Delta E_{\text{system}} = \Delta K + \Delta E_{\text{int}} = 0$$

$$-f_k d + \Delta E_{\text{int}} = 0$$

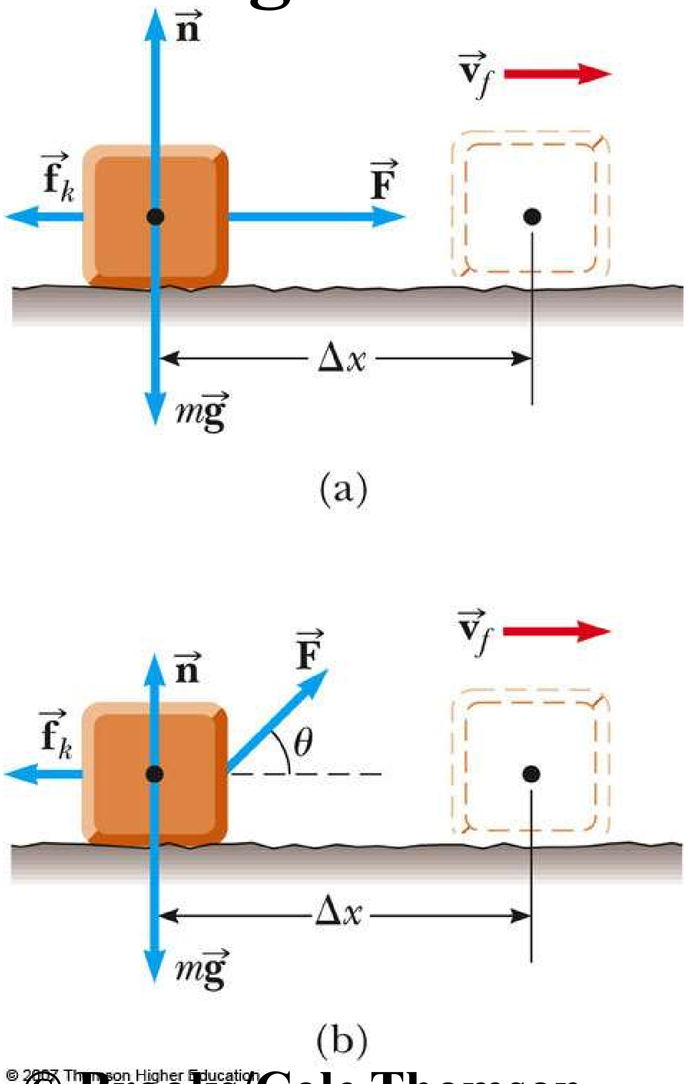
$$\Delta E_{\text{int}} = f_k d$$

The increase in internal energy of the system is equal to the product of the friction force and the path length through which the block moves.

$$\sum W_{\text{other forces}} = W = \Delta K + \Delta E_{\text{int}}$$

Example 8.4 Block Pulled on Rough Surface

A block, mass **m = 6 kg**, is pulled by constant horizontal force **F = 12 N**. over a rough horizontal surface. Kinetic friction coefficient **μ_k = 0.15**. Moves a distance **Δx = 3 m**. Find the final speed.



$$\sum W_{\text{other forces}} = W_F = F\,\Delta x$$

$$\sum F_y = 0 \;\rightarrow\; n - mg = 0 \;\rightarrow\; n = mg$$

$$f_k = \mu_k n = \mu_k mg = (0.15)(6.0\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$$

$$F\Delta x = \Delta K + \Delta E_{\text{int}} = \left(\tfrac{1}{2}mv_f^2 - 0\right) + f_k d$$

$$v_f = \sqrt{\frac{2}{m}(-f_k d + F\,\Delta x)}$$

$$v_f = \sqrt{\frac{2}{6.0\text{ kg}}[-(8.82\text{ N})(3.0\text{ m}) + (12\text{ N})(3.0\text{ m})]} = \boxed{1.8\text{ m/s}}$$

Suppose the force $\vec{F}$ is applied at an angle θ as shown. At what angle should the force be applied to achieve the largest possible speed after the block has moved 3.0 m to the right?

