Ankara University
Engineering Faculty
Department of Engineering Physics

PEN207

Circuit Design and Analysis

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Chapter-1

Electric Circuits and Circuit Elements (2/2)

Circuit Elements

Receivers or **absorbers** in a circuit called **circuit elements.** These elements dissipate (absorb) or store energy.

Depending on the relation between voltage and current we can define three different type of circuit elements as follows:

Type-1: In this type of circuit elements the **voltage** is **linearly depends** on the **current** passing through it. These type of circuit elements are called ***resistor***. Energy is dissipated as a heat energy and it is irreversible.

$$\mathrm{v} = R.i$$

Type-2: In this type of circuit elements the **voltage** is **depends** on the variation of **current** in time (derivatives) passing through it. Proportional constant is called ***inductance***. Energy is conserved and stores as ***magnetic field.***

$$\mathrm{v} = L\frac{di}{dt}$$

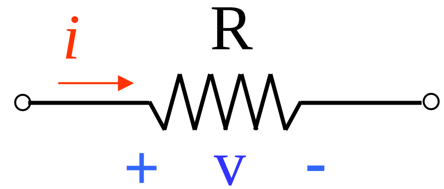
Type-3: In this type of circuit elements the **current** is **depends** on the variation of **voltage** in time (derivatives) accros it. Proportional constant is called ***capacitance***. Energy is conserved and stores as ***electric field.***

.

$$i = C\frac{d\mathrm{v}}{dt}$$

Ohm's Law: Resistance

If the current (i) passing through a circuit element is proportional with the potential difference (v) across is called **resistor**.



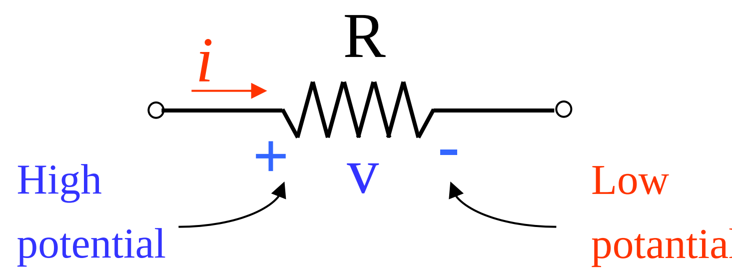
$$v = R.i$$

Resistance is the proportionality constant between current and voltage. The symbol **R** is used for resistance. Its unit in SI system is **ohm** (symbol Ω).

In general, linear relation between current and voltage is known as **Ohm's Law**

Symbol: R
Unit= Ohm (Ω) (1 Ω=1V/1A)
R=e/i => [ohm]=[volt] / [amper]
Circuit Symbol: 

When a current passes through a resistor there is voltage drop in the direction of current



The mechanical equivalent of the resistance is the frictional force. Resistance resists movement of electrical charges and the energy to overcome this resistance is lost as heat.

Power (p) loss on a resistor (R):

$$p = v.i = \left(R.i\right).i = Ri^2$$

In ters of current

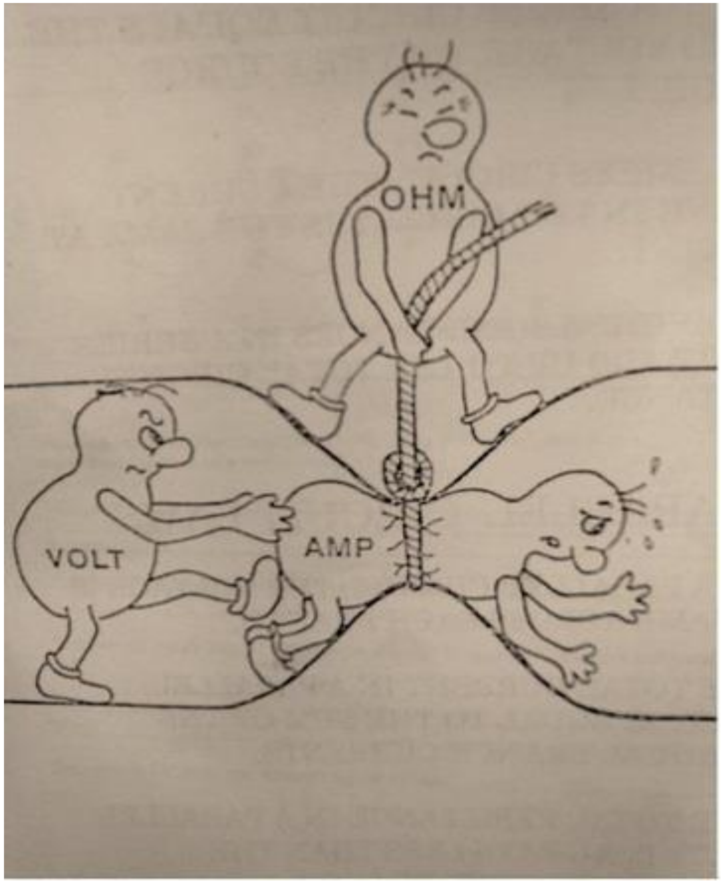
$$p = v.i = v\left(\frac{v}{R}\right) = \frac{v^2}{R}$$

In terms of voltage

Ohm's Law: Resistance

$$Voltage(V) = \mathrm{Re}\,sistor(R) \times Current(I)$$

$$V = R.I$$



Conductance, G

Sometime defining inverse of resistance can be practical to use.

DC Current-Voltage

$$V = IR$$
$$I = G.V$$

Ohm's Law in terms of Resistance (R): $\mathrm{v} = R.i$

Ohm's Law in terms of Conductance (G): $i = G.\mathrm{v}$

Here G, is the inverse of resistance and known as conductance

$$G \equiv \frac{1}{R}$$ Conductance

Unit of conductance is mho (1 mho=1/ohm)

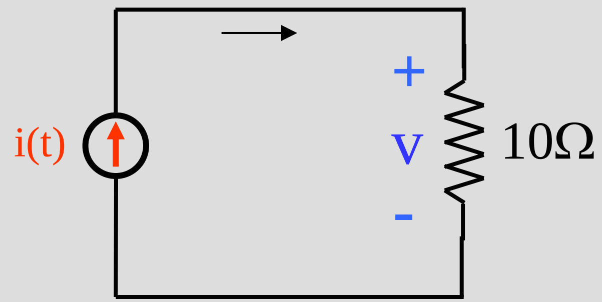
Power (p) loss on a resistor in terms of conductance (G):

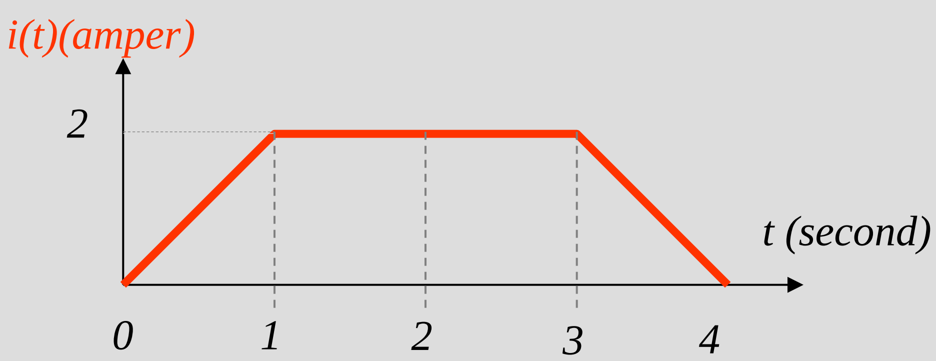
$$p = \mathrm{v}.i = \left(\frac{i}{G}.i\right).i = \frac{i^2}{G}$$ In ters of current

$$p = \mathrm{v}.i = \mathrm{v}.\left(G.\mathrm{v}\right) = G\mathrm{v}^2$$ In terms of voltage

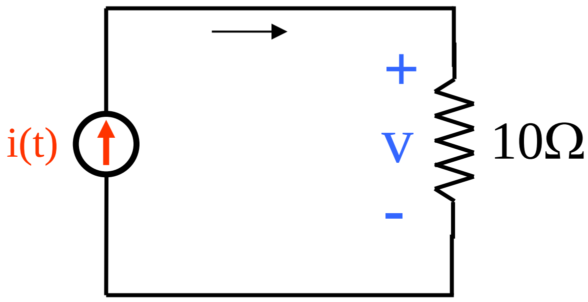
The unit of conductivity (G) is **mho** and has no special meaning. Since conductivity is the inverse of resistance, the unit of G is also written in reverse order of the unit of resistance, ohm.

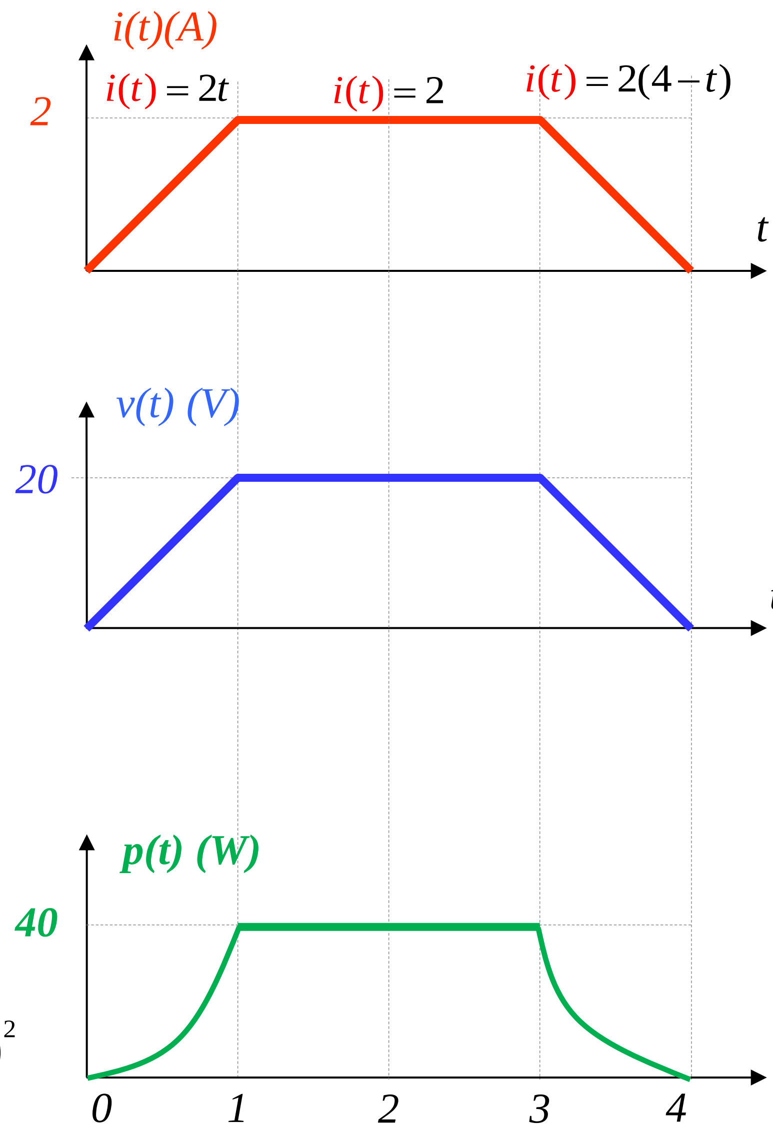
Example-1.0: The following circuit is excited by an ideal current source. The current curve as a function of time is given in the following figure. Draw the waveforms of the voltage v, the instantaneous power p on the resistor as a function of time.





Solution:





Voltage: $\mathrm{v} = R.i$

$$\mathrm{v}(t) = (10\Omega).i(t)$$

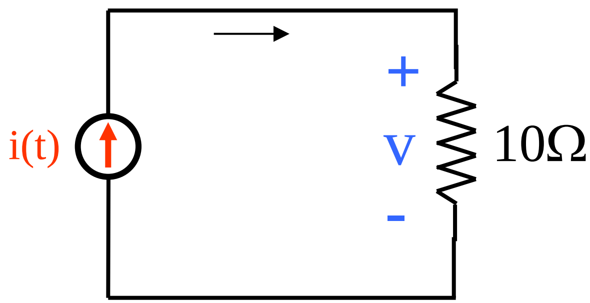
Power: $p = \mathrm{v}.i = \left(R.i\right).i = Ri^2$

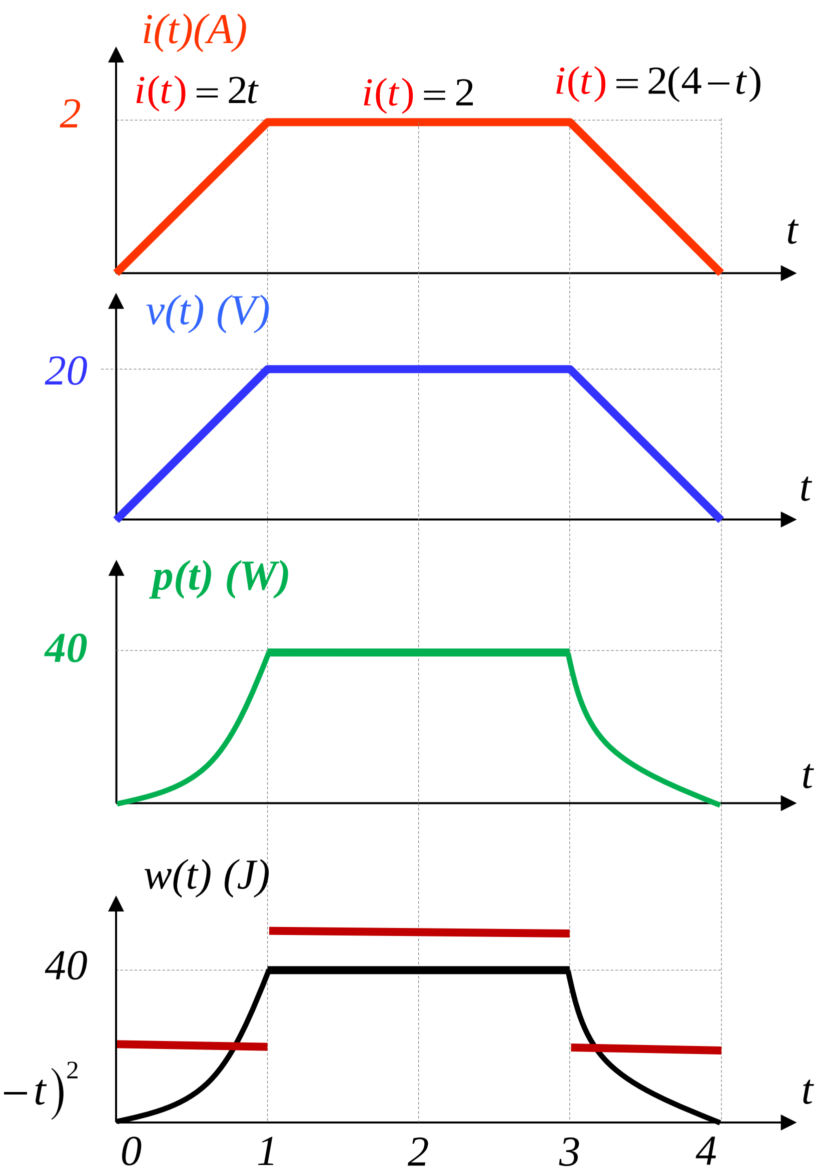
$$p = (10\Omega)i^2 = (40\Omega)t^2$$

$$p_1(t) = (10\Omega)(2t)^2 = (40\Omega)t^2$$

$p_1(t) = 40t^2$ $\quad p_2(t) = 40$ $\quad p_3(t) = 40\left(4-t\right)^2$

Solution: ***Check out this page!***





Voltage: $\mathrm{v} = R.i$

$$\mathrm{v}(t) = (10\Omega).i(t)$$

Power: $p = \mathrm{v}.i = \left(R.i\right).i = Ri^2$

$$p = (10\Omega)i^2 = (40\Omega)t^2$$

Energy? $w = \int_0^4 p(t)dt$

$$p_1(t) = (10\Omega)(2t)^2 = (40\Omega)t^2$$

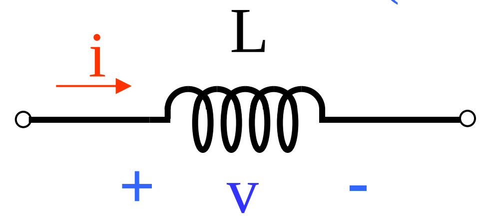
$$p_1(t) = 40t^2 \qquad p_2(t) = 40 \qquad p_3(t) = -\frac{1}{2}40\left(4-t\right)^2$$

$$w = \int_0^4 p(t)dt = \int_0^1 p(t)dt + \int_1^3 40dt + \int_3^4 p(t)dt$$

Inductance-1

The circuit element on which the voltage is directly proportional to the rate of change of the current passing through it is called inductor (Coil).

$$v = L\frac{di}{dt}$$



Inductance is the proportionality constant between change in current and voltage and it has symbol. Its unit is ***henry*** (Symbol **H**).

Inductance is a measure of resisting changes in current.

If we know voltage across an inductor the current passing through it is:

$$i = \frac{1}{L}\int v.dt \qquad i = \frac{1}{L}\int_{t=0}^{t=t} v.dt + i(0)$$

$i(0)$: Value of current before it changes

The effect of inductance is similar to the relationship between force and velocity (mass) in mechanics. Since F=dp/dt=m(dv/dt), in mechanical systems, mass is a measure of resistance to movement (making it difficult to accelerate of the stationary object and to stop the fast body). The inductance value L of the coil is likewise a measure of resisting the change in current.