$$\sum W_{\text{other forces}} = W_F = F\,\Delta x \cos\theta = Fd\cos\theta$$

$$\sum F_y = n + F\sin\theta - mg = 0$$

$$n = mg - F\sin\theta$$

$$W_F = \Delta K + \Delta E_{\text{int}} = (K_f - 0) + f_k d \rightarrow K_f = W_F - f_k d$$

$$K_f = Fd\cos\theta - \mu_k nd = Fd\cos\theta - \mu_k(mg - F\sin\theta)d$$

$$\frac{dK_f}{d\theta} = -Fd\sin\theta - \mu_k(0 - F\cos\theta)d = 0$$

$$-\sin\theta + \mu_k\cos\theta = 0$$

$$\tan\theta = \mu_k$$

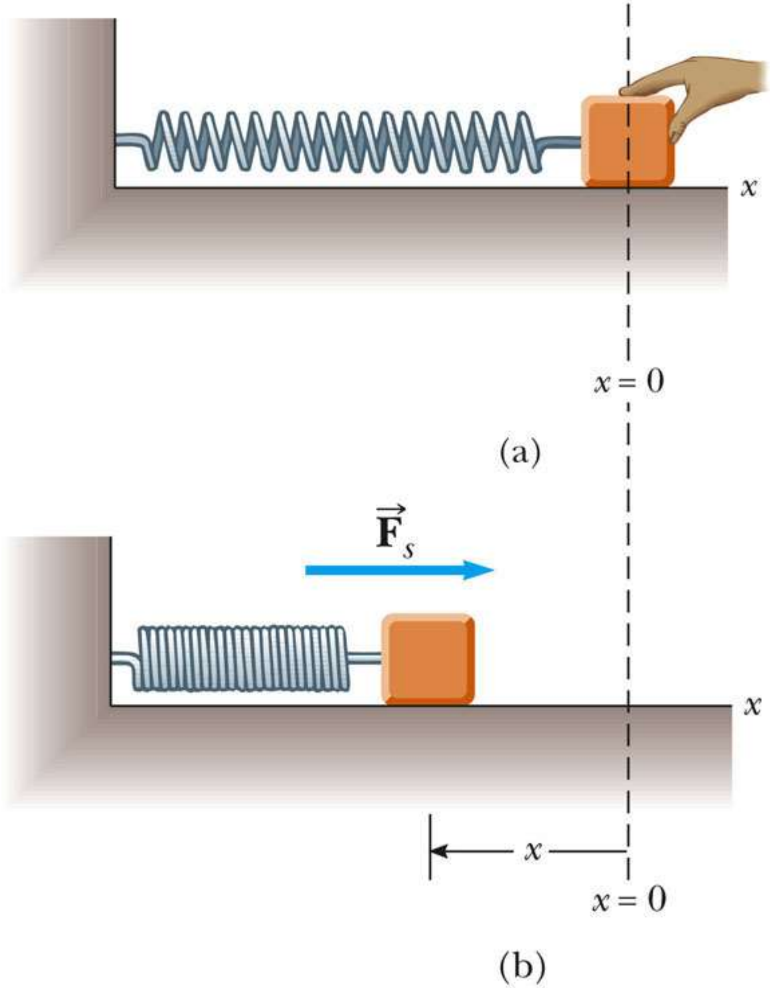
$$\theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.15) = 8.5°$$

Example 8.6 Block – Spring System

A mass **m = 1.6 kg**, is attached to ideal spring of constant **k = 1,000 N/m**. Spring is compressed **x = - 2.0 cm = - 2 × 10⁻² m** & is released from rest.

(a) Find the speed at **x = 0** if there is no friction.

(b) Find the speed at **x = 0** if there is a constant friction force **f_k = 4 N**.



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In this situation, the block starts with v_i=5 0 at x_i=5 22.0 cm, and we want to find v_f at x_f

$$W_s = \frac{1}{2}kx_{\max}^2$$

$$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}W_s} = \sqrt{v_i^2 + \frac{2}{m}\left(\frac{1}{2}kx_{\max}^2\right)}$$

$$v_f = \sqrt{0 + \frac{2}{1.6\text{ kg}}\left[\frac{1}{2}(1\,000\text{ N/m})(0.020\text{ m})^2\right]} = 0.50\text{ m/s}$$

$$W_s = \Delta K + \Delta E_{\text{int}} = \left(\frac{1}{2}mv_f^2 - 0\right) + f_k d$$

$$v_f = \sqrt{\frac{2}{m}(W_s - f_k d)} \qquad v_f = \sqrt{\frac{2}{m}\left(\frac{1}{2}kx_{\max}^2 - f_k d\right)}$$

$$v_f = \sqrt{\frac{2}{1.6\text{ kg}}\left[\frac{1}{2}(1\,000\text{ N/m})(0.020\text{ m})^2 - (4.0\text{ N})(0.020\text{ m})\right]} = 0.39\text{ m/s}$$