Inductance-2

Inductance resists to increasing current and helps decreasing current.

$$\mathrm{v}(t) = L\frac{di(t)}{dt}$$

Power in inductance: $$p = \mathrm{v}.i = \left(L\frac{di}{dt}\right).i = iL\frac{di}{dt}$$

Energy: $$w = \int p\,dt = \int Li\frac{di}{dt}dt = \int Li\,di = \frac{1}{2}Li^2$$

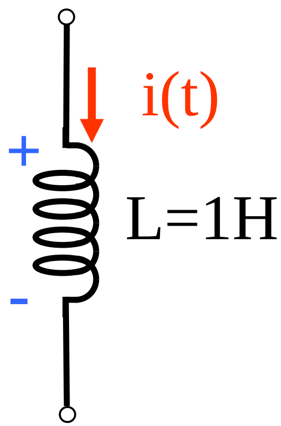
Inductive energy is conserved and reversible (energy in the resistor, however is non-conservative and lost as heat).

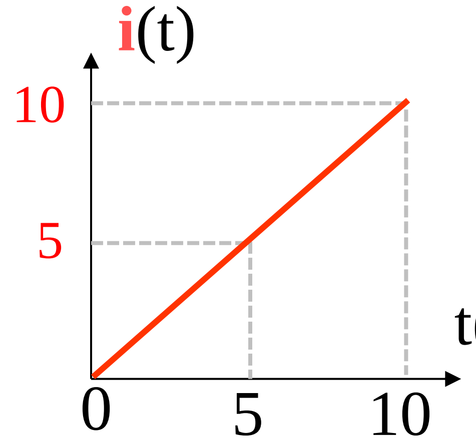
Inductance-3

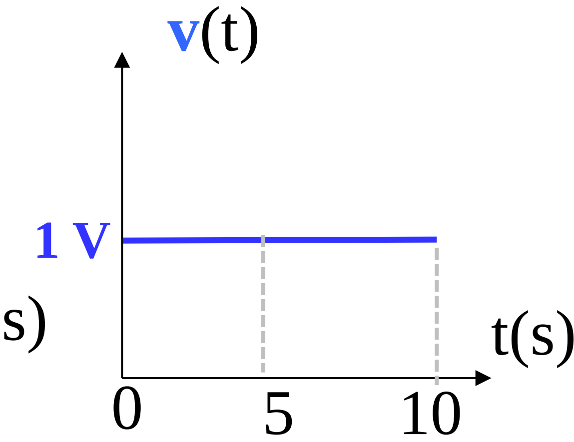
$$\mathrm{v}(t) = L\frac{di(t)}{dt} \qquad \mathrm{L=1\ H}$$

$i(t)=0\rightarrow 10A$

$i(t) = t$

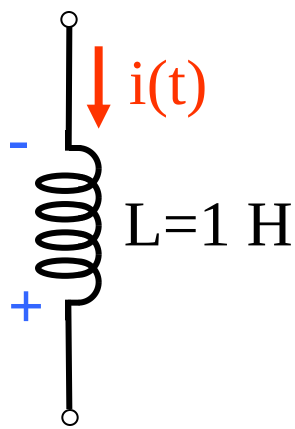


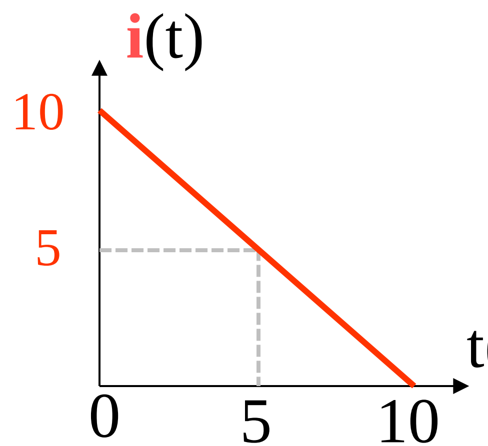


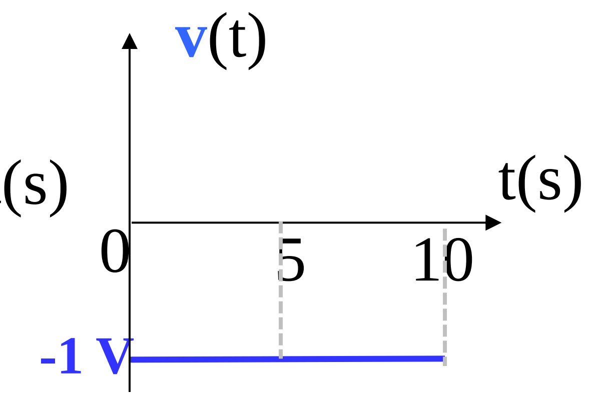


$i(t)=10\rightarrow 0A$

$i(t) = 10 - t$

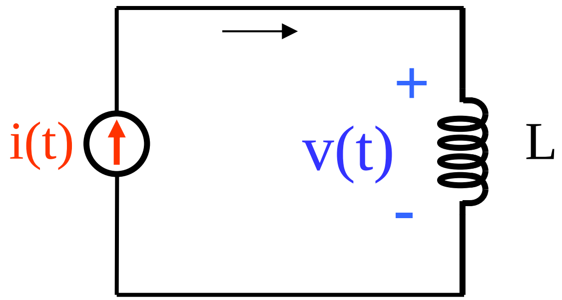


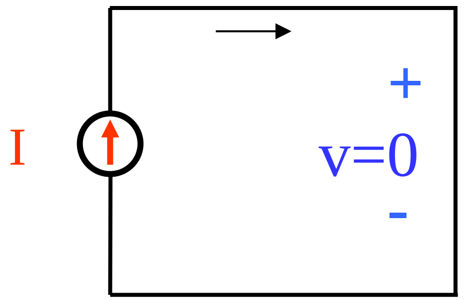




Important Note!

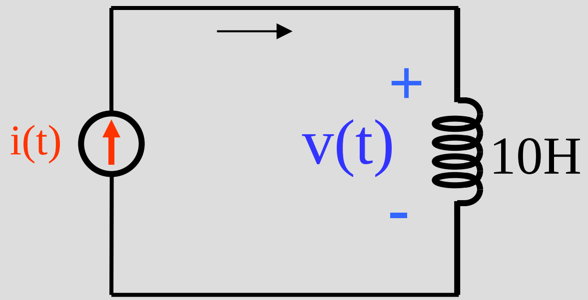
If the current is not an **alternative** or **time varying**, the voltage on the inductor is zero and the inductor acts as a **short circuit**.

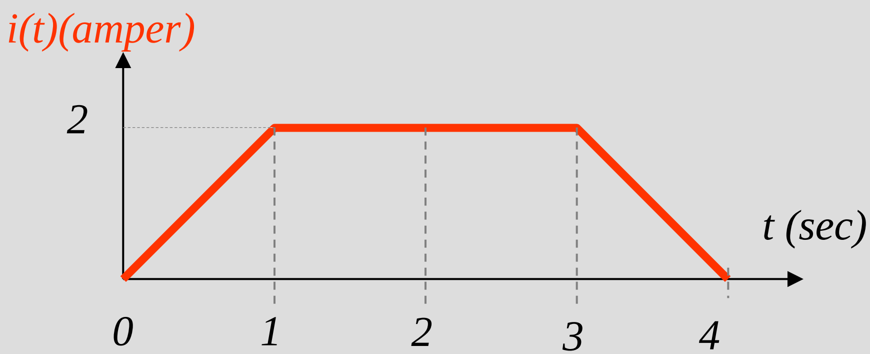




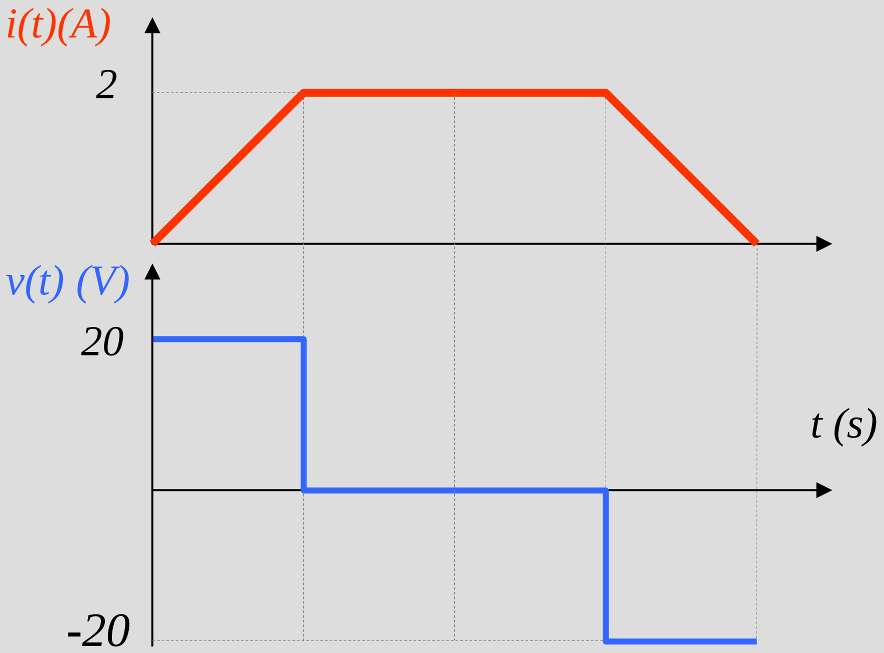
$$v = L\frac{di}{dt} \qquad i = sabit \Rightarrow \frac{di}{dt} = 0 \qquad v = L\frac{di}{dt} = 0$$

Example-1.3: An ideal current source suppleis energy ton the circuit below. The current curve as a function of time is given in the following figure. Plot the waveforms of voltage.





Solution



$$i(t) = \begin{cases} i(t) = 2t & 0 < t < 1 \\ i(t) = 2 & 1 < t < 3 \\ i(t) = 8 - 2t & 3 < t < 4 \end{cases}$$

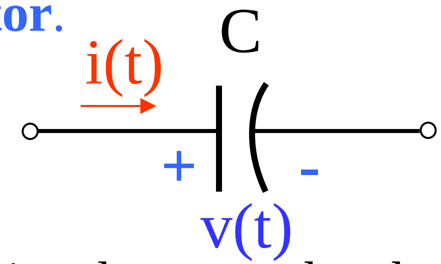
$$f'(t) = \frac{di(t)}{dt} = \begin{cases} f'(t) = 2 & 0 < t < 1 \\ f'(t) = 0 & 1 < t < 3 \\ f'(t) = -2 & 3 < t < 4 \end{cases}$$

$$v(t) = L\frac{di(t)}{dt} = Lf'(t) = \begin{cases} v(t) = 20 & 0 < t < 1 \\ v(t) = 0 & 1 < t < 3 \\ v(t) = -20 & 3 < t < 4 \end{cases}$$

Capacitance

Circuit element that is he current passing through it is proportional to the voltage change between its ends is called **Capacitor**.

$$i(t) = C\frac{d\mathrm{v}(t)}{dt}$$

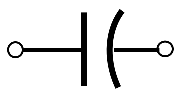


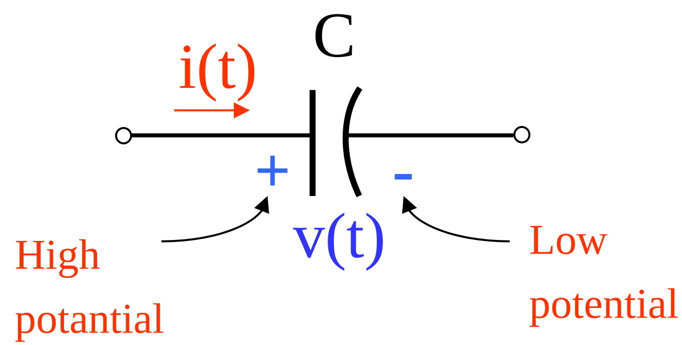
Capacitance is the proportionality coefficient between the change in voltage and the current, indicated by **C**. The unit of capacitance is **Farad** (symbol **F**).

Symbol: C

Unit= Farad (F)

[Farad]= [Ampere] / [Volt/s]

Circuit symbol: 



Voltage on capacitor if we know the current: $\mathrm{v}(t) = \frac{1}{C}\int i(t)dt$

In terms of charge: $\mathrm{v}(t) = \frac{1}{C}\left(\int i(t)dt\right) = \frac{q}{C} \Rightarrow q = C\mathrm{v}$

$q \equiv \int i(t)dt$

Capacitance-2

Power on capacitor:

$$p = \mathrm{v}.i = \mathrm{v}\left(C\frac{d\mathrm{v}}{dt}\right) = C\mathrm{v}\frac{d\mathrm{v}}{dt} \quad watt$$

Depo edilen enerji

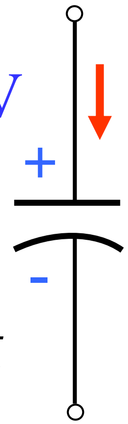
$$w = \int p\,dt = \int C\mathrm{v}\frac{d\mathrm{v}}{dt}dt = \int C\mathrm{v}\,d\mathrm{v} = \frac{1}{2}C\mathrm{v}^2 \quad joule$$

The energy on capacitor is similiar to the energy stored in spring. The value of this energy depends only on the magnitude of the voltage; it is independent of how it reaches this value.

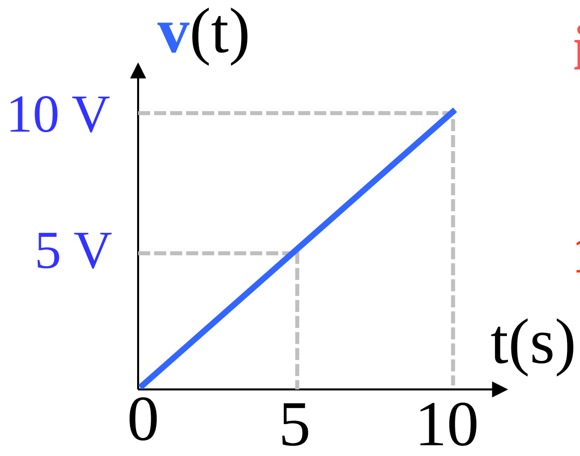
Capacitance-3

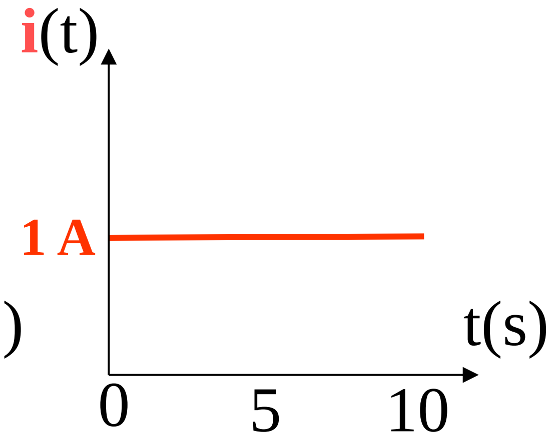
$$i(t) = C\frac{d\mathbf{v}(t)}{dt}$$

v(t)=0→10V

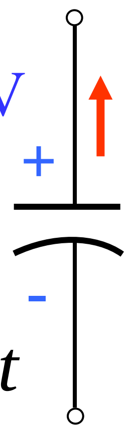


$$\mathbf{v}(t) = t$$

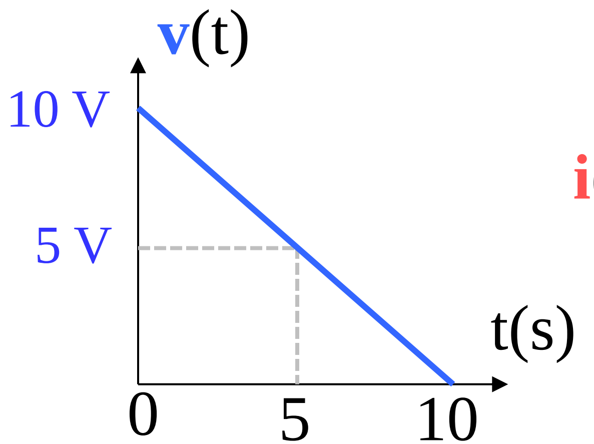


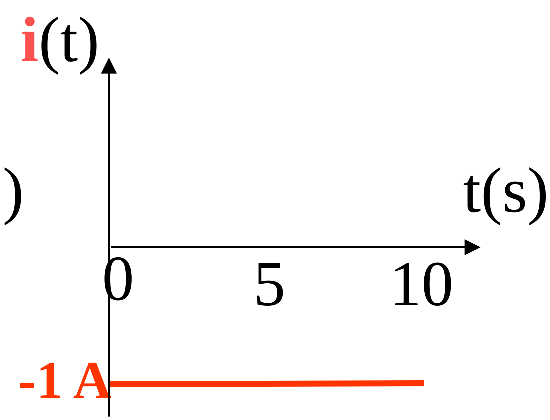


v(t)=10→0V



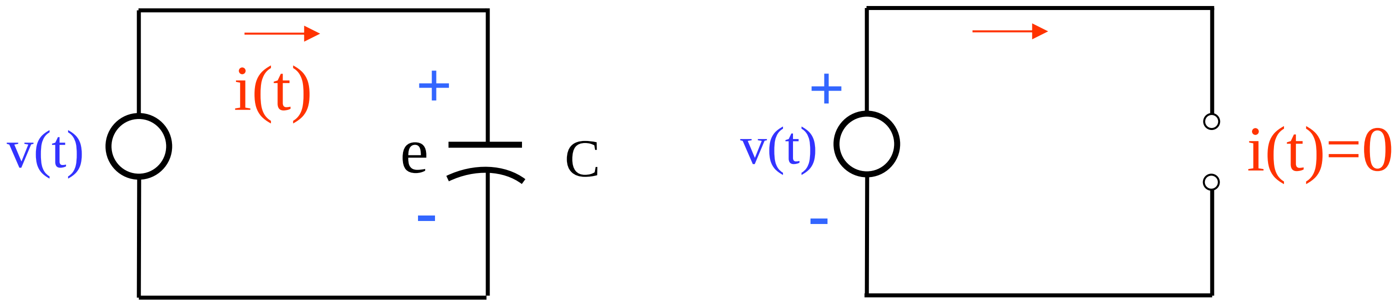
$$\mathbf{v}(t) = 10 - t$$





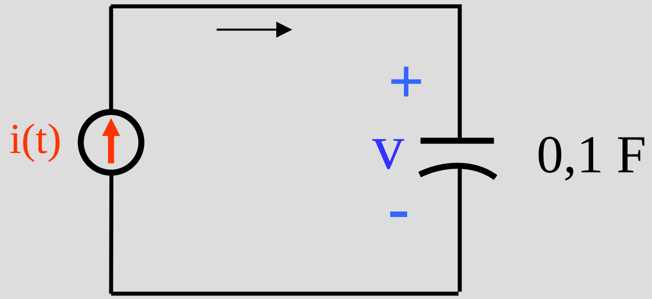
İmportant Note!

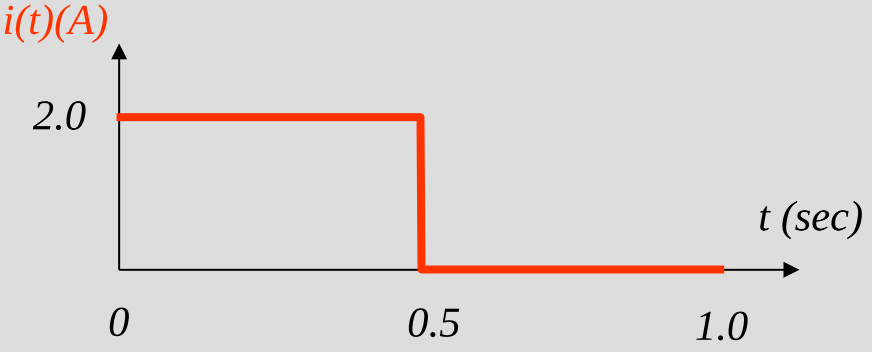
If the voltage is not an **alternative** or **time varying**, the current on the capacitor is zero and the capacitor acts as a **open circuit**.



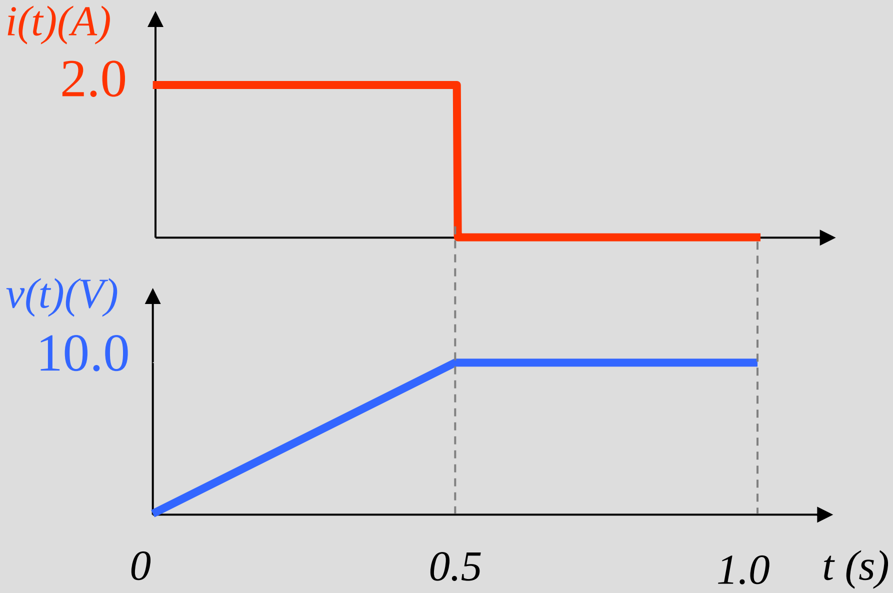
$$i = C\frac{dv}{dt} \qquad v = constant \Rightarrow \frac{dv}{dt} = 0 \quad i = C\frac{dv}{dt} = 0$$

Example-1.4: In the circuit below, a capacitor of 0.1F is excited by an ideal current source. The current curve as a function of time is given in the following figure. Plot the waveforms of the voltage v.





Solution



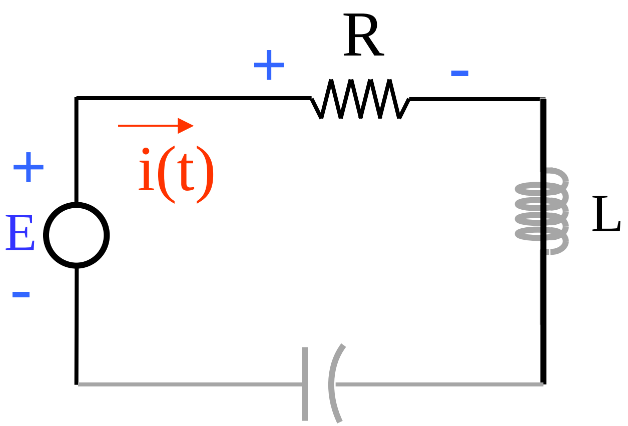
$$i(t) = \begin{cases} i(t) = 2 & 0 < t < 0.5 \\ i(t) = 0 & 0.5 < t < 1.0 \end{cases}$$

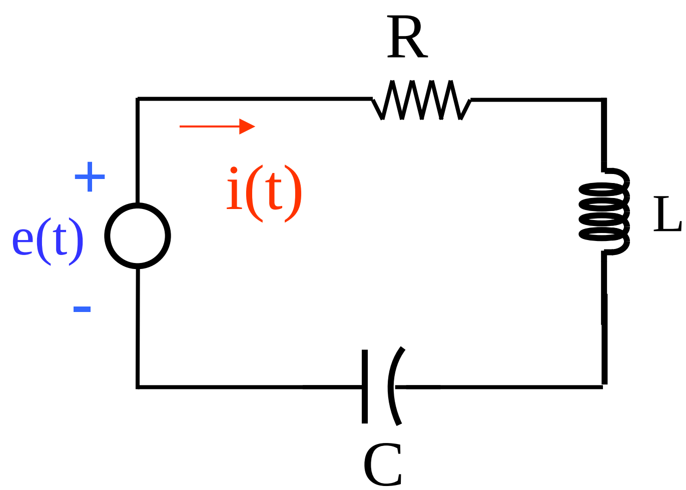
$$f(t) = \int i(t)dt = \begin{cases} f(t) = 2t + c & 0 < t < 0.5 \\ f(t) = c & 0.5 < t < 1.0 \end{cases}$$

$$\text{v}(t) = \frac{1}{C}\int i(t)dt = \frac{1}{C} f(t) = \begin{cases} \text{v}(t) = 20t & 0 < t < 0.5 \\ \text{v}(t) = 10 & 0.5 < t < 1.0 \end{cases}$$

L and C are ineffective in DC circuits

(by making **L** short and **C** open)





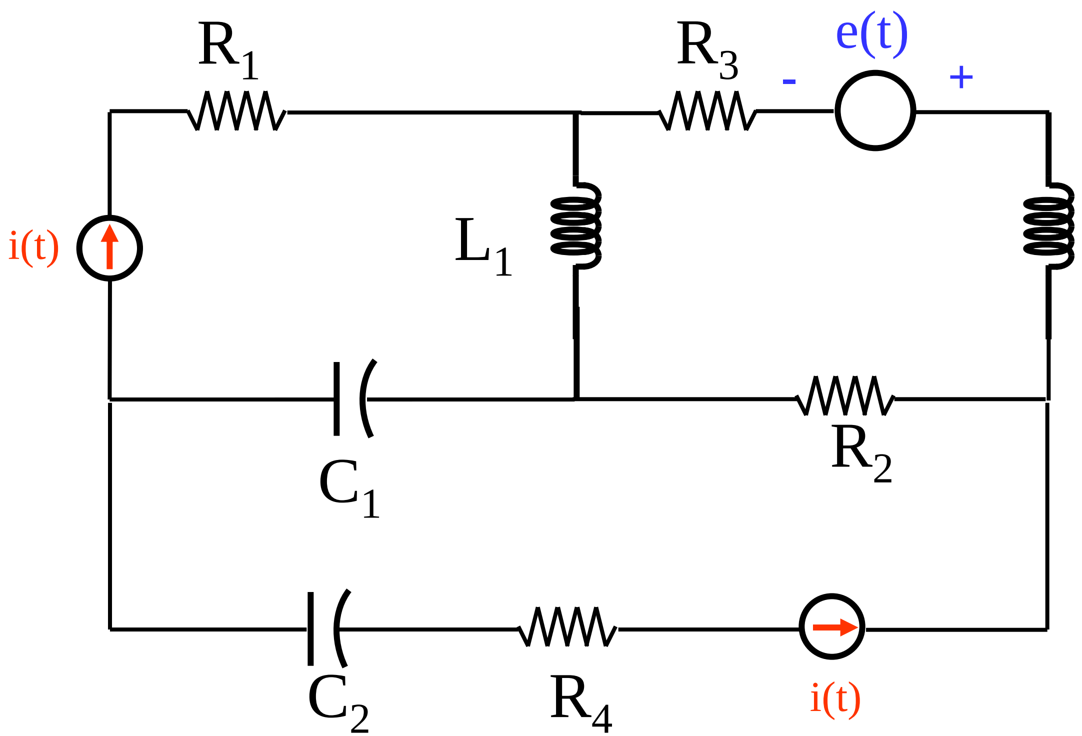
$$i(t) = \frac{1}{L}\int e(t).dt$$

$$i(t) = C\frac{de(t)}{dt}$$

Circuit Analysis

Consider the circuit below

- What is the voltage difference (or current) between R_2 ?
- How much source is needed to circulate the current (to do work)?



Fundamental Circuit Laws: Kirchhof's Law

The basic laws of electrical circuits are derived from the properties of electrical circuit elements. These basic laws allow for the systematic examination and analysis of complex electrical circuits. These laws are known as **Kirchhoff's Laws** and consist of **two basic laws**:

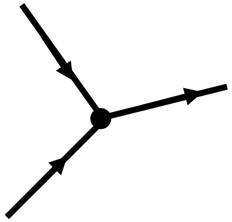
1- Kirchhoff's Current Law (KCL) (Conservation of charge)

2- Kirchhoff's Voltage Law (KVL) (Conservation of Energy)

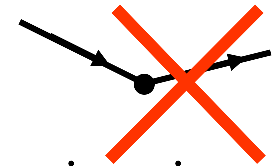
Kirchhoff's Current Law (KCL)

The algebraic sum of all currents directed towards a junction (node) is zero

A junction (or a node) is a point where three or more connections (branch) are met



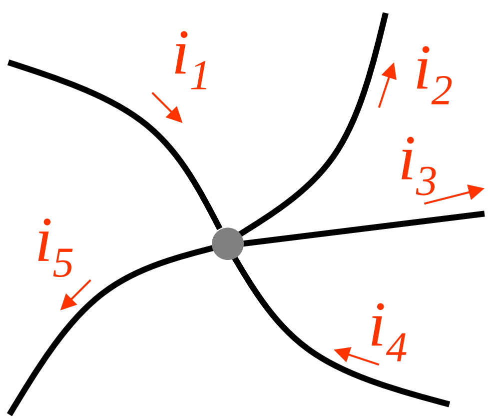
Junction (node)



Not a junction point!

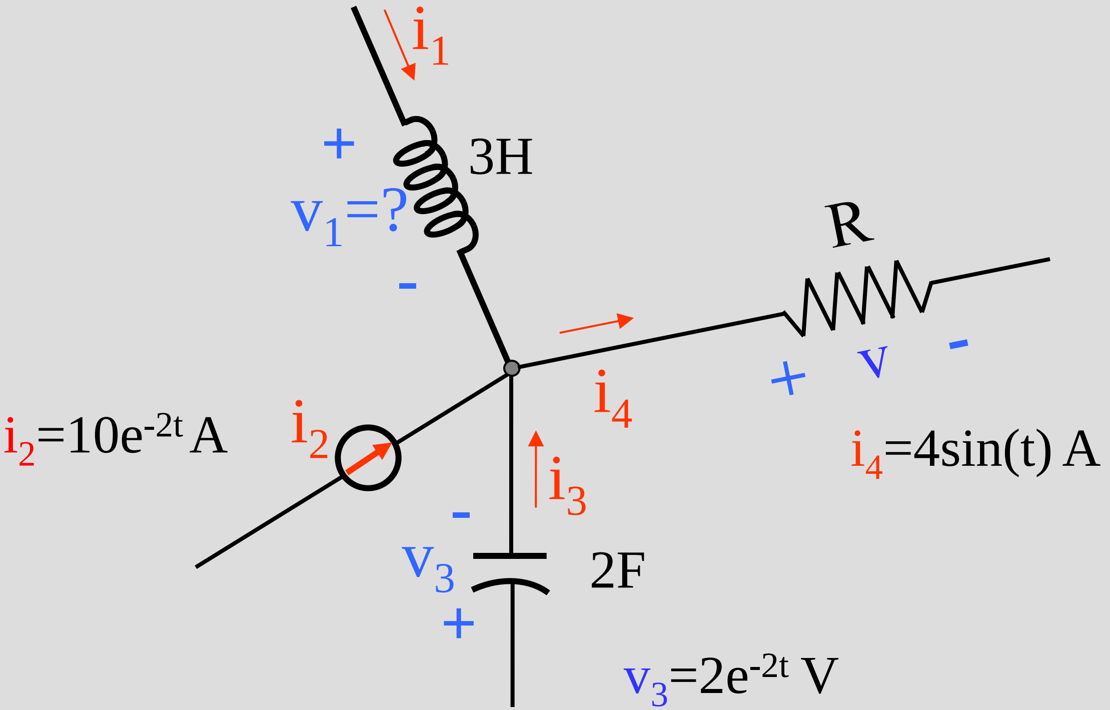
At a junction point:

- Entering currents are (+) positive
- Leaving currents are (-) negative



$$+i_1 - i_2 - i_3 + i_4 - i_5 = 0$$

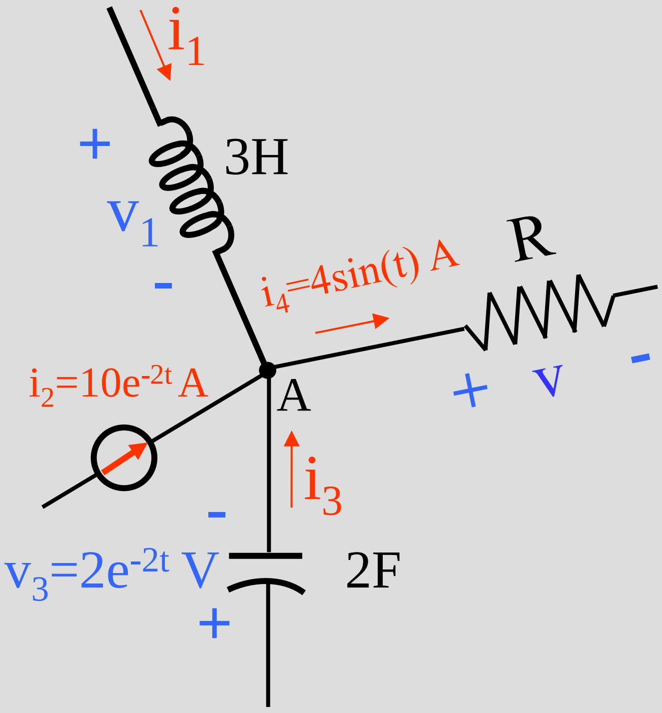
Example 1.5: In the following circuits currents and voltages are known: $i_2=10e^{-2t}$ A, $i_4=4\sin(t)$ A and $v_3=2e^{-2t}$ V. What is v_1 potential?



Solution:

Since the algebraic sum of the currents at the point A must be zero (from KCL)

$$+i_1 + i_2 + i_3 - i_4 = 0$$



i_2 and i_4 currents are known, i_3 can be find.

$$i_3 = C\frac{dv_3}{dt} = 2\frac{d}{dt}(2e^{-2t}) = 4(-2)e^{-2t} = -8e^{-2t}\ A$$

i_1 current:

$$i_1 = i_4 - i_2 - i_3$$

$$i_1 = 4\sin t - 10e^{-2t} + 8e^{-2t} = 4\sin t - 2e^{-2t}$$

v_1 potential:

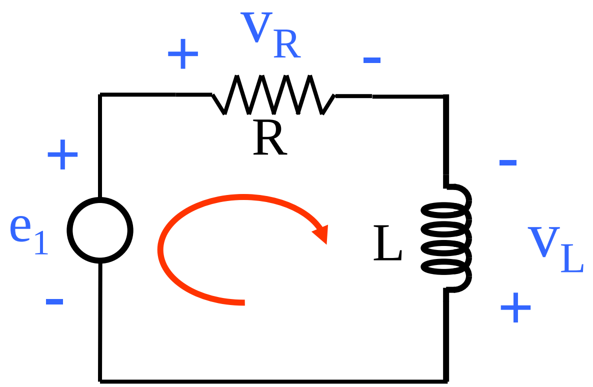
$$v_1 = L\frac{di_1}{dt} = 3\frac{d}{dt}(4\sin t - 2e^{-2t})$$

$$= 12\cos t + 12e^{-2t}\ V$$ is found.

Kirchhoff's Voltage Law (KVL)

The algebraic sum of the potentials around a closed loop (or closed path) is zero.

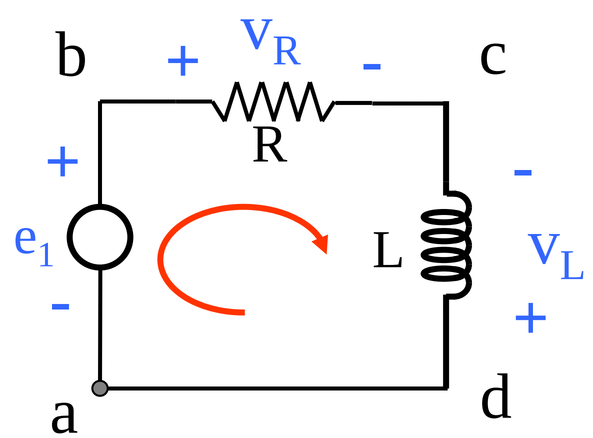
Closed loop (or path) refers to a way to leave a point and get back to it without leaving the network.



Closed loop (path)

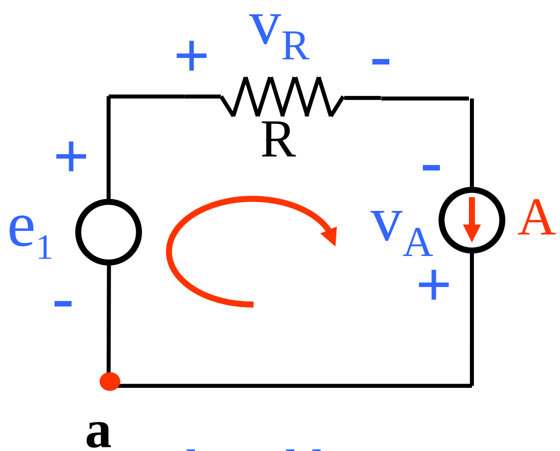
Along a closed loop (or path) in a determined direction (CW or CCW):

- Potential rises are (from - to +) positive;
- Potential drops are (from + to -) negative



$$+e_1 - v_R + v_L = 0$$

Kirchhoff's Voltage Law (KVL) -2



Closed loop

$$+e_1 - v_R + v_A = 0$$

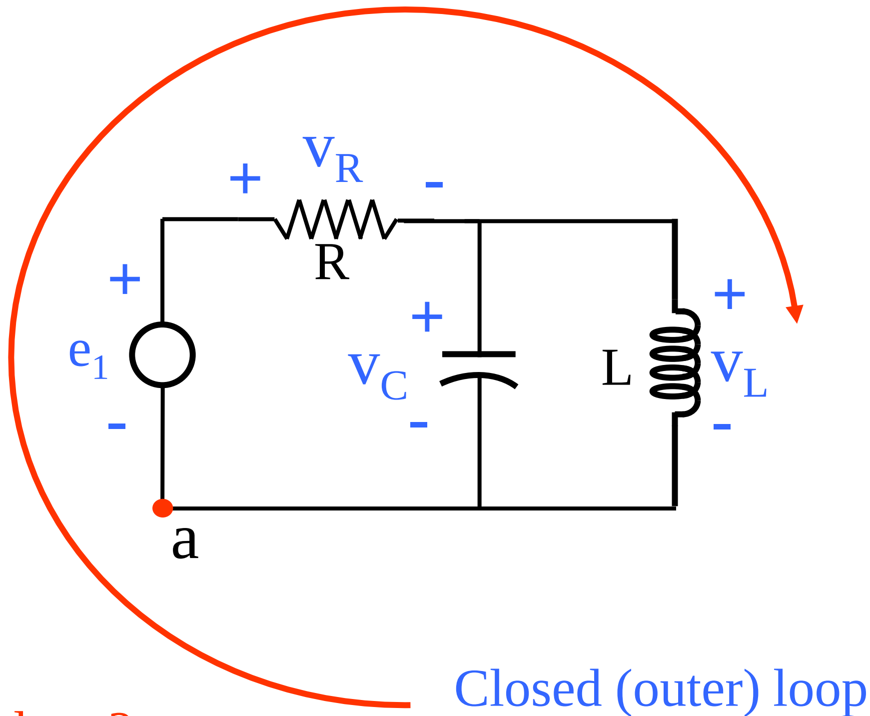
Closed loop (or path); refers to a way to leave a point (starting point -a) and get back to it without leaving the network.

Starting from point **a** and back to point **a**, the algebraic sum of the voltages is zero,

In the direction of rotation, the voltage increase is taken as (+) and the decrease is taken as (-)

Choosing a Loop-1

Similarly, the algebraic sum of the voltages is also zero if the point at which it is started is reached when traveling outside the loop.

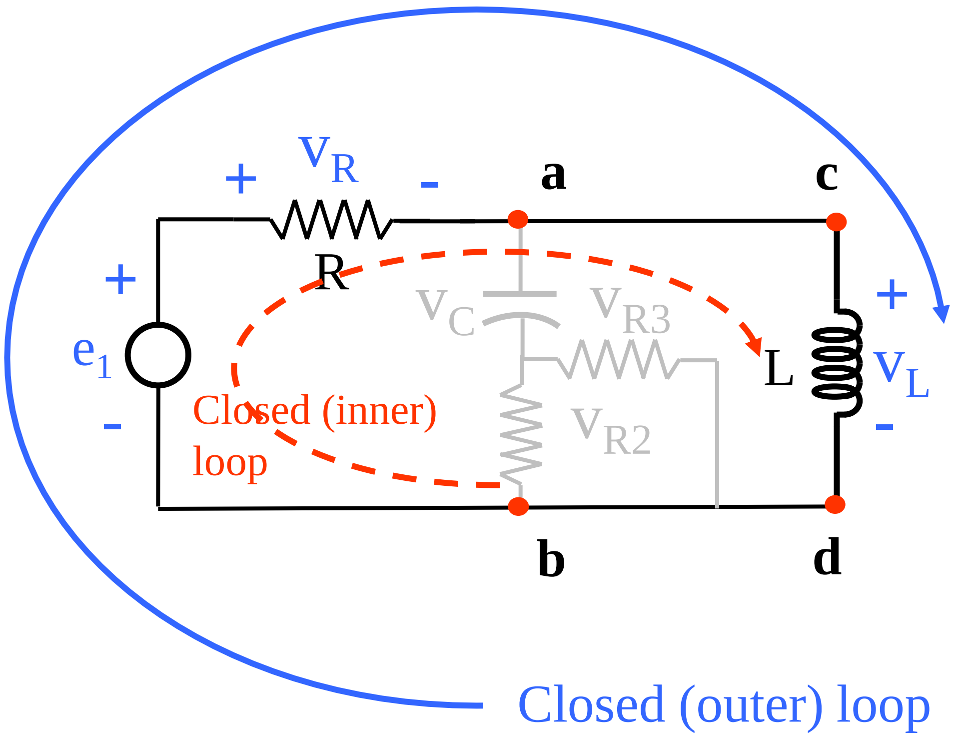


Why (when) the outer loop?

If sometimes it gives us additional information, we may want to write the voltages along the closed (outer) loop.

$$+e_1 - v_R - v_L = 0$$

Choosing a Loop-2



V_{ab}=V_{cd}

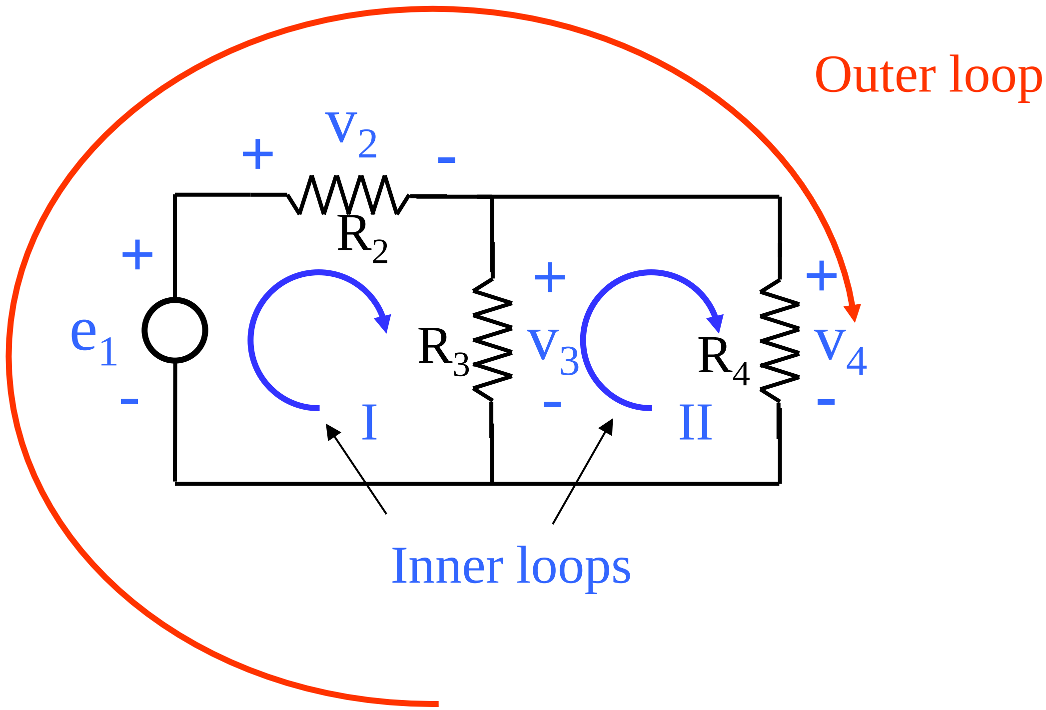
It doesn't matter how many loop there are.

Closed (inner) loop=Closed (outer) loop

$$+e_1 - v_R - v_L = 0$$

Choosing a Loop-3

KVL is also for closed outer loops: The algebraic sum of the voltages is zero even when traveling outside a loop.



Inner Loops

I. loop $+e_1 - v_2 - v_3 = 0$

II. loop $+v_3 - v_4 = 0$

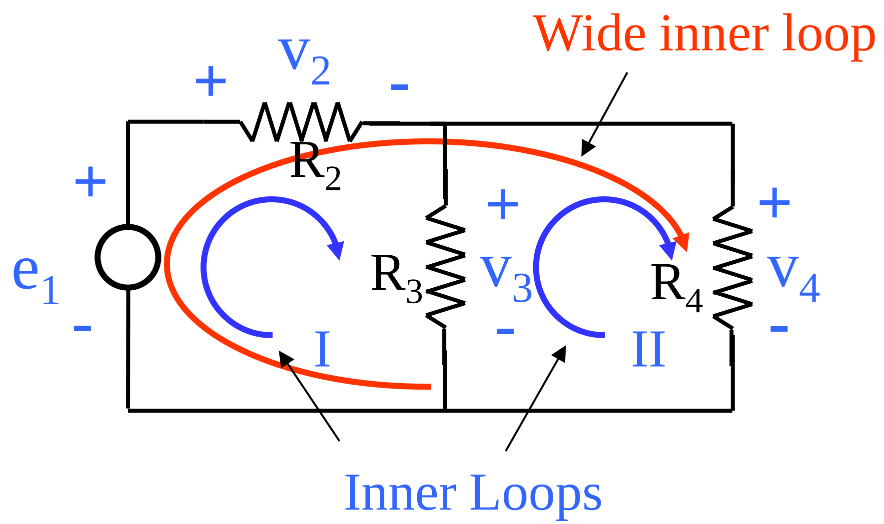
$$+e_1 - v_2 - (v_4) = 0$$

Outer loop

$$+e_1 - v_2 - v_4 = 0$$

Choosing a Loop-4

A loop of two inner loops can also be taken as a single loop and the KVL can be written.



Inner loops

I. loop $+e_1 - v_2 - v_3 = 0$

II. loop $+v_3 - v_4 = 0$

$$+e_1 - v_2 - (v_4) = 0$$

Wide inner loop

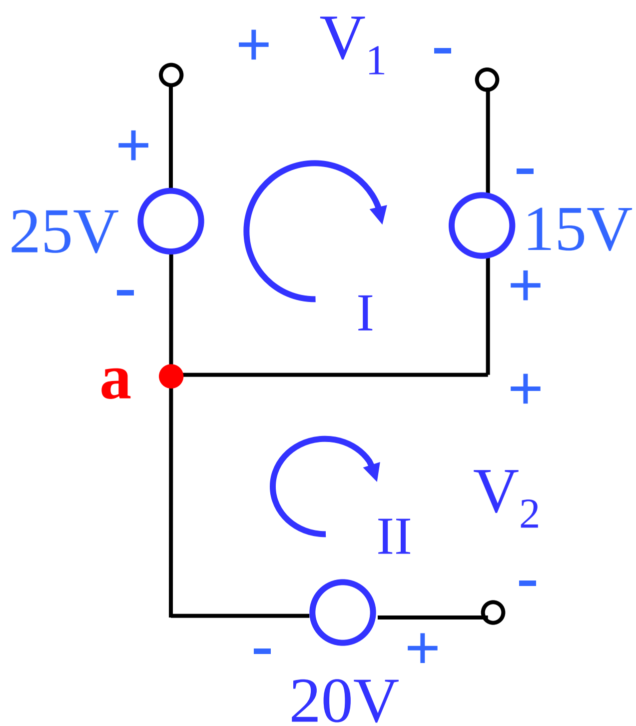
$$+e_1 - v_2 - v_4 = 0$$

$$+e_1 - v_2 - (v_4) = 0$$

Choosing a Loop-5

There is no requirement that the followed path have charge flow or current.

Starting from point a:



I. loop

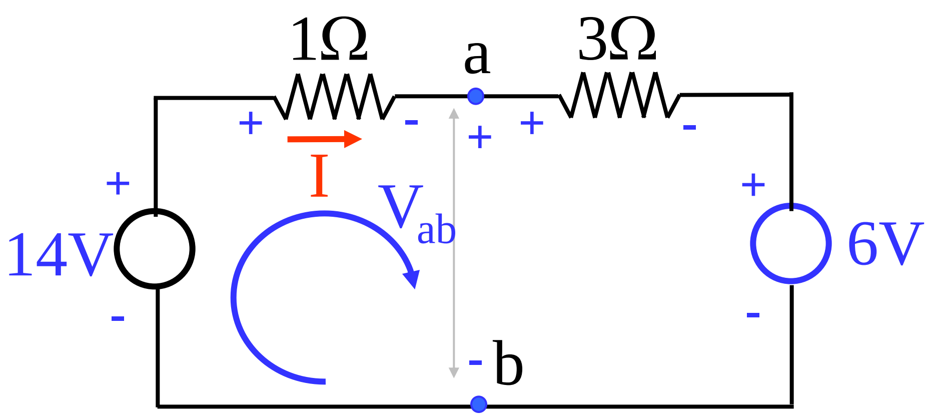
$$+25V - V_1 + 15V = 0$$

$$V_1 = 40V$$

II. loop

$$-V_2 - 20V = 0$$

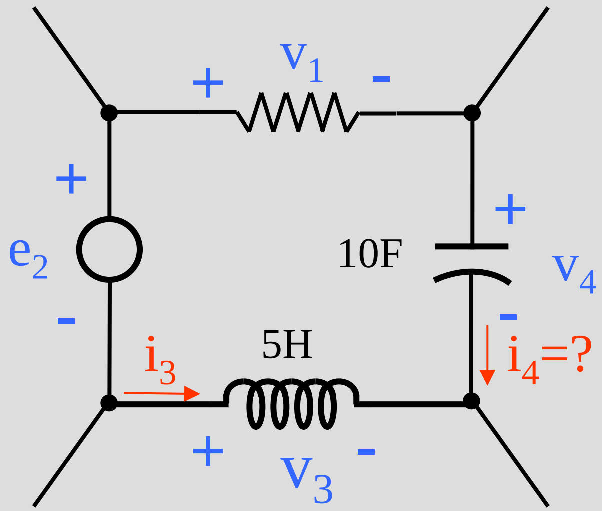
$$V_2 = -20V$$



$$+14V - I(1\Omega) - V_{ab} = 0$$

$$V_{ab} = 14V - (2A)(1\Omega) + = 12V$$

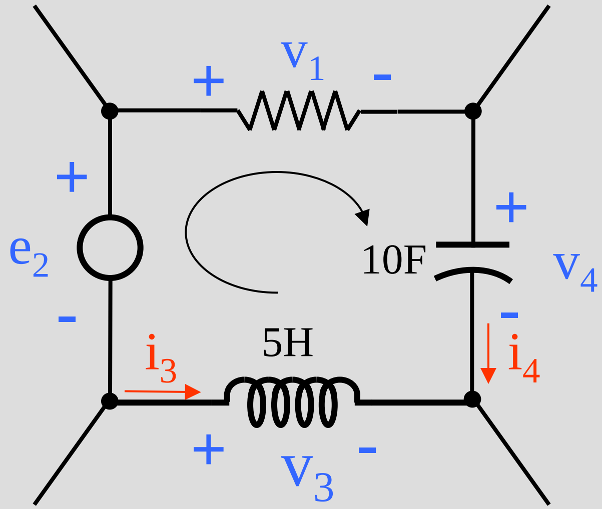
Example 1.6: The following circuit shows a section of an electrical circuit. At this section; If v_1=4 V, e_2=3cos(2t) V and $i_3=2e^{-t/5}$ A find i_4 current.



Solution:

The algebraic sum of the voltages throughout a closed loop is zero (KVL)

$$+v_3 + e_2 - v_1 - v_4 = 0$$



If we know v_1 and e_2 potentials, we can find v_3

$$v_3 = L\frac{di_3}{dt} = 5\frac{d}{dt}(2e^{-t/5}) = -2e^{-t/5}\,V$$

v_4 potential

$$v_4 = v_3 + e_2 - v_1$$

$$v_4 = -2e^{-t/5} + 3\cos(2t) - 4\ V$$

Current i_4

$$i_4 = C\frac{dv_4}{dt} = 10\frac{d}{dt}(-2e^{-t/5} + 3\cos(2t) - 4)$$

$$= 4e^{-t/5} - 60\sin(2t)\ A \quad \text{found}$$

What is Next?..

- Basic circuit elements (Resistor, Inductor, Capacitor) and their I-V (voltage-current relationship) characteristics were learned.

- The analysis of a circuit can be obtained from **Kirchhoff's current** and **voltage** laws.

- **Circuit Theory**, which is the application of these rules and thus facilitates the solution of specific problems, will be examined in detail in the next section.